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Complex Analysis

Third Edition

Springer

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(1930–2007)

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Preface to the Third Edition

Beginning with the first edition of *Complex Analysis*, we have attempted to present the classical and beautiful theory of complex variables in the clearest and most intuitive form possible. The changes in this edition, which include additions to ten of the nineteen chapters, are intended to provide the additional insights that can be obtained by seeing a little more of the "big picture". This includes additional related results and occasional generalizations that place the results in a slightly broader context.

The Fundamental Theorem of Algebra is enhanced by three related results. Section 1.3 offers a detailed look at the solution of the cubic equation and its role in the acceptance of complex numbers. While there is no formula for determining the roots of a general polynomial, we added a section on Newton's Method, a numerical technique for approximating the zeroes of any polynomial. And the Gauss-Lucas Theorem provides an insight into the location of the zeroes of a polynomial and those of its derivative.

A series of new results relate to the mapping properties of analytic functions. A revised proof of Theorem 6.15 leads naturally to a discussion of the connection between critical points and saddle points in the complex plane. The proof of the Schwarz Reflection Principle has been expanded to include reflection across analytic arcs, which plays a key role in a new section (14.3) on the mapping properties of analytic functions on closed domains. And our treatment of special mappings has been enhanced by the inclusion of Schwarz-Christoffel transformations.

A single interesting application to number theory in the earlier editions has been expanded into a new section (19.4) which includes four examples from additive number theory, all united in their use of generating functions.

Perhaps the most significant changes in this edition revolve around the proof of the prime number theorem. There are two new sections (17.3 and 18.2) on Dirichlet series. With that background, a pivotal result on the Zeta function (18.10), which seemed to "come out of the blue", is now seen in the context of the analytic continuation of Dirichlet series. Finally the actual proof of the prime number theorem has been considerably revised. The original independent proofs by Hadamard and de la Vallée Poussin were both long and intricate. Donald Newman's 1980 article presented a dramatically simplified approach. Still the proof relied on several nontrivial number-theoretic results, due to Chebychev, which formed a separate appendix in the earlier editions. Over the years, further refinements of Newman's approach have been offered, the most recent of which is the award-winning 1997 article by Zagier. We followed Zagier's approach, thereby eliminating the need for a separate appendix, as the proof relies now on only one relatively straightforward result due of Chebychev.

The first edition contained no solutions to the exercises. In the second edition, responding to many requests, we included solutions to all exercises. This edition contains 66 new exercises, so that there are now a total of 300 exercises. Once again, in response to instructors' requests, while solutions are given for the majority of the problems, each chapter contains at least a few for which the solutions are not included. These are denoted with an asterisk.

Although Donald Newman passed away in 2007, most of the changes in this edition were anticipated by him and carry his imprimatur. I can only hope that all of the changes and additions approach the high standard he set for presenting mathematics in a lively and "simple" manner.

In an earlier edition of this text, it was my pleasure to thank my former student, Pisheng Ding, for his careful work in reviewing the exercises. In this edition, it is an even greater pleasure to acknowledge his contribution to many of the new results, especially those relating to the mapping properties of analytic functions on closed domains. This edition also benefited from the input of a new generation of students at City College, especially Maxwell Musser, Matthew Smedberg, and Edger Sterjo. Finally, it is a pleasure to acknowledge the careful work and infinite patience of Elizabeth Loew and the entire editorial staff at Springer.

Joseph Bak
City College of NY
April 2010

Preface to the Second Edition

One of our goals in writing this book has been to present the theory of analytic functions with as little dependence as possible on advanced concepts from topology and several-variable calculus. This was done not only to make the book more accessible to a student in the early stages of his/her mathematical studies, but also to highlight the authentic complex-variable methods and arguments as opposed to those of other mathematical areas. The minimum amount of background material required is presented, along with an introduction to complex numbers and functions, in Chapter 1.

Chapter 2 offers a somewhat novel, yet highly intuitive, definition of analyticity as it applies specifically to polynomials. This definition is related, in Chapter 3, to the Cauchy-Riemann equations and the concept of differentiability. In Chapters 4 and 5, the reader is introduced to a sequence of theorems on entire functions, which are later developed in greater generality in Chapters 6–8. This two-step approach, it is hoped, will enable the student to follow the sequence of arguments more easily. Chapter 5 also contains several results which pertain exclusively to entire functions.

The key result of Chapters 9 and 10 is the famous Residue Theorem, which is followed by many standard and some not-so-standard applications in Chapters 11 and 12.

Chapter 13 introduces conformal mapping, which is interesting in its own right and also necessary for a proper appreciation of the subsequent three chapters. Hydrodynamics is studied in Chapter 14 as a bridge between Chapter 13 and the Riemann Mapping Theorem. On the one hand, it serves as a nice application of the theory developed in the previous chapters, specifically in Chapter 13. On the other hand, it offers a physical insight into both the statement and the proof of the Riemann Mapping Theorem.

In Chapter 15, we use "mapping" methods to generalize some earlier results. Chapter 16 deals with the properties of harmonic functions and the related theory of heat conduction.

A second goal of this book is to give the student a feeling for the wide applicability of complex-variable techniques even to questions which initially do not seem to belong to the complex domain. Thus, we try to impart some of the enthusiasm apparent in the famous statement of Hadamard that "the shortest route between two truths in the real domain passes through the complex domain." The physical applications of Chapters 14 and 16 are good examples of this, as are the results of Chapter 11. The material in the last three chapters is designed to offer an even greater appreciation of the breadth of possible applications. Chapter 17 deals with the different forms an analytic function may take. This leads directly to the Gamma and Zeta functions discussed in Chapter 18. Finally, in Chapter 19, a potpourri of problems–again, some classical and some novel–is presented and studied with the techniques of complex analysis.

The material in the book is most easily divided into two parts: a first course covering the materials of Chapters 1–11 (perhaps including parts of Chapter 13), and a second course dealing with the later material. Alternatively, one seeking to cover the physical applications of Chapters 14 and 16 in a one-semester course could omit some of the more theoretical aspects of Chapters 8, 12, 14, and 15, and include them, with the later material, in a second-semester course.

The authors express their thanks to the many colleagues and students whose comments were incorporated into this second edition. Special appreciation is due to Mr. Pi-Sheng Ding for his thorough review of the exercises and their solutions. We are also indebted to the staff of Springer-Verlag Inc. for their careful and patient work in bringing the manuscript to its present form.

Joseph Bak
Donald J. Newmann

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