INTUITIVE PROBABILITY AND RANDOM PROCESSES USING MATLAB®

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To my wife
Cindy,
whose love and support
are without measure

and to my daughters
Lisa and Ashley,
who are a source of joy

NOTE TO INSTRUCTORS

As an aid to instructors interested in using this book for a course, the solutions to the exercises are available in electronic form. They may be obtained by contacting the author at kay@ele.uri.edu.

Preface

The subject of probability and random processes is an important one for a variety of disciplines. Yet, in the author's experience, a first exposure to this subject can cause difficulty in assimilating the material and even more so in applying it to practical problems of interest. The goal of this textbook is to lessen this difficulty. To do so we have chosen to present the material with an emphasis on conceptualization. As defined by Webster, a *concept* is "an abstract or generic idea generalized from particular instances." This embodies the notion that the "idea" is something we have formulated based on our past experience. This is in contrast to a *theorem*, which according to Webster is "an idea accepted or proposed as a demonstrable truth". A theorem then is the result of many *other* persons' past experiences, which may or may not coincide with our own. In presenting the material we prefer to first present "particular instances" or examples and then generalize using a definition/theorem. Many textbooks use the opposite sequence, which undeniably is cleaner and more compact, but omits the motivating examples that initially led to the definition/theorem. Furthermore, in using the definition/theorem-first approach, for the sake of mathematical correctness multiple concepts must be presented at once. This is in opposition to human learning for which "under most conditions, the greater the number of attributes to be bounded into a single concept, the more difficult the learning becomes"[1]. The philosophical approach of specific examples followed by generalizations is embodied in this textbook. It is hoped that it will provide an alternative to the more traditional approach for exploring the subject of probability and random processes.

To provide motivating examples we have chosen to use MATLAB[2], which is a very versatile scientific programming language. Our own engineering students at the University of Rhode Island are exposed to MATLAB as freshmen and continue to use it throughout their curriculum. Graduate students who have not been previously introduced to MATLAB easily master its use. The pedagogical utility of using MATLAB is that:

1. Specific computer generated examples can be constructed to provide motivation for the more general concepts to follow.

[1]Eli Saltz, *The Cognitive Basis of Human Learning*, Dorsey Press, Homewood, IL, 1971.
[2]Registered trademark of TheMathWorks, Inc.

2. Inclusion of computer code within the text allows the reader to interpret the mathematical equations more easily by seeing them in an alternative form.

3. Homework problems based on computer simulations can be assigned to illustrate and reinforce important concepts.

4. Computer experimentation by the reader is easily accomplished.

5. Typical results of probabilistic-based algorithms can be illustrated.

6. Real-world problems can be described and "solved" by implementing the solution in code.

Many MATLAB programs and code segments have been included in the book. In fact, most of the figures were generated using MATLAB. The programs and code segments listed within the book are available in the file `probbook_matlab_code.tex`, which can be found at http://www.ele.uri.edu/faculty/kay/New%20web/Books.htm. The use of MATLAB, along with a brief description of its syntax, is introduced early in the book in Chapter 2. It is then immediately applied to simulate outcomes of random variables and to estimate various quantities such as means, variances, probability mass functions, etc. *even though these concepts have not as yet been formally introduced.* This chapter sequencing is purposeful and is meant to expose the reader to some of the main concepts that will follow in more detail later. In addition, the reader will then immediately be able to simulate random phenomena to learn through doing, in accordance with our philosophy. In summary, we believe that the incorporation of MATLAB into the study of probability and random processes provides a "hands-on" approach to the subject and promotes better understanding.

Other pedagogical features of this textbook are the discussion of discrete random variables first to allow easier assimilation of the concepts followed by a parallel discussion for continuous random variables. Although this entails some redundancy, we have found less confusion on the part of the student using this approach. In a similar vein, we first discuss scalar random variables, then bivariate (or two-dimensional) random variables, and finally N-dimensional random variables, reserving separate chapters for each. All chapters, except for the introductory chapter, begin with a summary of the important concepts and point to the main formulas of the chapter, and end with a real-world example. The latter illustrates the utility of the material just studied and provides a powerful motivation for further study. It also will, hopefully, answer the ubiquitous question "Why do we have to study this?". We have tried to include real-world examples from many disciplines to indicate the wide applicability of the material studied. There are numerous problems in each chapter to enhance understanding with some answers listed in Appendix E. The problems consist of four types. There are "formula" problems, which are simple applications of the important formulas of the chapter; "word" problems, which require a problem-solving capability; and "theoretical" problems, which are more abstract and mathematically demanding; and finally, there are "computer" problems, which are either computer simulations or involve the application of computers to facilitate analytical solutions. A complete solutions manual for all the problems is available to instructors from the author upon request. Finally, we have provided warnings on how to avoid common errors as well as in-line explanations of equations within the derivations for clarification.

The book was written mainly to be used as a first-year graduate level course in probability and random processes. As such, we assume that the student has had some exposure to basic probability and therefore Chapters 3–11 can serve as a review and a summary of the notation. We then will cover Chapters 12–15 on probability and selected chapters from Chapters 16–22 on random processes. This book can also be used as a self-contained introduction to probability at the senior undergraduate or graduate level. It is then suggested that Chapters 1–7, 10, 11 be covered. Finally, this book is suitable for self-study and so should be useful to the practitioner as well as the student. The necessary background that has been assumed is a knowledge of calculus (a review is included in Appendix B); some linear/matrix algebra (a review is provided in Appendix C); and linear systems, which is necessary only for Chapters 18–20 (although Appendix D has been provided to summarize and illustrate the important concepts).

The author would like to acknowledge the contributions of the many people who over the years have provided stimulating discussions of teaching and research problems and opportunities to apply the results of that research. Thanks are due to my colleagues L. Jackson, R. Kumaresan, L. Pakula, and P. Swaszek of the University of Rhode Island. A debt of gratitude is owed to all my current and former graduate students. They have contributed to the final manuscript through many hours of pedagogical and research discussions as well as by their specific comments and questions. In particular, Lin Huang and Cuichun Xu proofread the entire manuscript and helped with the problem solutions, while Russ Costa provided feedback. Lin Huang also aided with the intricacies of LaTex while Lisa Kay and Jason Berry helped with the artwork and to demystify the workings of Adobe Illustrator 10.[3] The author is indebted to the many agencies and program managers who have sponsored his research, including the Naval Undersea Warfare Center, the Naval Air Warfare Center, the Air Force Office of Scientific Research, and the Office of Naval Research. As always, the author welcomes comments and corrections, which can be sent to kay@ele.uri.edu.

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[3]Registered trademark of Adobe Systems Inc.

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