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Quantum Mechanics

With 101 Illustrations

Springer

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Preface

This book is an outgrowth of lectures given at the University of Michigan at various times from 1966–1996 in a first-year graduate course on quantum mechanics. It is meant to be at a fairly high level. On the one hand, it should provide future research workers with the tools required to solve real problems in the field. On the other hand, the beginning graduate courses at the University of Michigan should be self-contained. Although most of the students will have had an undergraduate course in quantum mechanics, the lectures are intended to be such that a student with no previous background in quantum mechanics (perhaps an undergraduate mathematics or engineering major) can follow the course from beginning to end.

Part I of the course, Introduction to Quantum Mechanics, thus begins with a brief background chapter on the duality of nature, which hopefully will stimulate students to take a closer look at the two references given there. These references are recommended for every serious student of quantum mechanics. Chapter 1 is followed by a review of Fourier analysis before we meet the Schrödinger equation and its interpretation. The dual purpose of the course can be seen in Chapters 4 and 5, where an introduction to simple square well problems and a first solution of the one-dimensional harmonic oscillator by Fuchsian differential equation techniques are followed by an introduction to the Bargmann transform, which gives us an elegant tool to show the completeness of the harmonic oscillator eigenfunctions and enables us to solve some challenging harmonic oscillator problems, (e.g., the case of general n for problem 11). Early chapters (7 through 12) on the eigenvalue problem are based on the coordinate representation and include detailed solutions of the spherical harmonics and radial functions of the hydrogen atom, as well as many of the soluble, one-dimensional potential problems. These chapters are based on the factorization method. It is hoped the ladder step-up and step-down operator approach of this method will help to lead the student naturally from the Schrödinger equation approach to the more modern algebraic techniques of quantum mechanics, which are introduced in Chapters 13 to 19. The full Dirac bra, ket notation is introduced in Chapter 13. These chapters also give the full algebraic approach to the general angular momentum problem, SO(3) or SU(2), the harmonic oscillator algebra, and the SO(2,1) algebra. The solution for the latter is given in problem 23, which is used in considerable detail in later chapters. The problems often amplify the material of the course.

Part II of the course, Chapters 20 to 26, on time-independent perturbation theory, is based on Fermi's view that most of the important problems of quantum mechanics can be solved by perturbative techniques. This part of the course shows how various types of degeneracies can be handled in perturbation theory, particularly the case in which a degeneracy is not removed in lowest order of perturbation theory so that the lowest order perturbations do not lead naturally to the symmetry-adapted basis; a case ignored in many books on quantum mechanics and perhaps particularly important in the case of accidental near-degeneracies. Chapters 25 and 26 deal with magnetic-field perturbations, including a short section on the Aharanov–Bohm effect, and a treatment on fine structure and Zeeman perturbations in one-electron atoms.

Part III of the course, Chapters 27 to 35, then gives a detailed treatment of angular momentum and angular momentum coupling theory, including a derivation of the matrix elements of the general rotation operator, Chapter 29; spherical tensor operators, Chapter 31; the Wigner–Eckart theorem, Chapter 32; angular momentum recoupling coefficients and their use in matrix elements of coupled tensor operators in an angular-momentum-coupled basis, Chapter 34; as well as the use of an SO(2,1) algebra and the stretched Coulombic basis and its power in hydrogenic perturbation theory without the use of the infinite sum and continuum integral contributions of the conventional hydrogenic basis, Chapter 35.

Since the full set of chapters is perhaps too much for a one-year course, some chapters or sections, and, in particular, some mathematical appendices, are marked in the table of contents with an asterisk (*). This symbol designates that the chapter can be skipped in a first reading without loss of continuity for the reader. Chapters 34 and 35 are such chapters with asterisks. Because of their importance, however, an alternative is to skip Chapters 36 and 37 on the WKB approximation. These chapters are therefore placed at this point in the book, although they might well have been placed in Part II on perturbation theory.

Part IV of the lectures, Chapters 38 to 40, gives a first introduction to systems of identical particles, with the emphasis on the two-electron atom and a chapter on variational techniques.

Parts I through IV of the course deal with bound-state problems. Part V on scattering theory, which might constitute the beginning of a second semester, begins the treatment of continuum problems with Chapters 41 through 56 on scattering theory, including a treatment of inelastic scattering processes and rearrangement collisions, and the spin dependence of scattering cross sections. The polarization of particle beams and the scattering of particles with spin are used to introduce density matrices and statistical distributions of states.

Part VI of the course gives a conventional introduction to time-dependent perturbation theory, including a chapter on magnetic resonance and an application of the sudden and adiabatic approximations in the reversal of magnetic fields.

Part VII on atom–photon interactions includes an expansion of the quantized radiation field in terms of the full set of vector spherical harmonics, leading to a detailed derivation of the general electric and magnetic multipole-transition matrix elements needed in applications to nuclear transitions, in particular.

Parts V through IX may again be too much material for the second semester of a one-year course. At the University of Michigan, curriculum committees have at various times insisted that the first-year graduate course include *either* an introduction to Dirac theory of relativistic spin $\frac{1}{2}$-particles *or* an introduction to many-body theory. Part VIII of the course on relativistic quantum mechanics, Chapters 69 through 77, and Part IX, an introduction to many-body theory, Chapters 78 and 79, are therefore written so that a lecturer could choose *either* Part VIII *or* Part IX to complete the course.

The problems are meant to be an integral part of the course. They are often meant to build on the material of the lectures and to be real problems (rather than small exercises, perhaps to derive specific equations). They are, therefore, meant to take considerable time and often to be somewhat of a challenge. In the actual course, they are meant to be discussed in detail in problem sessions. For this reason, detailed solutions for a few key problems, particularly in the first part of the course, are given in the text as part of the course (e.g., the results of problem 23 are very much used in later chapters, and problem 34, actually a very simple problem in perturbation theory is used to illustrate how various types of degeneracies can be handled properly in perturbation theory in a case in which the underlying symmetry leading to the degeneracy *might not* be easy to recognize). In the case of problem 34, the underlying symmetry *is* easy to recognize. The solution therefore also shows how this symmetry should be exploited.

The problems are not assigned to specific chapters, but numbered 1 through 55 for Parts I through IV of the course, and, again, 1 through 51 for Parts V through IX, the second semester of the course. They are placed at the point in the course where the student should be ready for a particular set of problems.

The applications and assigned problems of these lectures are taken largely from the fields of atomic and molecular physics and from nuclear physics, with a few examples from other fields. This selection, of course, shows my own research interests, but I believe, is also because these fields are fertile for the applications of nonrelativistic quantum mechanics.

I first of all want to acknowledge my own teachers. I consider myself extremely fortunate to have learned the subject from David M. Dennison and George E. Uhlenbeck. Among the older textbooks used in the development of these lectures, I acknowledge the books by Leonard I. Schiff, *Quantum Mechanics*, McGraw-Hill, 1949; Albert Messiah, *Quantum Mechanics, Vol. I and II*, John Wiley and Sons, 1965; Eugene Merzbacher, *Quantum Mechanics*, John Wiley and Sons, 1961; Kurt

Gottfried, *Quantum Mechanics*, W. A. Benjamin, Inc., 1966; and L. D. Landau and E. M. Lifshitz, *Quantum Mechanics. Nonrelativistic Theory. Vol.* 3. *Course of Theoretical Physics*, Pergamon Press 1958. Hopefully, the good features of these books have found their way into my lectures.

References to specific books, chapters of books, or research articles are given throughout the lectures wherever they seemed to be particularly useful or relevant. Certainly, no attempt is made to give a complete referencing. Each lecturer in a course on quantum mechanics must give the student his own list of the many textbooks a student should consult. The serious student of the subject, however, must become familiar with the two classics: P. A. M. Dirac, *The Principles of Quantum Mechanics*, Oxford University Press, first ed. 1930; and Wolfgang Pauli, *General Principles of Quantum Mechanics*, Springer-Verlag, 1980, (an English translation of the 1933 Handbuch der Physik article in Vol. 24 of the Handbuch).

Finally, I want to thank the many students at the University of Michigan who have contributed to these lectures with their questions. In fact, it was the encouragement of former students of this course that has led to the idea these lectures should be converted into book form. I also thank Prof. Yasuyuki Suzuki for his many suggestions after a careful reading of an early version of the manuscript. Particular thanks are due to Dr. Sudha Swaminathan and Dr. Frank Lamelas for their great efforts in making all of the figures.

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