Appendix

A.1. Matrix Inversion

This brief section is included only to help you understand Eq. (1.8.5) in the main text and is by no means comprehensive.

Consider the inversion of a 3 × 3 matrix

$$M=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \tag{A.1.1}$$

The elements of M have been named in this way rather than as M_{ij}, for in the following discussion we will treat the rows as components of the vectors **A**, **B**, and **C**, i.e., in the notation of vector analysis (which we will follow in this section),

$$\mathbf{A}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k} \text{ and so on}$$

Consider next a triplet of vectors

$$\begin{aligned} \mathbf{A}_R &= \mathbf{B}\times\mathbf{C} \\ \mathbf{B}_R &= \mathbf{C}\times\mathbf{A} \\ \mathbf{C}_R &= \mathbf{A}\times\mathbf{B} \end{aligned} \tag{A.1.2}$$

which are said to be *reciprocal* to **A**, **B**, and **C**. In general,

$$\mathbf{A}\cdot\mathbf{A}_R\neq 0, \qquad \mathbf{A}\cdot\mathbf{B}_R=\mathbf{A}\cdot\mathbf{C}_R=0 \qquad \text{and cyclic permutations} \tag{A.1.3}$$

If we construct now a matrix $\bar{\text{M}}$ (called the *cofactor* transpose of M) whose *columns* are the reciprocal vectors,

$$\bar{\text{M}}=\begin{bmatrix}(a_R)_1 & (b_R)_1 & (c_R)_1\\(a_R)_2 & (b_R)_2 & (c_R)_2\\(a_R)_3 & (b_R)_3 & (c_R)_3\end{bmatrix}$$

then

$$M\cdot\bar{\text{M}}=\begin{bmatrix}\mathbf{A}\cdot\mathbf{A}_R & \mathbf{A}\cdot\mathbf{B}_R & \mathbf{A}\cdot\mathbf{C}_R\\\mathbf{B}\cdot\mathbf{A}_R & \mathbf{B}\cdot\mathbf{B}_R & \mathbf{B}\cdot\mathbf{C}_R\\\mathbf{C}\cdot\mathbf{A}_R & \mathbf{C}\cdot\mathbf{B}_R & \mathbf{C}\cdot\mathbf{C}_R\end{bmatrix}=\begin{bmatrix}\mathbf{A}\cdot\mathbf{A}_R & 0 & 0\\0 & \mathbf{B}\cdot\mathbf{B}_R & 0\\0 & 0 & \mathbf{C}\cdot\mathbf{C}_R\end{bmatrix} \tag{A.1.4}$$

Now all three diagonal elements are equal:

$$\begin{aligned}\mathbf{A}\cdot\mathbf{A}_R&=\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})=\mathbf{B}\cdot\mathbf{B}_R=\mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})=\mathbf{C}\cdot\mathbf{C}_R\\&=\det M\end{aligned} \tag{A.1.5}$$

where the last equality follows from the fact that the cross product may be written as a determinant:

$$\mathbf{B}\times\mathbf{C}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\b_1 & b_2 & b_3\\c_1 & c_2 & c_3\end{vmatrix} \tag{A.1.6}$$

(We shall follow the convention of using two vertical lines to denote a determinant.)

Hence the inverse of the matrix M is given by

$$M^{-1}=\frac{\bar{\text{M}}}{\det M} \tag{A.1.7}$$

When does det M vanish? If one of the vectors, say **C**, is a linear combination of the other two; for if

$$\mathbf{C}=\alpha\mathbf{A}+\beta\mathbf{B}$$

then

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{A}\cdot(\mathbf{B}\times\alpha\mathbf{A})+\mathbf{A}\cdot(\mathbf{B}\times\beta\mathbf{B})=\mathbf{B}\cdot(\alpha\mathbf{A}\times\mathbf{A})=0$$

Thus the determinant vanishes if the rows of the matrix are not linearly independent (LI) and vice versa. If the matrix is used to represent three simultaneous equations, it means not all three equations are independent. The method can be generalized for inverting $n \times n$ matrices, with real or complex elements. One defines a cross product of $n-1$ vectors as

$$\mathbf{A}_1 \times \mathbf{A}_2 \times \cdots \mathbf{A}_{n-1} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} & \cdots & \\ (a_1)_1 & (a_1)_2 & & \cdots & \\ & & & \vdots & \\ (a_{n-1})_1 & (a_{n-1})_2 & & \cdots & (a_{n-1})_n \end{vmatrix} \tag{A.1.8}$$

The resulting vector is orthogonal to the ones in the product, changes sign when we interchange any two of the adjacent ones, and so on, just like its three-dimensional counterpart. If we have a matrix M, whose n rows may be identified with n vectors, $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n$, then the cofactor transpose has as its columns the reciprocal vectors $\mathbf{A}_{1R}, \ldots, \mathbf{A}_{nR}$, where

$$\mathbf{A}_{jR} = \mathbf{A}_{j+1} \times \mathbf{A}_{j+2} \times \cdots \mathbf{A}_n \times \mathbf{A}_1 \times \cdots \mathbf{A}_{j-1} \tag{A.1.9}$$

One tricky point: the cross product is defined to be orthogonal to the vectors in the product with respect to an inner product

$$\mathbf{A} \cdot \mathbf{B} = \sum A_i B_i$$

and *not*

$$\mathbf{A} \cdot \mathbf{B} = \sum A_i^* B_i$$

even when the components of $\mathbf{A}$ are complex. There is no contradiction here, for the vectors $\mathbf{A}_1, \ldots, \mathbf{A}_n$ are fictitious objects that enter a mnemonic and not the elements of the space $\mathbb{V}^n(C)$ on which the operator acts.

Exercise A.1.1. Using the method described above, show that

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -5 & 4 \\ -1 & 3 & -2 \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 2 \\ 0 & -1 & 2 \end{bmatrix}^{-1} = \frac{1}{12} \begin{bmatrix} -4 & 5 & 1 \\ 8 & -4 & -8 \\ 4 & -2 & 2 \end{bmatrix}$$

Theorem A.1.1. If $\Omega|V\rangle=|0\rangle$ implies $|V\rangle=|0\rangle$ then Ω^{-1} exists.

Proof. Let $|V_1\rangle, \ldots, |V_n\rangle$ be a LI basis in $\mathbb{V}^n$. Then another LI basis is generated by the action of Ω, i.e., $\Omega|V_1\rangle, \ldots, \Omega|V_n\rangle$ is also a LI basis. To see this, let us assume the contrary, that there exists a relation of the form

$$\sum_i \alpha_i \Omega|V_i\rangle = 0$$

with not all $\alpha_i=0$. Upon pulling out Ω, because it is linear, we get

$$\Omega\left(\sum_i \alpha_i|V_i\rangle\right)=0$$

which, when combined with the assumed property of Ω, implies that

$$\sum_i \alpha_i|V_i\rangle=|0\rangle$$

with not all $\alpha_i=0$, which is not true. So we can conclude that every vector $|V'\rangle$ in $\mathbb{V}^n$ may be written as a *unique* linear combination in the new basis generated by Ω as

$$|V'\rangle=\sum_i \alpha_i\Omega|V_i\rangle$$

In terms of $|V\rangle=\sum \alpha_i|V_i\rangle$, we see that *every* $|V'\rangle$ in $\mathbb{V}^n$ may be written as

$$|V'\rangle=\Omega|V\rangle$$

where $|V\rangle$ is *unique*. In other words, we can think of *every* $|V'\rangle$ in $\mathbb{V}^n$ as arising from a *unique* source $|V\rangle$ in $\mathbb{V}^n$ under the action of Ω. Define an operator Λ whose action on any vector $|V'\rangle$ in $\mathbb{V}^n$ is to take it back to its unique source $|V\rangle$. (If the source of $|V'\rangle$ were not unique—say, because there are two vectors $|V_1\rangle$ and $|V_2\rangle$ that are mapped into $|V'\rangle$ by Ω—then we could not define Λ, for acting on $|V'\rangle$, it would not know whether to give $|V_1\rangle$ or $|V_2\rangle$.) The action of Λ is then

$$\Lambda|V'\rangle=|V\rangle, \quad \text{where } |V'\rangle=\Omega|V\rangle$$

We may identify Λ as the inverse of Ω,

$$\Lambda=\Omega^{-1} \quad \text{or} \quad \Lambda\Omega=I$$

since for any $|V'\rangle$ in $\mathbb{V}^n$

$$\Lambda|V'\rangle=\Lambda\Omega|V\rangle=|V\rangle \quad \text{Q.E.D.}$$

A.2. Gaussian Integrals

We discuss here all the Gaussian integrals that we will need. Consider

$$I_0(\alpha)=\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx, \qquad \alpha>0 \tag{A.2.1}$$

This integral cannot be evaluated by conventional methods. The trick is to consider

$$I_0^2(\alpha)=\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx \int_{-\infty}^{\infty} e^{-\alpha y^2}\,dy=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\alpha(x^2+y^2)}\,dx\,dy$$

Switching to polar coordinates in the x–y plane,

$$\begin{aligned} I_0^2(\alpha)&=\int_0^{\infty}\int_0^{2\pi} e^{-\alpha\rho^2}\rho\,d\rho\,d\phi \\ &=\pi/\alpha \end{aligned}$$

Therefore

$$I_0(\alpha)=(\pi/\alpha)^{1/2} \tag{A.2.2}$$

By differentiating with respect to α we can get all the integrals of the form

$$I_{2n}(\alpha)=\int_{-\infty}^{\infty} x^{2n}\,e^{-\alpha x^2}\,dx$$

For example,

$$\begin{aligned} I_2(\alpha)&=\int_{-\infty}^{\infty} x^2\,e^{-\alpha x^2}\,dx=-\frac{\partial}{\partial\alpha}\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx \\ &=-\frac{\partial}{\partial\alpha}I_0(\alpha)=\frac{1}{2\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2} \end{aligned} \tag{A.2.3}$$

The integrals $I_{2n+1}(\alpha)$ vanish because these are integrals of odd functions over an even interval $-\infty$ to $+\infty$. Equations (A.2.2) and (A.2.3) are valid even if α is purely imaginary.

Consider next

$$I_0(\alpha,\beta)=\int_{-\infty}^{\infty} e^{-\alpha x^2+\beta x}\,dx \tag{A.2.4}$$

By completing the square on the exponent, we get

$$I_0(\alpha, \beta) = e^{\beta^2/4\alpha} \int_{-\infty}^{\infty} e^{-\alpha(x-\beta/2\alpha)^2}\, dx = e^{\beta^2/4\alpha} \left(\frac{\pi}{\alpha}\right)^{1/2} \tag{A.2.5}$$

These results are valid even if α and β are complex, provided Re $\alpha > 0$. Finally, by applying to both sides of the equation

$$\int_0^\infty e^{-\alpha r}\, dr = \frac{1}{\alpha}$$

the operator $(-d/d\alpha)^n$, we obtain

$$\int_0^\infty r^n e^{-\alpha r}\, dr = \frac{n!}{\alpha^{n+1}}$$

Consider this integral with $\alpha = 1$ and n replaced by $z - 1$, where z is an arbitrary complex number. This defines the *gamma function* $\Gamma(z)$

$$\Gamma(z) = \int_0^\infty r^{z-1} e^{-r}\, dr$$

For real, positive and integral z,

$$\Gamma(z) = (z-1)!$$

A.3. Complex Numbers

A complex variable z can be written in terms of two real variables x and y, and $i = (-1)^{1/2}$, as

$$z = x + iy \tag{A.3.1}$$

Its *complex conjugate* z^* is defined to be

$$z^* = x - iy \tag{A.3.2}$$

One may invert these two equations to express the *real and imaginary parts*, x and y, as

$$x = \tfrac{1}{2}(z + z^*), \qquad y = (z - z^*)/2i \tag{A.3.3}$$

The *modulus squared* of z, defined to be zz^*, equals

$$zz^* \equiv |z|^2 = (x+iy)(x-iy) = x^2+y^2 \tag{A.3.4}$$

You may verify that $z=z'$ implies that $x=x'$ and $y=y'$ by considering the modulus of $z-z'$.

From the power-series expansions

$$\sin x = x - x^3/3! + x^5/5! - \cdots$$
$$\cos x = 1 - x^2/2! + x^4/4! - \cdots$$

one can deduce that

$$e^{ix} = \cos x + i \sin x \tag{A.3.5}$$

It is clear that e^{ix} has unit modulus (x is real).

The expression $z=x+iy$ gives z in *Cartesian form*. The *polar form* is

$$\begin{aligned} z = x+iy &= (x^2+y^2)^{1/2}\left[\frac{x}{(x^2+y^2)^{1/2}} + i\frac{y}{(x^2+y^2)^{1/2}}\right] \\ &= \rho(\cos\theta + i\sin\theta) \\ &= \rho\, e^{i\theta} \end{aligned}$$

where

$$\rho = (x^2+y^2)^{1/2} \quad \text{and} \quad \theta = \tan^{-1}(y/x) \tag{A.3.6}$$

Clearly

$$|z| = \rho \tag{A.3.7}$$

Each complex number $z=x+iy$ may be visualized as a point (x, y) in the x–y plane. This plane is also called the *complex z plane*.

A.4. The $i\varepsilon$ Prescription

We will now derive and interpret the formula

$$\frac{1}{x \mp i\varepsilon} = \mathscr{P}\frac{1}{x} \pm i\pi\delta(x) \tag{A.4.1}$$

where $\varepsilon \to 0$ is a positive infinitesimally small quantity. Consider an integral of the form

$$I = \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \frac{f(x)\, dx}{x - i\varepsilon}. \tag{A.4.2}$$

Viewing this as the integral on the real axis of the complex $z = x + iy$ plane, we see that the integrand has an explicit pole at $z = i\varepsilon$ in addition to any singularities f might have. We assume f has no singularities on or infinitesimally close to the real axis. As long as ε is fixed, there is no problem with the integral. For example, if f has some poles in the upper half-plane and vanishes fast enough to permit our closing the contour in the upper half-plane, the integral equals $2\pi i$ times the sum of the residues of the poles of f and the pole at $z = i\varepsilon$. Likewise, if we change the sign of the ε term, we simply drop the contribution from the explicit pole, which is now in the lower half-plane.

What if $\varepsilon \to 0$? Now the pole is going to ram (from above) into our contour which runs along the x-axis. So we prepare for this as follows. Since the only singularity near the real axis is the explicit pole as $z = i\varepsilon$, we make the following deformation of the contour without changing the value of I: the contour runs along the real axis from $-\infty$ to $-\varepsilon'$, (ε' is another positive infinitesimal) goes around counterclockwise, below the origin in a semicircle of radius ε', and resumes along the real axis from $x = \varepsilon'$ to ∞. The nice thing is that we can now set $\varepsilon = 0$, which brings the pole to the origin. The three parts of the integration contour contribute as follows:

$$\begin{aligned} I &= \lim_{\varepsilon' \to 0} \left[\int_{-\infty}^{-\varepsilon'} \frac{f(x)\, dx}{x} + \int_{\varepsilon'}^{\infty} \frac{f(x)\, dx}{x} + i\pi f(0) \right] \\ &\equiv \mathscr{P} \int_{-\infty}^{\infty} \frac{f(x)\, dx}{x} + i\pi f(0). \end{aligned} \tag{A.4.3}$$

The sum of the two integrals in the limit $\varepsilon' \to 0$ is defined as the *principal value integral* denoted by the symbol $\mathscr{P}$. In the last term, which is restricted to the infinitesimal neighbourhood of the origin, we have set the argument of the smooth function f to zero and done the integral of dz/z counterclockwise around the *semicircle* to get $i\pi$.

Eq. (A.4.1) is a compact way to say all this. It is understood that Eq. (A.4.1) is to be used inside an integral only and that inside an integral the factor $1/(x - i\varepsilon)$ leads to two terms: the first, $\mathscr{P}(1/x)$, leads to the principal value integral, and the second, $i\pi\delta(x)$, leads to $i\pi f(0)$.

It is clear that if we reverse the sign of the ε term, we change the sign of the delta function since the semicircle now goes around the pole in the clockwise direction. The principal part is not sensitive to this change of direction and is unaffected.

It is clear that if we replace x by $x - a$ the pole moves from the origin to $x = a$ and $f(0)$ gets replaced by $f(a)$ so that we may write

$$\frac{1}{(x-a) \mp i\varepsilon} = \mathscr{P} \frac{1}{(x-a)} \pm i\pi\delta(x-a) \tag{A.4.4}$$

It is clear that the limits on x need not be $\pm\infty$ for the formula to work.

Finally, note that according to Eq. (A.4.4) the difference between the integrals with two signs of ε is just $2\pi i f(a)$. This too agrees with the present analysis in terms of the integral I in Eq. (A.4.2) since in the difference of the two integrals the contribution along the real axis cancels due to opposite directions of travel except for the part near the pole where the difference of the two semicircles (one going above and going below the pole) is a circle around the pole.

Answers to Selected Exercises

Chapter 1

1.8.1. (1) $|\omega=1\rangle \to \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad |\omega=2\rangle \to \frac{1}{(30)^{1/2}} \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}, \quad |\omega=4\rangle \to \frac{1}{(10)^{1/2}} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

(2) No, no.

1.8.2. (1) Yes

(2) $|\omega=0\rangle \to \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad |\omega=1\rangle \to \frac{1}{2^{1/2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad |\omega=-1\rangle \to \frac{1}{2^{1/2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

1.8.10. $\omega=0, 0, 2; \lambda=2, 3, -1.$

Chapter 4

4.2.1. (1) 1, 0, −1

(2) $\langle L_x \rangle = 0, \langle L_x^2 \rangle = 1/2, \Delta L_x = 1/2^{1/2}$

(3) $|L_x=1\rangle \to \begin{bmatrix} 1/2 \\ 1/2^{1/2} \\ 1/2 \end{bmatrix}, \quad |L_x=0\rangle \to \begin{bmatrix} -1/2^{1/2} \\ 0 \\ 1/2^{1/2} \end{bmatrix},$

$|L_x=-1\rangle \to \begin{bmatrix} 1/2 \\ -1/2^{1/2} \\ 1/2 \end{bmatrix}$

(4) $P(L_x=1)=1/4, \quad P(L_x=0)=1/2, \quad P(L_x=-1)=1/4$

(5) $|\psi\rangle \to \frac{1}{(1/4+1/2)^{1/2}}\begin{bmatrix}1/2\\0\\1/2^{1/2}\end{bmatrix}$ = projection of $|\psi\rangle$ on the $L_z^2=1$ eigenspace. $P(L_z^2=1)=3/4$. If L_z is measured $P(L_z=1)=1/3$, $P(L_z=-1)=2/3$. Yes, the state changes.

(6) No. To see this right away note that if $\delta_1=\delta_2=\delta_3=0$, $|\psi\rangle=1|L_x=1\rangle$ and if $\delta_1=\delta_3=0$ and $\delta_2=\pi$, $|\psi\rangle=|L_x=-1\rangle$. [See answer to part (3).] The vectors $|\psi\rangle$ and $e^{i\theta}|\psi\rangle$ are physically equivalent only in the sense that they generate the same probability distribution for any observable. This does not mean that when the vector $|\psi\rangle$ appears as a part of a linear combination it can be multiplied by an arbitrary phase factor. In our example one can only say, for instance, that

$$|\psi\rangle'=e^{-i\delta_1}|\psi\rangle$$
$$=\frac{1}{2}|L_z=1\rangle+\frac{e^{i(\delta_2-\delta_1)}}{2^{1/2}}|L_z=0\rangle+\frac{e^{i(\delta_3-\delta_1)}}{2}|L_z=-i\rangle$$

is physically equivalent to $|\psi\rangle$. Although $|\psi\rangle'$ has different coefficients from $|\psi\rangle$ in the linear expansion, it has the same "direction" as $|\psi\rangle$. In summary, then, the relative phases $\delta_2-\delta_1$ and $\delta_3-\delta_1$ are physically relevant but the overall phase is not, as you will have seen in the calculation of $P(L_x=0)$.

Chapter 5

5.4.2. (a) $R=(maV_0)^2/(\hbar^4k^2+m^2a^2V_0^2)$; $T=1-R$

(b) $T=(\cosh^2 2\kappa a+\alpha^2\sinh^2 2\kappa a)^{-1}$ where $i\kappa$ is the complex wave number for $|x|\le a$ and $\alpha=(V_0-2E)/[4E(V_0-E)]^{1/2}$.

Chapter 7

7.4.2. 0, 0, $(n+1/2)\hbar/m\omega$, $(n+1/2)m\omega\hbar$, $(n+1/2)\hbar$. Note that the recipe $m\omega\to(m\omega)^{-1}$ is at work here.

7.4.5. (1) $(1/2^{1/2})(|0\rangle e^{-i\omega t/2}+|1\rangle e^{-3i\omega t/2})$

(2) $\langle X(t)\rangle=(\hbar/2m\omega)^{1/2}\cos\omega t$, $\langle P(t)\rangle=-(m\omega\hbar/2)^{1/2}\sin\omega t$

(3) $\langle\dot{X}(t)\rangle=(i\hbar)^{-1}\langle[X,H]\rangle=\langle P(t)\rangle/m$, $\langle\dot{P}(t)\rangle=-m\omega^2\langle X(t)\rangle$. By eliminating $\langle P\rangle$ we can get an equation for $\langle X(t)\rangle$ and vice versa and solve it using the initial values $\langle X(0)\rangle$ and $\langle P(0)\rangle$, e.g., $\langle X(t)\rangle=\langle X(0)\rangle\cos\omega t+[\langle P(0)\rangle/m\omega]\sin\omega t$.

Chapter 10

10.3.2. $3^{-1/2}[|334\rangle + |343\rangle + |433\rangle]$

Chapter 12

12.6.1. $E = -\hbar^2/2\mu a_0^2, \qquad V = -\hbar^2/\mu a_0 r$

Chapter 13

13.3.1. Roughly 200 MeV.

13.3.2. Roughly 1 Å.

Chapter 14

14.3.5. $M = \left(\frac{\alpha+\delta}{2}\right)I + \left(\frac{\beta+\gamma}{2}\right)\sigma_x + i\left(\frac{\beta-\gamma}{s}\right)\sigma_y + \left(\frac{\alpha-\delta}{2}\right)\sigma_z$

14.3.7. (1) $2^{1/4}(\cos\pi/8 + i(\sin\pi/8)\sigma_x)$.

(2) $2/3I - 1/3\sigma_x$.

(3) σ_x

14.4.4. Roughly 2×10^{-9} second.

14.4.6. $(e\hbar/2mc)\tanh(e\hbar B/2mckT)\mathbf{k}$

14.5.2. (1) Roughly one part in a million.

(2) 10^{10} G.

14.5.3. 1/2, 1/4, 0.

14.5.4. $\left(\frac{1+\cos\theta}{2}\right)^2$

Chapter 15

15.2.2. (1) $\langle 1\,1, 1/2(-1/2)|3/2\;1/2\rangle = (1/3)^{1/2}$
$\langle 1\,0, 1/2\;1/2|3/2\;1/2\rangle = (2/3)^{1/2}$
$\langle 1\,1, 1/2(-1/2)|1/2\;1/2\rangle = (2/3)^{1/2}$
$\langle 1\,0, 1/2\;1/2|1/2\;1/2\rangle = -(1/3)^{1/2}$

(2) $|jm\rangle = |2, 1\rangle = 2^{-1/2}|m_1 = 1, m_2 = 0\rangle + 2^{-1/2}|m_1 = 0, m_2 = 1\rangle$
$|2, 0\rangle = 6^{-1/2}|1, -1\rangle + (\frac{2}{3})^{1/2}|0, 0\rangle + (\frac{1}{6})^{1/2}|-1, 1\rangle$
$|1, 1\rangle = 2^{-1/2}|1, 0\rangle - 2^{-1/2}|0, 1\rangle$
$|1, 0\rangle = 2^{-1/2}|1, -1\rangle - 2^{-1/2}|-1, 1\rangle$
$|0, 0\rangle = 3^{-1/2}|1, -1\rangle - 3^{-1/2}|0, 0\rangle + 3^{-1/2}|-1, 1\rangle$

The others are either zero, obvious, or follow from Eq. (15.2.11).

15.2.6. $\mathbb{P}_+ = \dfrac{(2\mathbf{L}\cdot\mathbf{S})/\hbar^2 + l + 1}{2l+1}, \qquad \mathbb{P}_- = \dfrac{l - (2\mathbf{L}\cdot\mathbf{S})/\hbar^2}{2l+1}$

Chapter 16

16.1.2. $E(a_0) = 10E_0/\pi^2$

16.1.3. $-ma_0^2 V_0^2/\pi\hbar^2$

16.1.4. $E(a_0) = \frac{1}{2}\hbar\omega(\frac{12}{11})^{1/2}$

16.2.4. Roughly 1.5×10^{17} seconds or 10^{10} years.

Table of Constants

$$\hbar c = 1973.3 \text{ eV Å}$$

$$\alpha = e^2/\hbar c = 1/137.04$$

$$mc^2 = 0.511 \text{ MeV} \quad (m \text{ is the electron mass})$$

$$Mc^2 = 938.28 \text{ MeV} \quad (M \text{ is the proton mass})$$

$$a_0 = \hbar^2/me^2 = 0.511 \text{ Å}$$

$$e\hbar/2mc = 0.58 \times 10^{-8} \text{ eV/G} \quad \text{(Bohr magneton)}$$

$$k = 8.62 \times 10^{-5} \text{ eV/K}$$

$$kT \simeq 1/40 \text{ eV at } T = 300 \text{ K} \quad \text{(room temperature)}$$

$$1 \text{ eV} = 1.6 \times 10^{-12} \text{ erg}$$

Mnemonics for Hydrogen

In the ground state,

$$v/c \equiv \beta = \alpha$$

$$E_1 = -T = -\tfrac{1}{2}mv^2 = -\tfrac{1}{2}mc^2\alpha^2$$

$$mva_0 = \hbar$$

In higher states, $E_n = E_1/n^2$.

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