Chapter 12
Fatigue and Scatter

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12.1 Introduction

Scatter is an inherent characteristic of mechanical properties of structures and materials. This also applies to fatigue properties. The fatigue lives of similar specimens or structures under the same fatigue load can be significantly different. Also, the fatigue limit of one and the same material is subjected to scatter. In the literature, statical aspects of fatigue of structures and materials are well recognized, but the implications for engineering problems are not always clear. In this chapter, various sources for scatter of fatigue are discussed first (Section 12.2). These sources can be essentially different for the crack initiation period and crack growth period. The description of the statistical variability is addressed in Section 12.3, including how to obtain experimental information about scatter. A special issue is scatter of the fatigue limit. Various engineering aspects of scatter are discussed in Section 12.4. The major topics of this chapter are recalled in Section 12.5.

12.2 Sources of scatter

The fatigue life covers a crack initiation period and a crack growth period. Differences between the fatigue mechanisms in the two periods are discussed

Table 12.1 Various sources of scatter.

Aspects considered	Possible sources of scatter	
	Laboratory test series	Structures in service
Material	Material structure	Material from different batches and manufacturers
Production	Specimen production Specimen surface quality	Production of structures over years Surface quality of fatigue critical notches in structure
Fatigue load	Type of fatigue load (CA, VA) Test frequency Accuracy of test equipment	Load spectra in service Different users of structure Fatigue life covers years
Environment	Laboratory environment, controlled temperature and humidity	Service environment, possibly aggressive
Personal aspects	Skill of laboratory technicians	Different users

in Chapter 2. It was shown that crack initiation, including the first microcrack growth, is primarily a material surface phenomenon. As a result, the crack initiation period is strongly dependent on the material surface conditions. In the second period, the crack growth period, crack extension is hardly depending on the material surface condition, but predominantly on the crack growth resistance of the material as a bulk property of the material. It should thus be expected that sources of scatter are different for the crack initiation period and the crack growth period. It is generally recognized that the fatigue life of the initiation period is much more sensitive to various influences. Scatter of this life period can be large. Fatigue crack growth in the second period shows a limited variability. In the present chapter, scatter of fatigue life is considered, but some attention is also paid to scatter of crack growth.

Statistical information about fatigue properties is mainly coming from laboratory investigations, and not from service experience. It thus is useful to consider various potential sources for scatter for both types of circumstances. A survey is given in Table 12.1.

More aspects could have been listed in the table, but the purpose here is to illustrate that essentially different circumstances can occur in service and in the laboratory. As a result, the variability of fatigue properties in service cannot be simply related to scatter observed in the laboratory. Actually, in laboratory investigations, it is generally tried to eliminate various sources of scatter because the purpose is to study a fatigue problem without obscuring the findings by scatter of the data. It implies that specimen production is done most carefully from a single batch of material, aiming at a uniform and fine surface quality. Furthermore, fatigue tests are carried out under closely controlled conditions.

Effects of different circumstances of industrial production and structures in service have been studied in laboratory investigations, e.g. the effect of the surface finish quality. Such investigations have contributed to understand various practical sources of scatter. The understanding of scatter goes back to the fatigue mechanisms discussed in Chapter 2.

12.3 Description of scatter

Some statistical concepts used in this chapter are: distribution function, probability density function, mean value, standard deviation and probability level. These concepts are discussed in text books on statistics. Definitions are also presented in ASTM STP 91-A [1] and the ESDU data sheets [2].

Fatigue life results of an old investigation [3] with a large number of tests on unnotched rotating beam specimens ($S_m = 0$) are presented in Figure 12.1. Scatter is very large in these test series, and it may well be

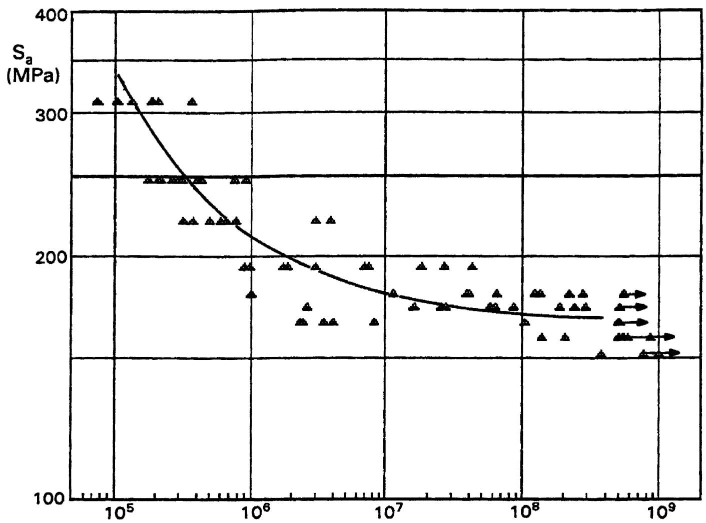


Fig. 12.1 Much scatter in an older test program of rotating beam fatigue tests on unnotched 7075-T6 specimens [3].

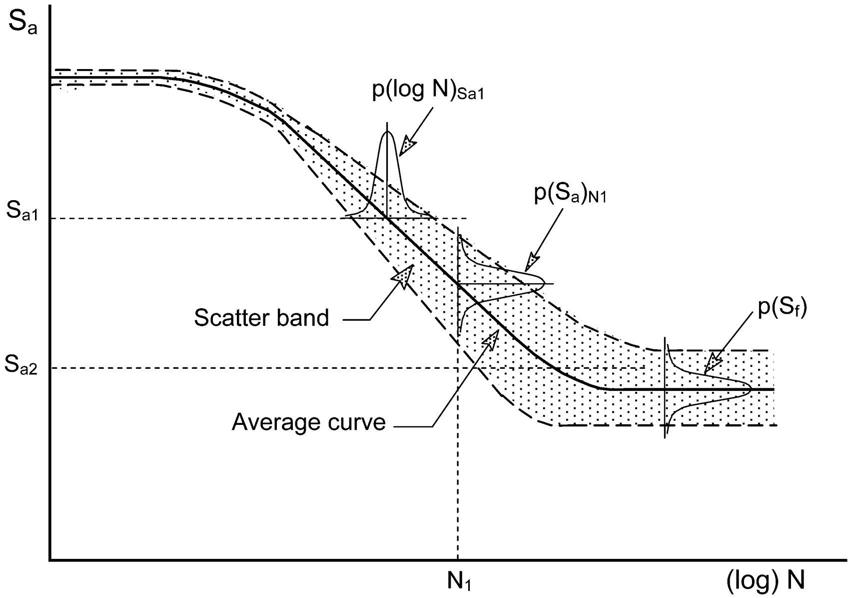


Fig. 12.2 Narrow scatter band at a high S_a, wide scatter band at a low S_a.

expected that several unfavorable conditions have contributed. Results of a much smaller number of experiments, previously shown in Figure 6.3, do not show a similarly large scatter. However, both figures show that scatter is less at high stress amplitudes, and larger at low stress amplitudes. In other words, the scatter band is narrower at a high S_a and wider at a low S_a. The wider band near the fatigue limit is also the result of specimens that do not fail after a very high number of load cycles (so-called run-outs). This scatter behavior is depicted in Figure 12.2. With reference to Chapter 2, the variation of scatter can be understood. At a high S_a-value, surface condition are less important for crack nucleation because microcracks are initiated early in the fatigue life. It is followed immediately by further crack growth. As a result, scatter will be relatively low. However, at a low S_a-value, crack nucleation and the first microcrack growth is meeting with material structural barriers. Nucleation can be dependent on local surface inhomogeneities and small surface irregularities, or even slight surface damage. These surface conditions can vary from specimen to specimen, and have a significant effect on the duration of the initiation period. As a result, more scatter is found at high endurances.

Statistical Distribution of Fatigue Life

Information on the distribution function of the fatigue life, N (or $\log N$) should be obtained by carrying out a large number of similar experiments at the same stress level. The probability density function, $p(\log N)_{S_{a1}}$ in Figure 12.2, represents the distribution of $\log(N)$ at the selected stress amplitude S_{a1}. Similarly, $p(S_a)_{N_1}$, also depicted in Figure 12.2, is the probability density function of the fatigue strength at the selected fatigue life N_1. A special function of the latter type is the distribution function of the fatigue limit, S_f. Scatter of the fatigue limit is of engineering interest if all cycles of a load spectrum have to be kept below the fatigue limit.

An intriguing question is: what is the distribution function of the fatigue strength? Especially, the distribution function of fatigue life has received considerable attention in the literature. Unfortunately, the function cannot be derived on the basis of physical arguments. In general, the function is simply assumed, or adjusted to experimental data of large test series. Constants in the function are derived from experimental data. Two popular distributions are the normal or Gaussian distribution and the Weibull distribution. The well-known normal distribution function is

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}[\frac{v-\mu}{\sigma}]^2} \mathrm{d}v \tag{12.1}$$

with μ as the mean value and σ as the standard deviation of the variable x, while v is the integral variable. Consideration of scatter of fatigue lives usually start with assuming that the variable of interest is the logarithm of the fatigue life, $x = \log(N)$. The values of μ and σ are estimated from the results of a series of m similar experiments (a statistical sample) by calculating

$$\mu = \frac{1}{m}\sum_{1}^{m} x_i \quad \text{and} \quad \sigma = \sqrt{\frac{\sum_{i}^{m}(x_i - \mu)^2}{m-1}} \tag{12.2}$$

The normal distribution covers an interval from $-\infty$ to $+\infty$. The lower limit implies $x = \log(N) = -\infty$, or $N = 0$. A zero fatigue life is physically impossible. This discrepancy does not occur in the three-parameter Weibull distribution function:

$$P(x) = 1 - \mathrm{e}^{-[\frac{x-x_0}{a}]^b} \tag{12.3}$$

In this equation, x_0 is the location parameter (values of x lower than x_0 are impossible, x_0 is the lower limit), a is the scale parameter, and b is the shape parameter. These three constants are determined from the test results by optimizing the correlation between the test results and Eq. (12.3). It requires an iterative calculation procedure [4].

The normal distribution function and the Weibull distribution function both have an upper limit at $x = \infty$, which is physically a strange result. However, the upper limit is of less practical interest. The lower limit, where probabilities of failure are low, is more significant for engineering problems associated with safety factors.

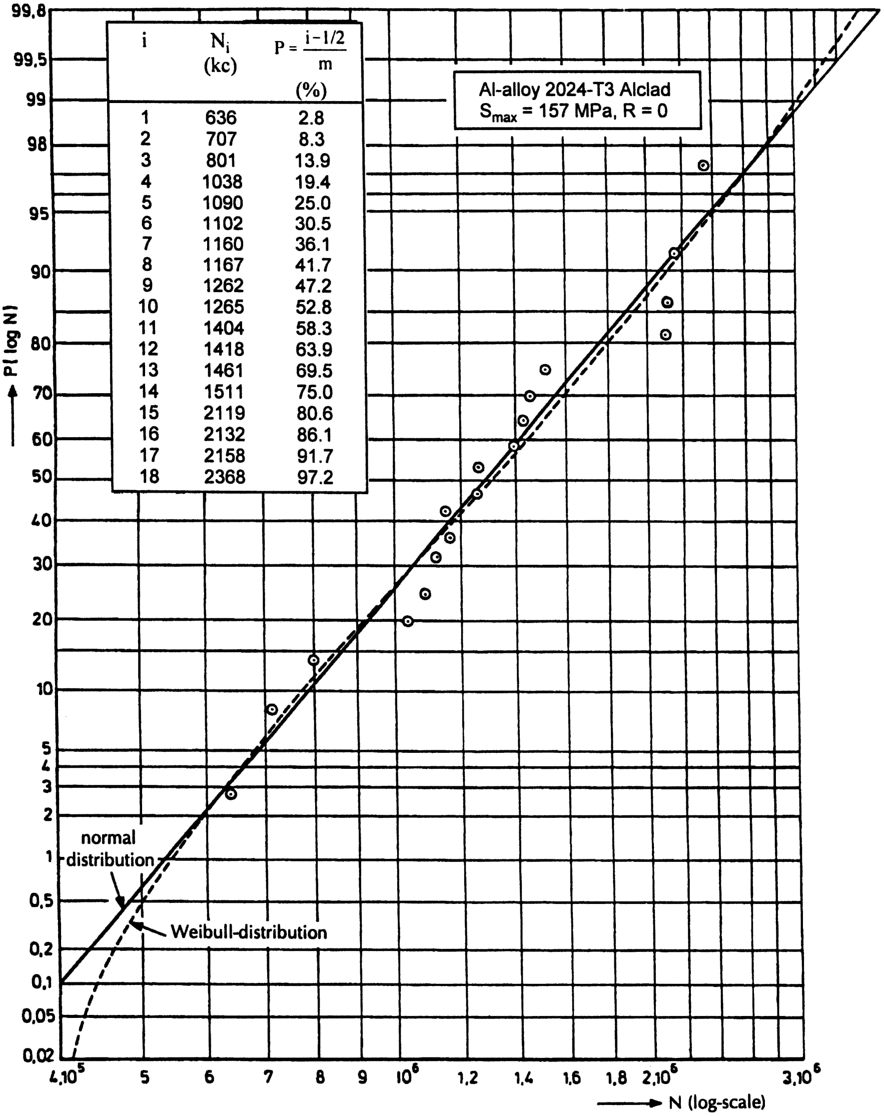
The validation of the two above distribution functions requires large test series. It is difficult to discriminate between the two distribution functions if the number of test data of nominally similar tests is small. An illustration is presented in Figure 12.3 with results of a series of 18 similar experiments on an unnotched specimen. A probability of failure $P(x)$ must be allotted to each result. For that purpose, the results are ranked in an increasing order of magnitude with rank numbers from $i = 1$ to $i = m$, see the inset table in Figure 12.3. The statistical estimate for $P(x)$ is:[16]

$$P(x_i) = \frac{i - \frac{1}{2}}{m} \tag{12.4}$$

The results are plotted in Figure 12.3 on so-called normal probability paper. The vertical scale of this paper is transformed to obtain the normal distribution as a linear relation with a slope dependent on the standard deviation σ. The test results in Figure 12.3 agree reasonably well with the normal distribution function. However, the results agree equally well with the Weibull distribution function also shown in the same figure by a dotted line. Differences between the two functions occur for $P(\log N) < 0.01$ and $P(\log N) > 0.99$, i.e. in the tails of the distribution function. In Figure 12.3, considering only 18 tests, the number of results is too low to discriminate between the two functions. The distribution function is of special interest at very low probabilities of failure, where the Weibull distribution shows a lower limit, whereas the normal distribution does not. If larger numbers of tests are carried out, it might be expected that a tendency to a lower limit of the distribution function becomes more evident. Even then, the question remains how to generalize this observation to practical conditions.

Some more warning comments must be made. First, experiments can never prove that a certain distribution function is applicable. At best, it can be shown that such a function agrees with test data. Second, in different test series, different distribution functions may give the best fit to

[16] A slightly better estimate, proposed by Rossow [6], is $P(x_i) = (3i - 1)/(3m + 1)$.



i	N_i (kc)	$P = \frac{i-1/2}{m}$ (%)
1	636	2.8
2	707	8.3
3	801	13.9
4	1038	19.4
5	1090	25.0
6	1102	30.5
7	1160	36.1
8	1167	41.7
9	1262	47.2
10	1265	52.8
11	1404	58.3
12	1418	63.9
13	1461	69.5
14	1511	75.0
15	2119	80.6
16	2132	86.1
17	2158	91.7
18	2368	97.2

Fig. 12.3 Fatigue lives obtained in a series of 18 similar tests on unnotched specimens, plotted on normal probability paper [5].

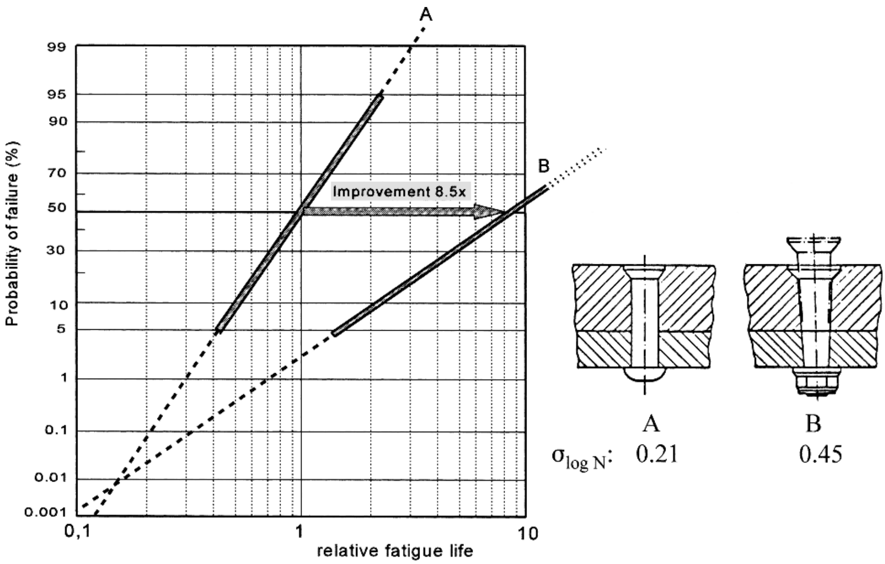


Fig. 12.4 Statistical fatigue life results in comparative tests on two types of fasteners [7].

the test data. Third, the Weibull distribution function is more flexible for data fitting because it has three constants instead of two for the normal distribution function. The third constant, x_0, accounts for the lower boundary of statistically possible fatigue lives, which seems to be more realistic. A lower boundary, N_0, could also be added as a third variable to the normal distribution by defining $x = \log(N - N_0)$ as the statistical variable instead of $x = \log(N)$. However, this is rarely done. Finally, it is physically unrealistic that a fatigue life distribution is continuous until $x \rightarrow \infty$, although this is not so much of practical interest. Low fatigue lives, rather than infinite fatigue lives, are the leading issue in statistical considerations on the fatigue performance of structures in view of the probability of failure of the structure.

Two Case Histories

If a substantial number of similar tests is carried out, the log mean value of the fatigue life is assumed to be a characteristic average value, and the standard deviation of $\log(N)$ gives an indication on scatter in the tests. In comparative test series, the mean values can show different effects for the variables investigated. However, statistical problems can arise if scatter is large. In a comparison between two types of fasteners, see Figure 12.4,

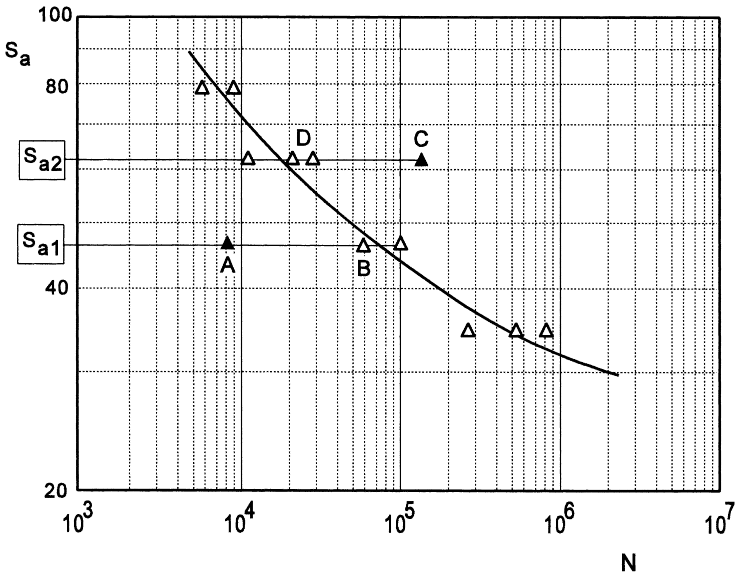


Fig. 12.5 An exceptionally low and high test result (A and C, respectively) in a series of experiments.

scatter was large indeed, see the standard deviations ($S_{\log N}$) in this figure. The taper-lok fastener (B) was developed to be an improvement of the conventional fastener (A). And indeed, the mean life in a test series of 10 specimens was significantly improved, 8.5 times. The results plotted on normal probability paper suggested a reasonable agreement with the normal distribution function, but in view of the number of 10 specimens in each series, this covers a range of probabilities of failure from 5 to 95% only. Extrapolations to lower probability of fatigue are made by the dashed lines in Figure 12.4. They intersect at $P = 0.01\%$, which implies that the taper-lok fastener then should become inferior. Obviously, this conclusion entirely depends on the validity of the assumed normal distribution function, which is questionable. Moreover, was the scatter observed in these test series really representative for scatter to be expected in mass-production? The major lesson to be learned from this test series is that the observed scatter is unacceptable. As long as this scatter cannot be reduced, a conclusion on improvements cannot and should not be drawn.

An other problem associated with scatter is illustrated by the results in Figure 12.5. Test results are plotted with the purpose to arrive at an average S-N curve. The curve in this figure was drawn by hand, while disregarding data points A and C, which are far outside the scatter band of the other results. These unusual data are a matter of concern. The low endurance of data point A could be due to incidental surface damage of the specimen, which would invalidate the test result. This should be checked by examining the specimen fracture surface, e.g. under a binocular optical microscope at magnifications from 10 to 50 times. The high endurance of data point C is not easily explained. If the specimen is a notched specimen, it could be due to an unintentionally applied high load at the beginning of the fatigue test. In statistical terms, it is hard to believe that results A and C are part of the population of the other fatigue lives. Such results can be omitted. An alternative approach is to consider median results of similar tests. In Figure 12.5, the median at stress amplitude S_{a1} is result B, and at S_{a2}, it is about D. Note that approximately the same S-N curve would be drawn when using these median values. Again, statistics is not a tool to solve problems, but to describe scatter. Problems as discussed above must be judiciously handled with understanding of possible influences.

Statistical Distribution of the Fatigue Strength and the Fatigue Limit

If a sufficiently large number of specimens is tested at the same cyclic stress level, a $P(\log N)$ curve can be plotted, a curve as shown in Figure 12.3. Fatigue lives, corresponding to specific $P(\log N)$-values, can be read from such graphs. Characteristic values of $P(\log N)$, used in discussions on failure probabilities, are $P(\log N) = 0.01$, 0.05, 0.10 and 0.50, corresponding to probabilities of failure of 1, 5, 10 and 50% (mean value) respectively; or to probabilities of survival of 99, 95, 90 and 50% respectively. If this is done at several stress levels, S-N curves for certain probabilities of failure can be drawn, see Figure 12.6. Such curves are referred to as P-S-N curves. These curves can only be drawn if large numbers of fatigue tests are carried out at several stress levels, which is expensive. Usually, an average S-N curve is drawn through a limited number of data, see Figure 6.3 as an example. The average curve is associated with $P = 50\%$, i.e. the average probability of failure.

A vertical cross plot of P-S-N curves gives a distribution function of the fatigue strength, $P(S_a)_N$ for a certain fatigue life N. The probability density function of such a function was already depicted in Figure 12.2 for N_1. It should be noted that a direct experimental determination of $P(S_a)_N$ is impossible. The result of a fatigue test is a fatigue life for an applied stress level, but it cannot be a stress amplitude for a selected life.

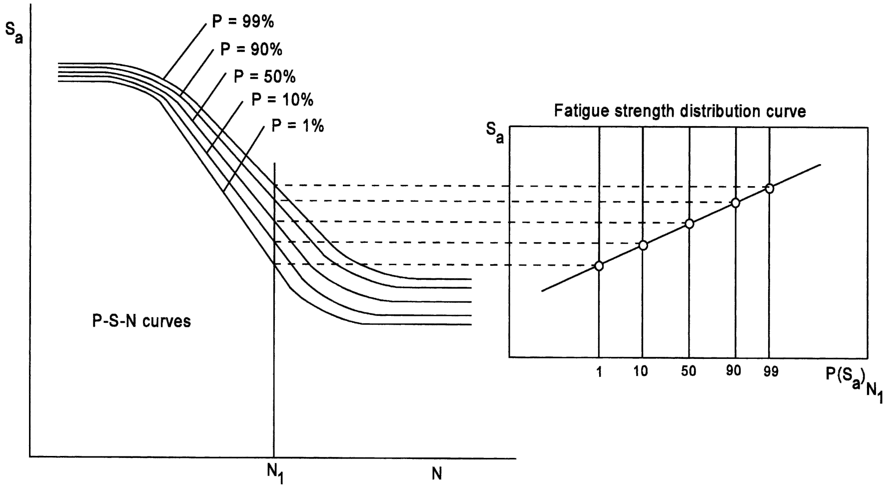


Fig. 12.6 P-S-N curves with a cross plot to obtain a fatigue strength distribution for a constant fatigue life.

Test results, 51 specimens

S_a (MPa)	Number of tests	Number of specimens failed before $N = 10^7$ cycles	Probability of failure (%)
280	15	1	6.7
290	8	2	25
300	5	2	.40
310	8	6	75
320	15	14	93.3

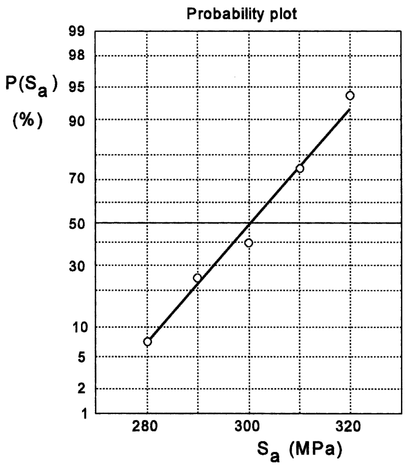


Fig. 12.7 The Probit method to determine the distribution function of the fatigue limit [1].

The P-S-N curves for low S_a-values, close to the fatigue limit, are difficult to obtain by such a cross plot. Several specimens with a very high endurance are not tested until failure, but stopped at a selected high number of cycles, e.g. 10^7. As a result, the distribution function of the fatigue limit cannot be derived in this way. An alternative procedure is the Probit method [1]. Specimens are tested at a number of stress levels around the anticipated fatigue limit. Tests are stopped at a high N-value to be associated with the fatigue limit, say $N = 10^7$ cycles. Results in the table of Figure 12.7 are

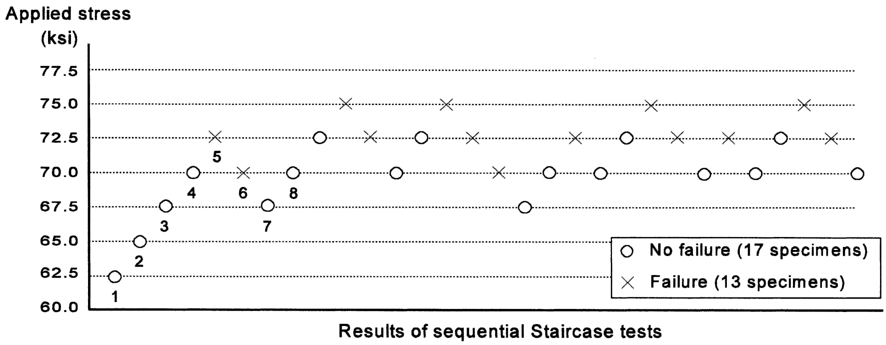


Fig. 12.8 The Staircase method to determine the fatigue limit [1].

shown as an example. At five stress levels the percentage of failed specimens is recorded, which is the estimated probability of failure at that stress level. These probabilities are then plotted on probability paper to obtain the distribution function of the fatigue limit associated with $N = 10^7$ cycles. The approximately linear relation indicates a mean value at $P(S_f) = 50\%$, equal to 300 MPa. The standard deviation derived from the slope is 14 MPa, which is 4.7% of the mean value.

The fatigue limit can also be determined by the so-called Staircase method which requires fewer specimens than the Probit method. Specimens are tested until a high number of cycles to be associated with the fatigue limit, e.g. $N_{sf} = 10^7$ cycles. The first specimen is tested at the estimated stress level of the fatigue limit. If failure does not occur, the test is stopped after N_{sf} cycles, and a second specimen is tested at a higher stress level. However, if failure occurs before N_{sf} cycles are applied, then the second test is carried out at a lower stress level. This procedure is sequentially followed with a number of specimens, see Figure 12.8. The increment between the stress levels is constant (2.5 ksi = 17 MPa in Figure 12.8). A non-failure test is followed by a test at a higher stress level, but if failure does occur, it is followed by a test at a lower stress level. As a result, the stress levels used are going up an down around the fatigue limit for N_{sf} cycles. The mean value and the standard deviation of the fatigue limit can ban be calculated from the test results, see [1]. The accuracy of the mean value is reasonable if some 20 to 30 specimens are used, but this is not true for the standard deviation. The number of specimens is still fairly large although smaller that for the Probit method. A rough estimate of the fatigue limit can be obtained with just a few specimens with a method described in Chapter 13 on testing techniques (see Figure 13.2).

It should be mentioned here that a determination of P-S-N curves requires many specimens. The same is true for the determination of the distribution function of the fatigue limit. It implies that such test programs are very expensive.

12.4 Some practical aspects of scatter

As discussed in Section 12.2, scatter in laboratory experiments and in service can occur for essentially different reasons. Furthermore, data on scatter in laboratory experiments are almost exclusively associated with CA loading whereas load spectra in service frequently apply to VA loading. Some practical consequences are illustrated in this section:

1. The fatigue limit and safety factors.
2. Scatter and VA loading.
3. Scatter of fatigue crack growth.
4. Scatter in different structures of the same type.
5. Symptomatic or incidental fatigue failures in service.
6. Scatter depending on how a structure is used.

Fully rational answers to practical questions about scatter cannot be given, but understanding the fatigue behavior of a structure offers some indications about the significance of scatter.

The Fatigue Limit and Safety Factors

Scatter of the fatigue life is mainly depending on the crack initiation period as explained before. If crack initiation can easily occur, scatter should be expected to be small. This applies to structures with sharp notches (high K_t, poor design). However, if crack initiation is difficult, scatter may offer problems. This can apply to structural elements designed to be free from fatigue failures for a long service life. All cycles of the load spectrum should then remain below the fatigue limit with a certain margin of safety. The situation is relevant to parts of engines, machinery, helicopter components, etc. Such parts are usually made of high-strength alloys, which in general are sensitive to notches and the quality of surface finish. The prediction of fatigue limits was discussed in Chapter 7 with emphasis on accounting for surface and size effects. If these effects are included in the predictions, and perhaps supported by supplemental test verifications, it must be recommended to apply a safety factor on the fatigue limit in order to account for possible scatter. In such a way an allowable design stress level can be obtained. The problem is how to arrive at a reasonable safety factor. This is a delicate problem, and statistical knowledge is not really helpful to select a safety factor. It could be assumed that the fatigue limit has a normal distribution with an estimated standard deviation. A low probability of failure must then be adopted, e.g. $P = 0.001$ (0.1%). This value is often mentioned in safety analyses, but if a designer is asked whether it is acceptable that a failure of an element in a production of 1000 elements could occur, the answer will be negative.

For a normal distribution of the fatigue limit, $P = 0.001$ implies that the estimated fatigue limit must be reduced by 3.09 times the standard deviation σ_{S_f}. For $P = 0.0001$ (0.01%), it would be 3.72 times σ_{S_f}.[17] In the previous example of determining the distribution function of S_f (Figure 12.7), the standard deviation was about 5% of the fatigue limit. The fatigue limit should thus be reduced by $3.09 \times 5 = 15.5\%$ and $3.72 \times 5 = 18.6\%$ respectively. The latter value corresponds to a safety factor of $1/(1-0.186) = 1.23$ on the fatigue limit. This safety factor is not impressive. Engineering judgement is indispensable in arriving at a more realistic value. It must then be recognized which sources of scatter can occur, e.g. variations with respect to the quality of the material, production, surface treatments, and possibly other sources of variability. Furthermore, how sensitive is the component for incidental surface damage. Also, are failures absolutely unacceptable, or could a few failures be allowed economically? As a suggestion, a safety factor of 1.5 could be considered to be a conservative factor for high-quality material and production, provided that quality control is assured.

Scatter and VA Loading

An informative series of tests was carried out in the late fifties [8] with results on scatter in CA and VA tests of riveted lap joints. At that time, service-simulation fatigue tests could not yet be carried out. Several types of program fatigue tests, including tests with different types of OL cycles, were carried out. The results reported in Figure 10.6 were part of this investigation. Each type of test was repeated ten times for CA loading and seven times for VA loading. Scatter was characterized by the standard deviation of $\log(N)$. Values of $\sigma_{\log N}$ are shown as a function of the fatigue life in Figure 12.9. The open data points are from the CA tests. The value

[17] These factors can be found in a table on the normal distribution function.

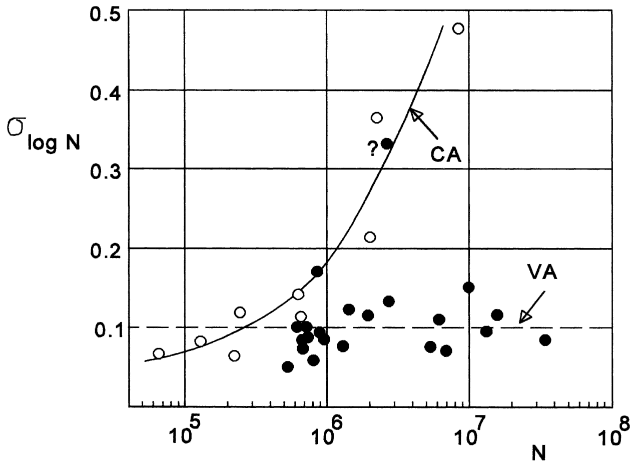


Fig. 12.9 Scatter in fatigue tests on riveted lap joints under CA loading and VA loading [8].

of $\sigma_{\log N}$ obviously increases for an increasing fatigue life, corresponding to a lower S_a-value. This observation agrees with the previous discussion on Figure 12.2. The results for the VA tests are remarkable. All values of $\sigma_{\log N}$ are centered around 0.1, without a noticeable dependence on the fatigue life. It is noteworthy that the average value of $\sigma_{\log N}$ for the VA tests is of the same order of magnitude as $\sigma_{\log N}$ in the CA tests for the largest amplitude of the VA loading. It suggests that the highest amplitude of the load spectrum is responsible for the amount of scatter. This observation is logical because scatter is mainly dependent on the variability of the crack initiation period. Cycles with the larger amplitudes of a VA load spectrum have a predominant effect on the occurrence of the initial crack nucleation. As a consequence, these more severe cycles regulate scatter in the VA tests.

Similar observations on scatter in tests with random or programmed sequences were reported in the literature. Mann [9] suggested that the maximum load of a spectrum will induce local plasticity, which will "smooth out" the influence of small inhomogeneities in the material, and cause load redistributions in the fatigue-critical regions of a built-up structure resulting in a more uniform fatigue response. As pointed out by Jacoby and Nowack [10], life predictions with the Miner rule overestimate the scatter under periodic VA-load histories and apparently gives too much weight to larger scatter of low stress amplitudes. Actually, scatter under VA loading cannot be calculated from scatter observed under CA loading.

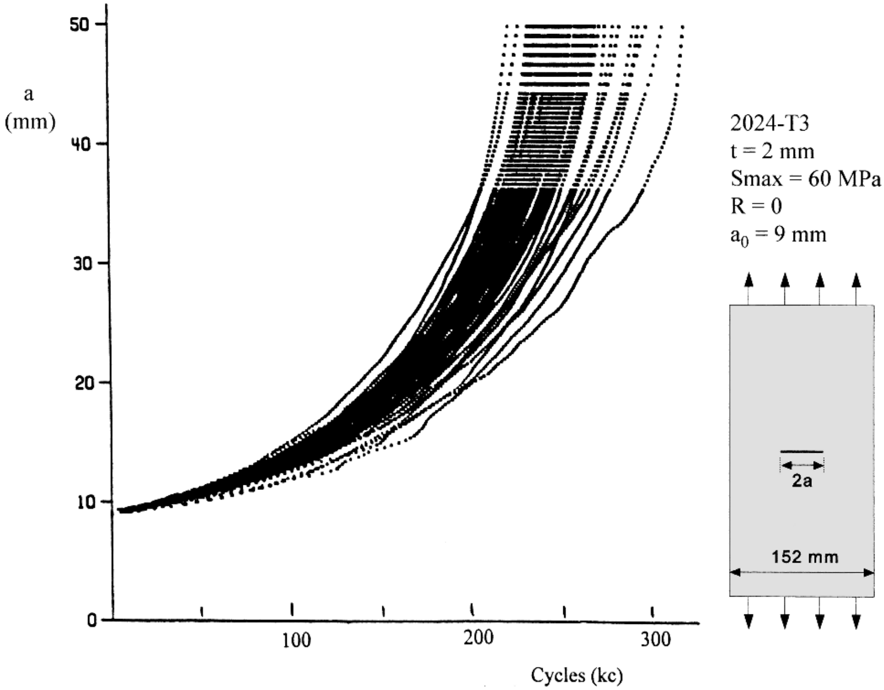


Fig. 12.10 Crack growth curves from 68 similar tests [11]. Scatter is small.

Scatter of Fatigue Crack Growth

Various publications on fatigue crack growth confirm that scatter of the fatigue life, including the crack nucleation period, can be large. By contrast, scatter of fatigue crack growth of visible cracks is generally found to be low. A frequently cited investigation was carried out by Virkler et al. [11], who carried out 68 similar crack growth curves on Al-alloy sheet specimens. The crack growth curves are shown in Figure 12.10. Cracks were started by a small spark eroded central notch. Scatter of the crack initiation was eliminated by normalizing the growth curves on an initial crack length of 9 mm. Figure 12.10 shows that most crack growth curves are concentrated in a narrow band with only a few curves for somewhat slower growth. Considering the shortest and longest crack growth lives in 68 tests, the standard deviation of log(crack growth life) is about 0.03. Standard deviations cited in the literature for the fatigue life, including the nucleation period, $\sigma_{\log(N)}$, are frequently in the range of 0.10 to 0.20 and even larger values are reported (see e.g. the values in Figure 12.9 for CA loading). The standard deviation for the 18 data in Figure 12.3 is $\sigma_{\log(N)} = 0.163$. Thus, the value $\sigma_{\log(N)} = 0.03$ for the crack growth results in Figure 12.10 must be considered to be small.

As explained earlier, scatter is low for fatigue crack growth because it is not dependent on surface conditions. Crack growth resistance then becomes a material bulk property, depending on the material structure. The crack growth resistance can be fairly uniform in a single plate or sheet, and even in plates or sheets from the same batch of material. Unfortunately, statistically significant differences have been found between crack growth lives of nominally similar material from different material manufacturers, and also between different batches of the same manufacturer. This problem was discussed in Chapter 8 and illustrated by the results in Figure 8.16.

The low scatter of fatigue crack growth is advantageous for investigations on crack growth. As little as two specimens for each test condition of a test program may be sufficient, if they show quantitatively the same crack growth curve. In case of doubt, testing of a third specimen is advisable.

Scatter in Different Structures of the Same Type

A service-simulation fatigue test on a full-scale aircraft structure of a transport aircraft with two turboprop engines indicated that some modifications of the wing structure had to be introduced. However, 13 wings were already manufactured and became available for a fatigue test program [12]. The center section of the tension skin (length 8.31 meters) was tested under various load sequences including VA load histories and CA loading. In general, two tension skins were tested under the same fatigue load history. A fatigue critical location occurred in the skin at the top of fingertip reinforcements. A total number of 40 similar fingertips loaded to the same stress level were present in the tension skin located near two skin joints. Inspections on fatigue crack nucleation were made by X-ray picture which allowed a determination of the crack initiation life until a crack with a length of 3 mm was present at the fingertips. These fatigue lives could be determined for a number of fingertips. Results of two similar tests are shown in Figure 12.11 plotted on log-normal probability paper. The tests could not be continued until cracks were present at all fingertips because cracks at other fingertips had grown too far and induced panel failure. However, the results in Figure 12.11 are sufficient to estimate $\sigma_{\log N}$-values of the two similarly tested tension skins. Both values are fairly low, 0.06 and 0.086. More important, the results also indicate that the fatigue lives of the fingertips in the two nominally similar skin structures are not

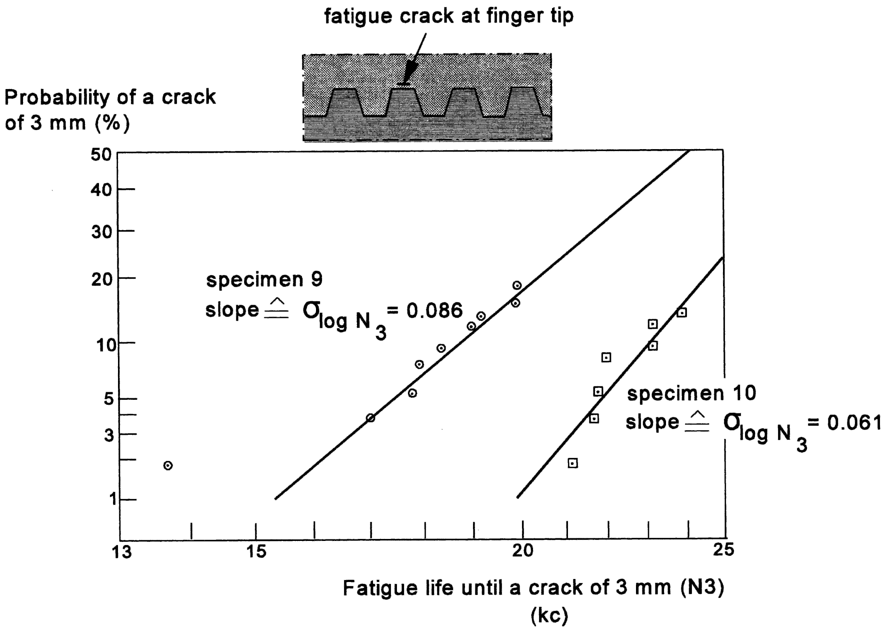


Fig. 12.11 Statistical results from two similar full-scale fatigue tests on an aircraft tension skin. The probability of failure for the crack initiation fatigue life of a crack (3 mm) at 40 similar finger tip reinformcementsin the tension skin. Scatter in each tension skin, and differences between two similar tension skins [12].

statistically identical. The fatigue lives in one tension skin (specimen 10) are systematically larger than for the other tension skin (specimen 9). In other words, the fatigue life distribution function of similar structural details in one structure was different from the distribution function of the other structure. The question then is why this could occur. The answer in this case is not known, but the difference may be due to the material (different batches or even different material producers) and also differences in the production of the two structures. These sources of scatter are avoided in simple specimens prepared for laboratory test programs, but they can apply to industrial products.

Symptomatic or Incidental Fatigue Failures in Service

Williams reported on a catastrophic fatigue failure at a bolt hole in the spar of a Freighter 170 aircraft after 13000 flights in 1957 [13]. Other aircraft of the same type were then inspected to see whether a similar fatigue crack

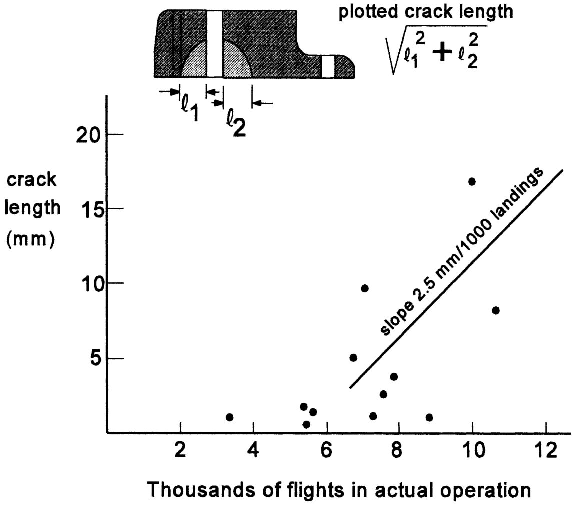


Fig. 12.12 The length of cracks found at a fatigue critical bolt hole in a lower wing spar cap of 12 aircraft [13].

occurred at the same bolt hole. Such an inspection is necessary, also because it should be made clear whether the fatigue crack is a symptomatic problem or an incidental case. Similar cracks were found in several other aircraft with a length shown in Figure 12.12. Apparently, the size of the cracks was larger for older aircraft, i.e. older in numbers of flights. Fatigue cracking thus was a symptomatic phenomenon, but scatter around the average trend can be seen in the graph. It is noteworthy that the average crack growth rate is about 2.5 mm per 1000 flights. In this case, scatter of fatigue crack nucleation and growth between similar aircraft can also be due to a different load spectrum depending on the utilization of the aircraft.

Scatter Depending on How a Structure Is Used

An example of different load spectra for the same aircraft structure is shown in Figure 12.13. The Fokker F28 aircraft of several operators were equipped with a counting accelerometer in the center of gravity of the aircraft. This apparatus counts the number of exceedings of certain acceleration levels which gives an indication of the severity of the load spectrum on the aircraft. The main loads on the aircraft are gust loads in turbulent air. The counting results in Figure 12.13 for four acceleration increments (Δn) are normalized in numbers per flight. Two design load spectra for different take-off weights

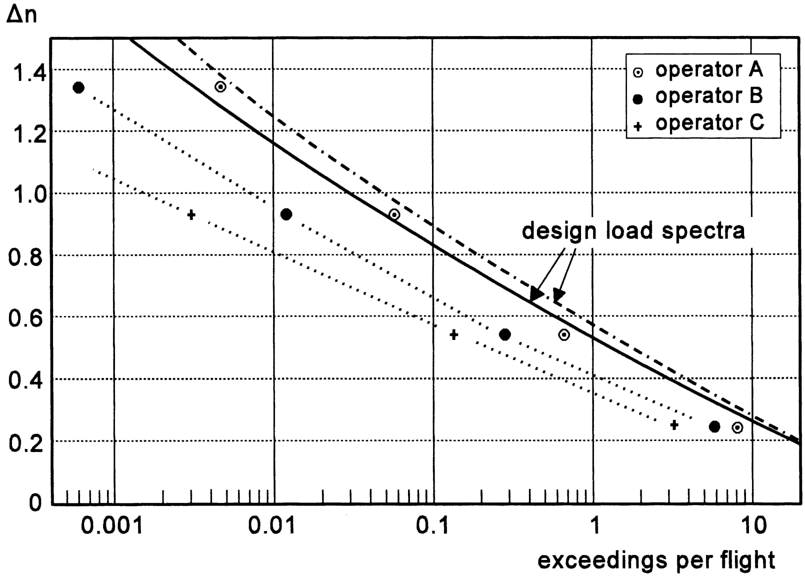


Fig. 12.13 Load spectra for the Fokker F28 aircraft measured in service by three operators with counting accelerometers. Comparison to design spectra.

of the aircraft are also shown in this graph. Counting results are presented for three aircraft operators. It turns out that operator A is encountering the most severe load spectrum which is close to the design load spectra. This operator was flying frequently over high mountains causing a severe load spectrum. Operator B is apparently flying in milder climatic conditions. Operator C meets the most benign load spectrum. Actually, operator C had one single aircraft which was used as an executive aircraft, and executives do not fly in poor weather conditions. Anyway, scatter depending on how a structure is used can be significant for many structures as pointed out in Chapter 9 on load spectra.

12.5 Major topics of the present chapter

1. Scatter of fatigue lives is mainly scatter of the crack initiation period. This period is easily affected by different conditions at the material surface. Scatter of macrocrack growth is significantly smaller.
2. Scatter of fatigue lives is less at high stress amplitudes and more significant at low amplitudes near the fatigue limit.

3. Several important sources of scatter in laboratory investigations and in service are essentially different (Table 12.1). Scatter in service is hardly predictable from scatter observed in laboratory investigations.
4. As a consequence of the previous conclusion, the application of safety factors on the fatigue limit of a structural component is a difficult question.
5. In a statistical analysis of a fatigue problem, a distribution function is usually assumed, but it is difficult to prove that application of the function is reliable.
6. Scatter under VA loading is predominantly controlled by the larger load cycles of the load spectrum. Scatter under VA loading cannot be deduced from scatter data obtained in CA tests.
7. Low scatter is promoted by sharp notches (poor design). Significant scatter is possible for long fatigue lives of carefully designed structural elements (low K_t-values) of high-strength materials. Accounting for scatter should then occur by adopting a suitable safety factor on the design stress level. Selection of this factor requires engineering judgement of all possible sources which can contribute to scatter of the structure in service.

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