Chapter 4
Residual Stress

4.1 Introduction

The significance of residual stresses for fatigue is important in various practical problems. Unintentional tensile residual stress can have an adverse effect on the fatigue resistance, while compressive residual stress can significantly improve the fatigue behavior. The existence of residual stress and the introduction of such stresses in components are the subjects of the present chapter. It is restricted to basic aspects, while some specific topics will return in later chapters.

By definition, residual stress refers to a stress distribution, which is present in a structure, component, plate or sheet, while there is no external load applied. In view of the absence of an external load, the residual stresses are sometimes labeled as internal stresses. The background of the terminology "residual stress" is that a residual stress distribution in a material is often left as a residue of inhomogeneous plastic deformation.

Residual tensile stress and residual compressive stress always occur together. A possible residual stress distribution is presented in Figure 4.1. If there is no external load, residual tensile stresses must be balanced by residual compressive stresses. More precisely, in view of the absence of an external load, the residual stress distribution must satisfy the equilibrium equation:

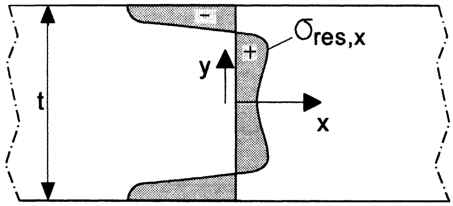

Fig. 4.1 A residual stress distribution is an equibibrium distribution.

$$\int_{-t/2}^{t/2} \sigma_x \, \mathrm{d}y = 0 \tag{4.1}$$

Similar, because of the absence of an external moment the following equation must also be satisfied:

$$\int_{-t/2}^{t/2} y \cdot \sigma_x \, \mathrm{d}y = 0 \tag{4.2}$$

An external load applied to a component will introduce a stress distribution in agreement with the external load and the geometry of the component. If the behavior is still elastic, the material will respond to the sum of the stress distribution of the external load and the residual stress distribution.

$$\sigma = \sigma_{\text{external load}} + \sigma_{\text{residual}} \tag{4.3}$$

If a cyclic fatigue load is applied, $\sigma_{\text{external load}}$ in the material is a cyclic stress with a certain stress amplitude (σ_a) and mean stress (σ_m). However, σ_{residual} is permanently present. It does not affect the stress amplitude, but it gives a shift to the mean stress:

$$\sigma_a = \sigma_{a,\text{external load}} \tag{4.4a}$$

$$\sigma_m = \sigma_{m,\text{external load}} + \sigma_{\text{residual}} \tag{4.4b}$$

If the local residual stress is positive, it increases σ_m (unfavorable for fatigue), and if it is negative, it reduces σ_m (favorable for fatigue). Residual stresses can be quite high. As a result of a high compressive residual stress it is possible that σ_{peak} is low, or even negative. In the latter case a microcrack can hardly grow. Because residual stresses do not affect the stress amplitude, cyclic slip at the material surface is still possible, and some microcrack nucleation can occur. However, if such microcracks are not opened at σ_{peak}, microcrack growth will not occur. If σ_{peak} including the compressive residual stress is positive, microcrack growth is possible, but the growth rate is reduced in view of the lower σ_{peak}.

Residual stresses have more consequences than for fatigue only. It is well known that tensile residual stresses can be most harmful if the material is sensitive to stress corrosion. Secondly, machining of the material with a residual stress distribution, can lead to distortions of the material (warpage). For instance, if a surface layer is removed from the plate in Figure 4.1 at one side only, the residual stress distribution does no longer satisfy the equilibrium equations (4.1) and (4.2) if warpage did not occur. As a consequence, warpage must occur (in this case plate bending) which changes the residual stress distribution until these equations are satisfied again.

It should be pointed out that the residual stresses discussed in this chapter occur on a macroscale. They have the same meaning as the stresses induced by an external load. On a much smaller scale, another type of residual stress can be present. Plastic deformation on a microscale is not a homogeneous process. It will be different from grain to grain, and even inside a single grain it may be concentrated into a few slip bands. Also in this case, equilibrium requires that the sum of the residual microstresses is zero. The microresidual stresses are significant for explaining the fatigue mechanism on a microlevel (Chapter 2) and also for the Bauschinger effect. These aspects are not considered in this chapter.

4.2 Different sources of residual stresses

Residual stresses can be present in a material as a result of different processes. In this section attention is paid to:

1. Inhomogeneous plastic deformation, in many cases at notches.
2. Production processes.
3. Shot peening.
4. Plastic hole expansion.
5. Heat treatment.
6. Assembling components

Inhomogeneous plastic deformation

A simple theoretical model will be discussed first. In Figure 4.2 two tension bars of different lengths are connected to the same infinitely stiff clampings at the ends. If a load is applied to this two-bar system, the elongation $\Delta\ell$ is the same for the two bars. As a consequence, the strain (ε) in the shorter

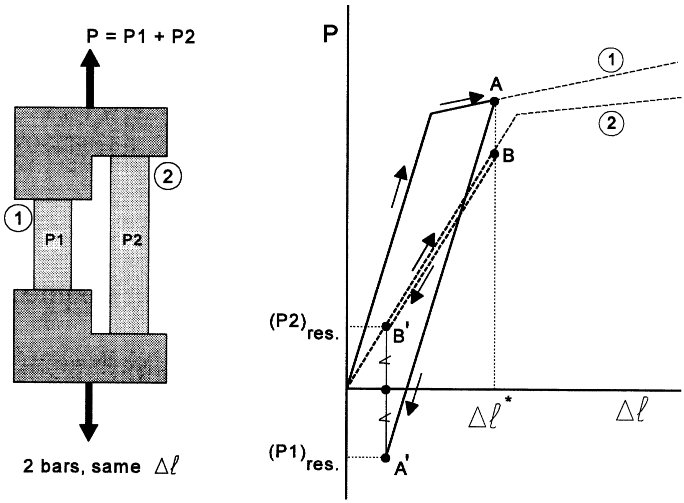

Fig. 4.2 A two-bar system to illustrate residual stress as a result of local plastic deformation.

bar will be larger because its length is shorter. In view of the larger ε, the stress will be higher and the bar thus will carry more load than the other bar. This is shown in the load-displacement ($P - \Delta\ell$) diagram in Figure 4.2. There is a load concentration in bar 1. Assuming that both bars are similar, it implies that permanent plastic deformation can occur in bar 1, while bar 2 is still fully elastic. This occurs at points A and B where $\Delta\ell = \Delta\ell^*$. During reversion of the loading direction elastic unloading occurs in both bars. After full unloading $P = 0$, which means that the sum of the residual loads in the two bars is zero; and thus $(P1)_{\text{res}} = -(P2)_{\text{res}}$, see Figure 4.2. Because of the plastic elongation of bar 1, this bar is longer than it was before. As a result, it will be in compression at $P = 0$ while bar 2 will be in tension. Residual stresses have been introduced as a result of plastic deformation in one part of the two-bar system.

A similarly inhomogeneous plastic deformation occurs in a strip with a hole loaded in tension, see Figure 4.3. If a high load is applied to the specimen, σ_{peak} at the edge of the hole exceeds the yield limit, and a small plastic zone is created at the root of the notch. As a consequence of the plastic deformation, σ_{peak} is smaller than $K_t\sigma_{\text{nom}}$. The peak of the stress distribution is flattened by local plastic yielding. In the plastic zone permanent plastic deformation has occurred. The plastic zone is elongated; it is larger than it was before. After removing the tensile load on the strip, i.e. in the unloaded condition, the elongated plastic zone will be under compression. It does no longer fit stress-free in the elastic surrounding which tries to constrain the

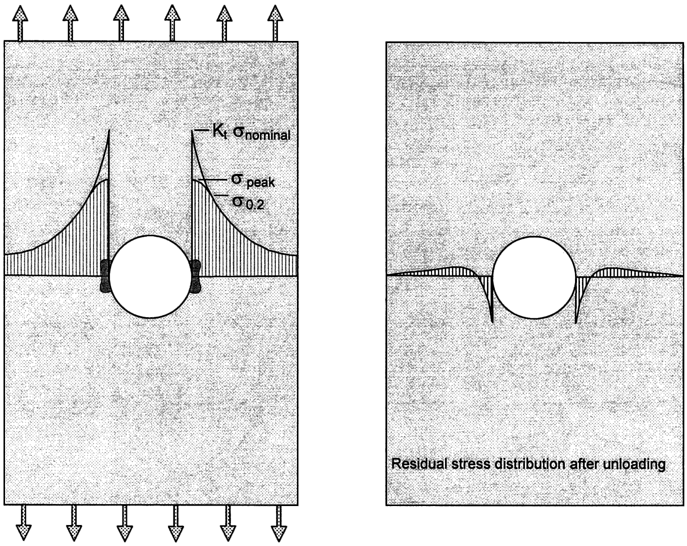

Fig. 4.3 Residual compressive stress at the root of a notch.

permanent plastic deformation. A residual stress distribution is introduced with a residual compressive stress at the root of the notch, which is the fatigue critical location under cyclic loading. The residual compressive stress is balanced by residual tensile stresses away from the notch. The residual compressive stress at the root of the notch can be most favorable for fatigue. In general, local plastic deformation causes an inhomogeneous residual stress distribution as illustrated by the right-hand picture in Figure 4.3. High residual stresses can be introduced up to a level approaching the compressive yield stress.

Production processes

Two common production processes are cold working and machining. Cold working implies that the material is plastically deformed, which should leave a residual stress distribution in the product. An elementary example is plastic bending. As illustrated by Figure 4.4, a bending moment will induce plastic deformation in the outer fibers of the material. After unloading, elastic spring back occurs, and a residual stress distribution as schematically presented in Figure 4.4 will remain in the material. The stress distribution should satisfy both equations (4.1) and (4.2).

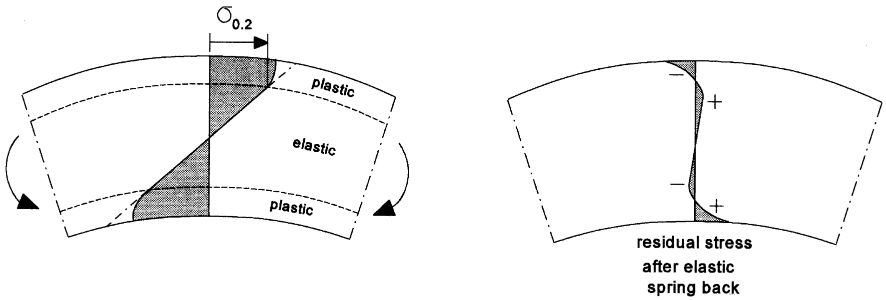

Fig. 4.4 Residual stresses as a result of a plastic bending operation.

In a similar way, residual stresses can exist after a variety of cold working processes. Forging in many cases is a hot-working process, which still can leave residual stresses. The same is true for rolling of plate and sheet material. Afterwards straightening is usually done at room temperature. A residual stress system may be present in the final product.

It is not always realized that machining operations can also introduce residual stresses. Metal cutting implies removal of a layer of material, which includes a failure process near the tip of the cutting tool. But the failure process is preceded by plastic deformation. Depending on machining conditions (sharpness of the cutter, feed rate, depth of cut, etc.) and also on the material, residual stresses can be significant in a thin surface layer.

Shot peening

Shot peening is a well-known process to introduce favorable residual stresses at the material surface of a component. In various practical cases it is applied to prevent fatigue or stress corrosion problems. The peening operation is plastically stretching the surface layer of a material. Because this layer must remain coherent with the elastic substrate material, residual compressive stresses are introduced at the surface. It can lead to warpage of the component, but dimensional distortions can sometimes be prevented by a symmetric peening operation.

The intensity of the peening operation can be checked by peening a so-called Almen strip, which is a steel strip (76 × 19 mm, 3 × 0.75 inch). The strip is fixed by bolts to a stiff foundation, and peened under well defined conditions from one side only, see Figure 4.5. After removing the bolts, the strip is curved. The arc height is measured, which gives a direct indication of the shot peening intensity [1]. An example of a residual stress distribution

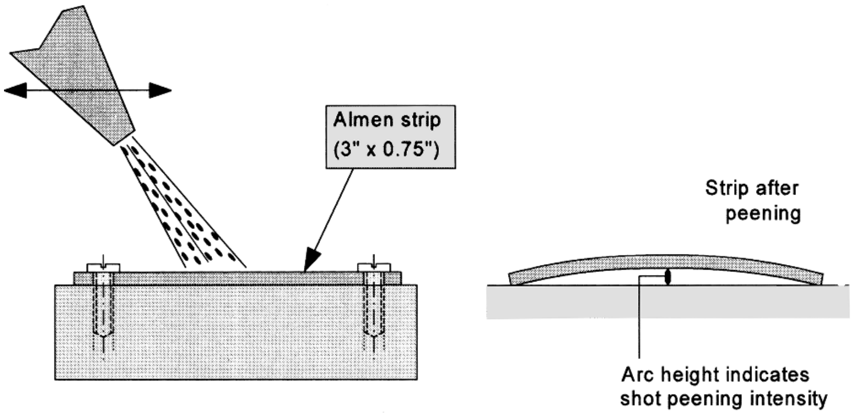

Fig. 4.5 Measurement of shot peening intensity with an Almen strip [1].

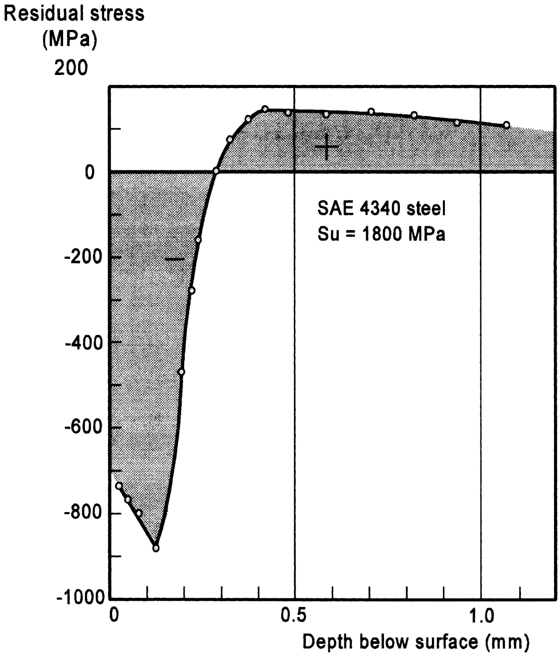

Fig. 4.6 Residual stress introduced by shot peening of SAE 4340 steel, heat treated to $S_U = 1800$ MPa [2].

obtained by shot peening is shown in Figure 4.6 for a high-strength steel, which is fatigue sensitive.

Surface rolling is another process to plastically deform the material surface, see the discussion in Chapter 14 (Figure 14.10). It can be applied

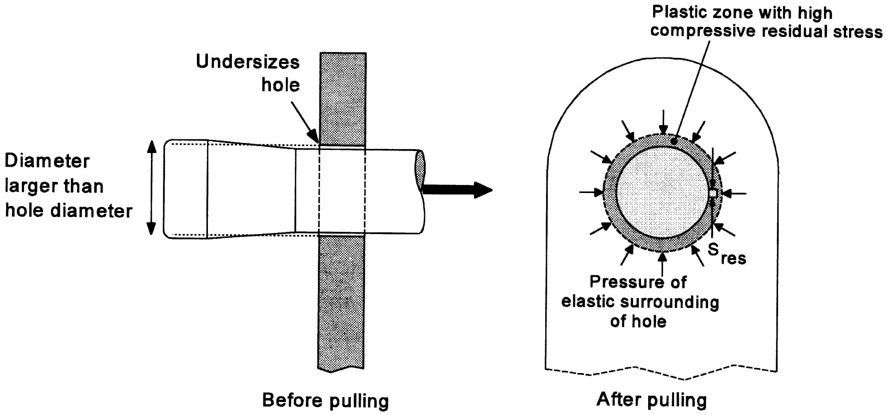

Fig. 4.7 Plastic hole expansion of a lug.

locally to the notch root area, e.g. to the root of fillet radii frequently present in axles. Rolling does not produce a rough surface.

Plastic hole expansion

Plastic hole expansion has been developed to improve the fatigue resistance of holes, also for bolted and riveted joints. The hole is drilled with a slightly undersized hole, e.g. with a diameter a few per cent smaller than the design value. A tapered pin is then pulled through the hole to expand the hole, see Figure 4.7. As a result, plastic deformation does occur around the hole. The plastic zone has been stretched tangentially because it was pushed outwards in the radial direction. The plastic zone has a larger diameter than before. It implies that the elastically strained material around this plastic zone will exert a pressure on the zone, see Figure 4.7. As a result tangential compressive stresses around the hole are introduced. The method is very effective for improving the fatigue resistance because the residual stresses can be high, i.e. almost in the order of the compressive yield stress. Moreover, the depth of the plastic zone can be a few millimeters (compare to the small depth in Figure 4.6). Small distortions of the cylindrical shape of the hole can be corrected afterwards by reaming, which hardly reduces the residual stress. Commercial apparatus has been developed for hole expansion, and a large favorable effect on fatigue can be obtained, see the discussion in Chapter 18.

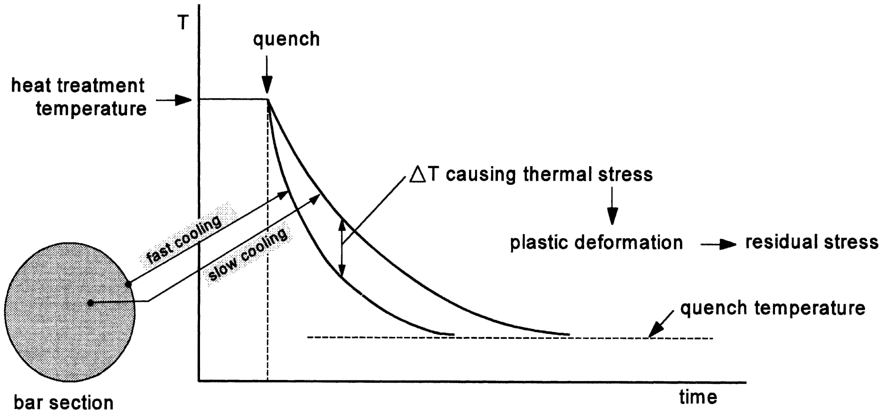

Fig. 4.8 Different cooling rates during quenching cause thermal stresses which can lead to a residual stress distribution.

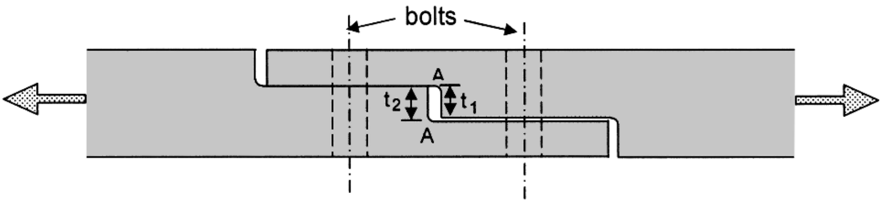

Fig. 4.9 Bolted joint with built-in stresses if $t1 \neq t2$.

Heat treatment

Quenching is an abrupt step of many heat treatments applied to alloys of various materials. Cooling usually occurs very fast at the outside of a component, see Figure 4.8, and significantly slower inside the material. The inhomogeneous cooling introduces thermal stresses. The faster thermal contraction at the outside causes local tensile stresses balanced by compressive stresses inside. At the still elevated temperature, the yield stress is low and plastic deformation can easily occur. Residual stresses are then introduced. In the rotational symmetric case of Figure 4.8 it should lead to the favorable situation of compressive residual stress at the outside balanced by tensile residual stress inside the material. Unfortunately, many components have complex shapes which makes it difficult to know the residual stress distribution obtained after quenching. Tensile residual stresses, also at the outside are possible. They can be reduced, or even reversed, by shot peening.

Assembling stresses

The previous examples of sources of residual stresses were associated with inhomogeneous plastic deformation. A completely different category of residual stresses in a structure is due to mounting of components to form a single structure. In many cases, bolted connections are involved. The residual stresses in the structure depend on the dimensional tolerances of the components. A simple example is shown in Figure 4.9. If $t1$ and $t2$ of this joint are not exactly equal, and the bolts are fastened, the misfit will introduce bending. Maximum internal stresses occur at the root of the fillet notches A in Figure 4.9 in the still unloaded joint. In this case the term "internal stresses" appears to be more correct. These stresses due to assembling a structure are also referred to as built-in stresses. The occurrence of the stresses can be avoided by a strict tolerance system.

In special cases, built-in stresses are desirable. This applies to bushes pushed with an interference fit into a hole, and to pretensioned bolts. These cases are discussed in Chapter 18.

4.3 Measurements or calculations of residual stresses

Residual stresses cannot directly be observed which is rather unfortunate because there are no simple techniques for measuring these stresses. A non-destructive measurement can be done by X-ray diffraction techniques, but it is a fairly elaborate method, which is not easily adopted on a routine basis. It becomes even more problematic if the residual stress has to be measured at the root of a notch, e.g. at the bore of a hole, where high strain gradients are present.

Destructive measurements are possible, but again it is not a simple routine procedure. Small strain gages are bonded to the locations of interest. Cuts in the material around the gages relax the residual strains. The strain variations indicated by the gages during this operation must be measured, and the residual stresses can then be calculated.

Calculations on residual stress distributions in notched elements can be made by FE analysis. A nonlinear elasto-plastic stress-strain relation must then be assumed. In the past simplification of the analysis was obtained by assuming that strain hardening did not occur. Later a linear strain hardening was adopted, and nowadays it is possible to employ a more realistic

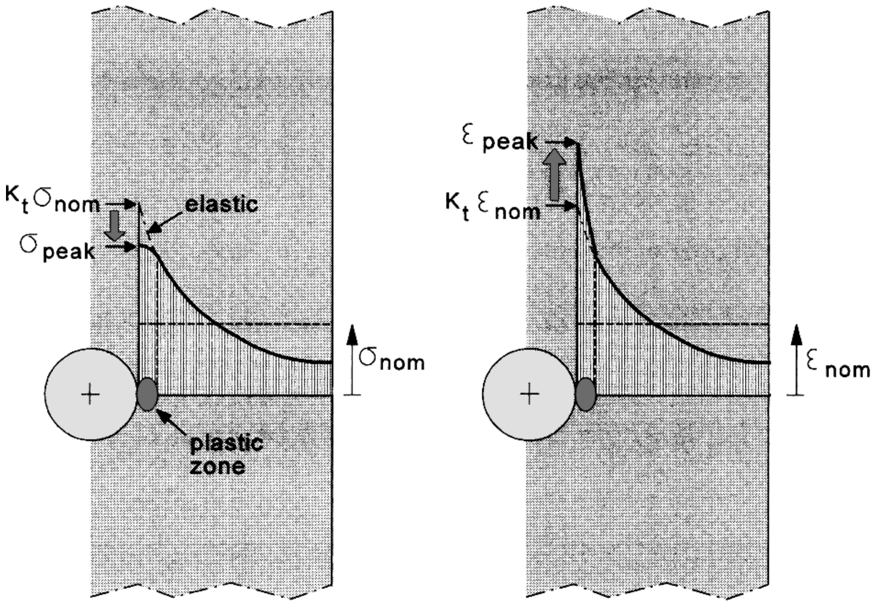

Fig. 4.10 Differences between the σ- and ε-distribution caused by a plastic zone at the root of a notch.

nonlinear strain hardening, e.g. the Ramberg–Osgood equation. However, elasto-plastic FE analysis requires expert experience.

Designers are not particularly fond of relying on good fatigue properties obtained by introducing favorable residual compressive stresses at fatigue critical locations. The argument is indeed that residual stresses cannot easily be measured. If residual compressive stresses at the material surface are still desirable, it is essential to have a closely controlled production process to introduce these stresses (e.g. by shot peening or hole expansion).

4.4 Estimation of the residual stress at a notch after a high load

In view of stress concentrations, it is possible that loads in service introduce a residual stress distribution at notches in the structure. These stresses are significant for crack nucleation, fatigue life, and fatigue damage accumulation in general. An analytical calculation of residual stresses around notches is practically impossible if plastic deformation occurs. Such calculations can be made with FE techniques, although they cannot be classified as simple calculations. However, a reasonable estimate of σ_{peak}

can be made with a simple procedure, based on a postulate of Neuber [3].[8] As long as plastic deformation does not occur, all strains are proportional to the applied load, which is Hooke's law. The shapes of the stress and strain distribution are not depending on the load. However, as soon as a plastic zone is created at the notch root, the shape of the two distributions will be changed. The stress at the notch root (σ_{peak}) is lower than the elastic prediction (Figure 4.10a) and the strain at the same location ($\varepsilon_{\text{peak}}$) is larger (Figure 4.10b). In other words:

$$\begin{aligned} \sigma_{\text{peak}} &< K_t \sigma_{\text{nom}} \\ \varepsilon_{\text{peak}} &> K_t \varepsilon_{\text{nom}} \end{aligned} \tag{4.5}$$

The fact that σ_{peak} is smaller than the elastic prediction is related to the other fact that $\varepsilon_{\text{peak}}$ is larger than the elastic prediction. According to the postulate of Neuber the product $\sigma_{\text{peak}}\varepsilon_{\text{peak}}$ still agrees with the elastic prediction:

$$\sigma_{\text{peak}}\varepsilon_{\text{peak}} = K_t^2 \sigma_{\text{nom}}\varepsilon_{\text{nom}} \tag{4.6}$$

It implies that in the product, σ_{peak} being smaller than predicted is compensated by $\varepsilon_{\text{peak}}$ being larger than predicted. Defining plastic concentration factors K_σ and K_ε as:

$$\begin{aligned} K_\sigma &= \sigma_{\text{peak}}/\sigma_{\text{nom}} \quad (< K_t) \\ K_\varepsilon &= \varepsilon_{\text{peak}}/\varepsilon_{\text{nom}} \quad (> K_t) \end{aligned} \tag{4.7}$$

the postulate of Neuber becomes:

$$K_\sigma K_\varepsilon = K_t^2 \tag{4.8}$$

Neuber has proven that the postulate is correct for a hyperbolic notch under shear loading. He then assumed that it will be approximately correct for other types of notches and loading. This was more or less confirmed empirically, provided the plastic zone is small.

Substitution of $\varepsilon_{\text{nom}} = \sigma_{\text{nom}}/E$ into Equation (4.6) leads to:

$$\sigma_{\text{peak}}\varepsilon_{\text{peak}} = (K_t \sigma_{\text{nom}})^2/E \tag{4.9}$$

For a given load and K_t, the right-hand side of Equation (4.9) has a known constant value. The equation thus gives one (hyperbolic) relation between

[8] A different proposal for the same problem was made by Glinka [4]. His analysis is based on energy density considerations.

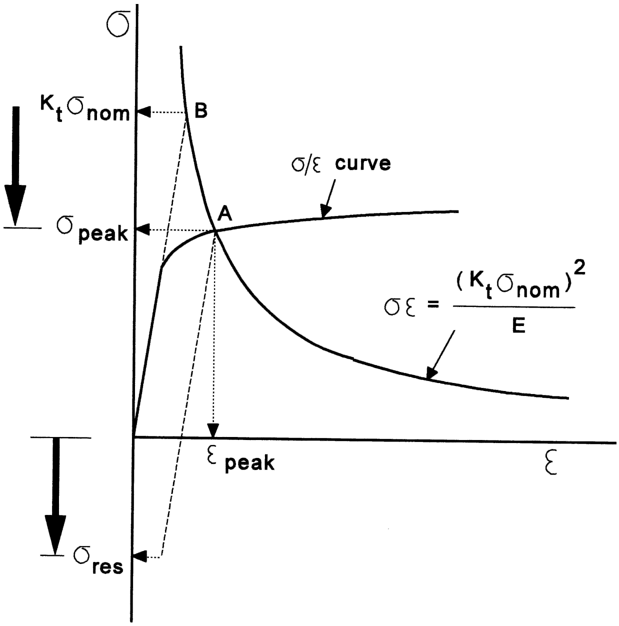

Fig. 4.11 Graphical method to determine σ_{peak} and σ_{residual}.

the unknown σ_{peak} and $\varepsilon_{\text{peak}}$. A second relation is necessary for which the stress-strain curve of the material obtained in a tensile test is used. The graphical solution then becomes as simple as shown in Figure 4.11. At the intersection point A of the two curves, the values of σ_{peak} and $\varepsilon_{\text{peak}}$ satisfy both relations. If no plasticity had occurred, the peak stress would have been found at point B. The difference between the two points B and A gives the reduction of the peak stress. After elastic unloading the residual stress is found as:

$$\sigma_{\text{residual}} = \sigma_A - \sigma_B = \sigma_A - K_t \sigma_{\text{nom}}$$

As a numerical example: $K_t = 2.5$, $\sigma_{\text{nom}} = 200$ MPa, $E = 210000$ (steel). Assume a bilinear σ-ε curve with a yield stress of 300 MPa and a plastic modulus $E_{\text{pl}} = E/20$.[9] According to the above equations the residual stress at the notch root becomes -176 MPa, which is a considerable compressive residual stress.

[9] It implies that the plastic σ-ε relation is: $\sigma - 300 = E/20 * (\varepsilon - 300/E)$. Substitution in Equation (4.9) leads to a second-order equation form which the solution is obtained as $\sigma_A = 323.6$ MPa. If the more realistic Ramberg–Osgood stress-strain relation is adopted the problem can also be solved numerically.

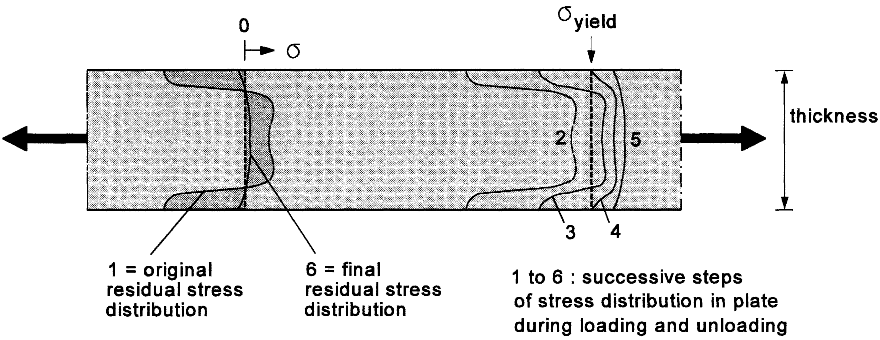

Fig. 4.12 Elimination of residual stress in a plate by plastic stretching.

4.5 How to remove residual stresses

It can be desirable to remove residual stresses. Two arguments are:

1. unfavorable residual tensile stresses can be harmful for fatigue and stress corrosion, and
2. dimensional distortions (warpage) can occur after machining.

A heat treatment may remove all residual stresses, especially if recrystallization occurs. The original permanent plastic deformation causing the residual stress system implies a high dislocation density. During a high temperature anneal, recrystallization removes the initial dislocation structure and a much lower dislocation density is obtained. The initial permanent plastic deformation (strain hardening) is removed, and as a consequence the residual stresses are eliminated. Annealing at a lower temperature will not always induce recrystallization, but certain rearrangements of the dislocation structure can reduce the residual stresses to some extent. Unfortunately, an annealing treatment is not always a feasible option because it can soften the material. The original heat treatment hardening may vanish.

As said before, unfavorable residual tensile stresses at the surface of a material can be reduced or reversed into favorable residual compressive stresses by shot peening. The surface quality obtained after peening can be rather rough, but usually the improvement by residual compressive stresses will prevail.

Another simple mechanical method to eliminate residual stresses from sheets and plates is to stretch the material to a small plastic strain. Figure 4.12 shows an originally inhomogeneous residual stress distribution, No. 1 in this figure. During stretching of the material to a stress level exceeding the yield stress plastic deformation leads to a more homogeneous stress distribution. Finally, when the yield stress is exceeded through the full thickness, the fairly homogeneous stress distribution No.5 is obtained. This distribution is retained during elastic unloading. After full unloading, the original residual stresses are practically eliminated. In the aluminum industry, such prestrained plates are produced. It is more expensive, but warpage problems in the workshop are prevented.

There is another interesting conclusion to be derived from Figure 4.12. If the stretching operation is continued until failure, the stress distribution before failure is approximately homogeneous. As a consequence, the strength of the material will not be affected by the original residual stresses. The same conclusion is also true for the static strength of a notched part. Actually pulling the central hole specimen in Figure 4.10 to failure is just a continuation of the high load that caused the residual stress and strain distribution of Figure 4.10 if unloading is done before failure.

4.6 Main topics of the present chapter

1. Residual stresses are usually caused by inhomogeneous plastic deformation. Due to permanent plastic deformation, the plastic zone no longer fits stress-free in the elastic surrounding, which introduces a residual stress system.
2. Residual stresses can be introduced on purpose (shotpeening, plastic hole expansion). They can also occur unintentionally (production processes, heat treatment). Another important source is assembling of components, which can cause significant built-in stresses.
3. A residual stress system is an equilibrium system. There is never a favorable compressive residual stress without an unfavorable tension residual stress at an other location.
4. Residual stresses can have a significant effect on fatigue and stress corrosion. During machining residual stresses can cause warpage.
5. Measuring of residual stresses at the root of a notch introduced by a high load is not a simple technique. An estimate can be obtained with a simple calculation technique. FE calculations of a residual stress distribution with an FE analysis is possible but it requires experience.

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