Computational Geometry

Third Edition

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Algorithms and Applications

Third Edition

Springer

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Preface

Computational geometry emerged from the field of algorithms design and analysis in the late 1970s. It has grown into a recognized discipline with its own journals, conferences, and a large community of active researchers. The success of the field as a research discipline can on the one hand be explained from the beauty of the problems studied and the solutions obtained, and, on the other hand, by the many application domains—computer graphics, geographic information systems (GIS), robotics, and others—in which geometric algorithms play a fundamental role.

For many geometric problems the early algorithmic solutions were either slow or difficult to understand and implement. In recent years a number of new algorithmic techniques have been developed that improved and simplified many of the previous approaches. In this textbook we have tried to make these modern algorithmic solutions accessible to a large audience. The book has been written as a textbook for a course in computational geometry, but it can also be used for self-study.

Structure of the book. Each of the sixteen chapters (except the introductory chapter) starts with a problem arising in one of the application domains. This problem is then transformed into a purely geometric one, which is solved using techniques from computational geometry. The geometric problem and the concepts and techniques needed to solve it are the real topic of each chapter. The choice of the applications was guided by the topics in computational geometry we wanted to cover; they are not meant to provide a good coverage of the application domains. The purpose of the applications is to motivate the reader; the goal of the chapters is not to provide ready-to-use solutions for them. Having said this, we believe that knowledge of computational geometry is important to solve geometric problems in application areas efficiently. We hope that our book will not only raise the interest of people from the algorithms community, but also from people in the application areas.

For most geometric problems treated we give just one solution, even when a number of different solutions exist. In general we have chosen the solution that is easiest to understand and implement. This is not necessarily the most efficient solution. We also took care that the book contains a good mixture of techniques like divide-and-conquer, plane sweep, and randomized algorithms. We decided not to treat all sorts of variations to the problems; we felt it is more important to introduce all main topics in computational geometry than to give more detailed information about a smaller number of topics.

Several chapters contain one or more sections marked with a star. They contain improvements of the solution, extensions, or explain the relation between various problems. They are not essential for understanding the remainder of the book.

Every chapter concludes with a section that is entitled *Notes and Comments*. These sections indicate where the results described in the chapter originated, mention other solutions, generalizations, and improvements, and provide references. They can be skipped, but do contain useful material for those who want to know more about the topic of the chapter.

At the end of each chapter a number of exercises is provided. These range from easy tests to check whether the reader understands the material to more elaborate questions that extend the material covered. Difficult exercises and exercises about starred sections are indicated with a star.

A course outline. Even though the chapters in this book are largely independent, they should preferably not be treated in an arbitrary order. For instance, Chapter 2 introduces plane sweep algorithms, and it is best to read this chapter before any of the other chapters that use this technique. Similarly, Chapter 4 should be read before any other chapter that uses randomized algorithms.

For a first course on computational geometry, we advise treating Chapters 1–10 in the given order. They cover the concepts and techniques that, according to us, should be present in any course on computational geometry. When more material can be covered, a selection can be made from the remaining chapters.

Prerequisites. The book can be used as a textbook for a high-level undergraduate course or a low-level graduate course, depending on the rest of the curriculum. Readers are assumed to have a basic knowledge of the design and analysis of algorithms and data structures: they should be familiar with big-Oh notations and simple algorithmic techniques like sorting, binary search, and balanced search trees. No knowledge of the application domains is required, and hardly any knowledge of geometry. The analysis of the randomized algorithms uses some very elementary probability theory.

Implementations. The algorithms in this book are presented in a pseudo-code that, although rather high-level, is detailed enough to make it relatively easy to implement them. In particular we have tried to indicate how to handle degenerate cases, which are often a source of frustration when it comes to implementing.

We believe that it is very useful to implement one or more of the algorithms; it will give a feeling for the complexity of the algorithms in practice. Each chapter can be seen as a programming project. Depending on the amount of time available one can either just implement the plain geometric algorithms, or implement the application as well.

To implement a geometric algorithm a number of basic data types—points, lines, polygons, and so on—and basic routines that operate on them are needed. Implementing these basic routines in a robust manner is not easy, and takes a lot

of time. Although it is good to do this at least once, it is useful to have a software library available that contains the basic data types and routines. Pointers to such libraries can be found on our Web site.

Web site. This book is accompanied by a Web site, which contains a list of errata collected for each edition of the book, all figures and the pseudo code for all algorithms, as well as some other resources. The address is

`http://www.cs.uu.nl/geobook/`

You can also use the address given on our Web site to send us errors you have found, or any other comments you have about the book.

About the third edition. This third edition contains two major additions: In Chapter 7, on Voronoi diagrams, we now also discuss Voronoi diagrams of line segments and farthest-point Voronoi diagrams. In Chapter 12, we have included an extra section on binary space partition trees for low-density scenes, as an introduction to realistic input models. In addition, a large number of small and some larger errors have been corrected (see the list of errata for the second edition on the Web site). We have also updated the notes and comments of every chapter to include references to recent results and recent literature. We have tried not to change the numbering of sections and exercises, so that it should be possible for students in a course to still use the second edition.

Acknowledgments. Writing a textbook is a long process, even with four authors. Many people contributed to the original first edition by providing useful advice on what to put in the book and what not, by reading chapters and suggesting changes, and by finding and correcting errors. Many more provided feedback and found errors in the first two editions. We would like to thank all of them, in particular Pankaj Agarwal, Helmut Alt, Marshall Bern, Jit Bose, Hazel Everett, Gerald Farin, Steve Fortune, Geert-Jan Giezeman, Mordecai Golin, Dan Halperin, Richard Karp, Matthew Katz, Klara Kedem, Nelson Max, Joseph S. B. Mitchell, René van Oostrum, Günter Rote, Henry Shapiro, Sven Skyum, Jack Snoeyink, Gert Vegter, Peter Widmayer, Chee Yap, and Günther Ziegler. We also would like to thank Springer-Verlag for their advice and support during the creation of this book, its new editions, and the translations into other languages (at the time of writing, Japanese, Chinese, and Polish).

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Contents