Part Three

LINEAR ALGEBRA and REPRESENTATIONS

We shall be concerned with modules and vector spaces, going into their structure under various points of view. The main theme here is to study a pair, consisting of a module, and an endomorphism, or a ring of endomorphisms, and try to decompose this pair into a direct sum of components whose structure can then be described explicitly. The direct sum theme recurs in every chapter. Sometimes, we use a duality to obtain our direct sum decomposition relative to a pairing, and sometimes we get our decomposition directly. If a module refuses to decompose into a direct sum of simple components, then there is no choice but to apply the Grothendieck construction and see what can be obtained from it.

The extension theme occurs only once, in Witt's theorem, in a brief counterpoint to the decomposition theme.