Undergraduate Texts in Mathematics

Undergraduate Texts in Mathematics

Undergraduate Texts in Mathematics are generally aimed at third- and fourth-year undergraduate mathematics students at North American universities. These texts strive to provide students and teachers with new perspectives and novel approaches. The books include motivation that guides the reader to an appreciation of interrelations among different aspects of the subject. They feature examples that illustrate key concepts as well as exercises that strengthen understanding.

For further volumes:
http://www.springer.com/series/666

William A. Adkins • Mark G. Davidson

Ordinary Differential Equations

Springer

William A. Adkins
Department of Mathematics
Louisiana State University
Baton Rouge, LA
USA

Mark G. Davidson
Department of Mathematics
Louisiana State University
Baton Rouge, LA
USA

ISSN 0172-6056
ISBN 978-1-4614-3617-1 ISBN 978-1-4614-3618-8 (eBook)
DOI 10.1007/978-1-4614-3618-8
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2012937994

Mathematics Subject Classification (2010): 34-01

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

This text is intended for the introductory three- or four-hour one-semester sophomore level differential equations course traditionally taken by students majoring in science or engineering. The prerequisite is the standard course in elementary calculus.

Engineering students frequently take a course on and use the Laplace transform as an essential tool in their studies. In most differential equations texts, the Laplace transform is presented, usually toward the end of the text, as an alternative method for the solution of constant coefficient linear differential equations, with particular emphasis on discontinuous or impulsive forcing functions. Because of its placement at the end of the course, this important concept is not as fully assimilated as one might hope for continued applications in the engineering curriculum. *Thus, a goal of the present text is to present the Laplace transform early in the text, and use it as a tool for motivating and developing much of the remaining differential equation concepts for which it is particularly well suited.*

There are several rewards for investing in an early development of the Laplace transform. The standard solution methods for constant coefficient linear differential equations are immediate and simplified. We are able to provide a proof of the existence and uniqueness theorems which are not usually given in introductory texts. The solution method for constant coefficient linear systems is streamlined, and we avoid having to introduce the notion of a defective or nondefective matrix or develop generalized eigenvectors. Even the Cayley–Hamilton theorem, used in Sect. 9.6, is a simple consequence of the Laplace transform. In short, the Laplace transform is an effective tool with surprisingly diverse applications.

Mathematicians are well aware of the importance of transform methods to simplify mathematical problems. For example, the Fourier transform is extremely important and has extensive use in more advanced mathematics courses. The wavelet transform has received much attention from both engineers and mathematicians recently. It has been applied to problems in signal analysis, storage and transmission of data, and data compression. We believe that students should be introduced to transform methods early on in their studies and to that end, the Laplace transform is particularly well suited for a sophomore level course in differential equations. It has been our experience that by introducing the Laplace transform near the beginning of the text, students become proficient in its use and comfortable with this important concept, while at the same time learning the standard topics in differential equations.

Chapter 1 is a conventional introductory chapter that includes solution techniques for the most commonly used first order differential equations, namely, separable and linear equations, and some substitutions that reduce other equations to one of these. There are also the Picard approximation algorithm and a description, without proof, of an existence and uniqueness theorem for first order equations.

Chapter 2 starts immediately with the introduction of the Laplace transform as an integral operator that turns a differential equation in t into an algebraic equation in another variable s. A few basic calculations then allow one to start solving some differential equations of order greater than one. The rest of this chapter develops the necessary theory to be able to efficiently use the Laplace transform. Some proofs, such as the injectivity of the Laplace transform, are delegated to an appendix. Sections 2.6 and 2.7 introduce the basic function spaces that are used to describe the solution spaces of constant coefficient linear homogeneous differential equations.

With the Laplace transform in hand, Chap. 3 efficiently develops the basic theory for constant coefficient linear differential equations of order 2. For example, the homogeneous equation $q(\boldsymbol{D})y = 0$ has the solution space $\mathcal{E}_q$ that has already been described in Sect. 2.6. The Laplace transform immediately gives a very easy procedure for finding the test function when teaching the method of undetermined coefficients. Thus, it is unnecessary to develop a rule-based procedure or the annihilator method that is common in many texts.

Chapter 4 extends the basic theory developed in Chap. 3 to higher order equations. All of the basic concepts and procedures naturally extend. If desired, one can simultaneously introduce the higher order equations as Chap. 3 is developed or very briefly mention the differences following Chap. 3.

Chapter 5 introduces some of the theory for second order linear differential equations that are not constant coefficient. Reduction of order and variation of parameters are topics that are included here, while Sect. 5.4 uses the Laplace transform to transform certain second order nonconstant coefficient linear differential equations into first order linear differential equations that can then be solved by the techniques described in Chap. 1.

We have broken up the main theory of the Laplace transform into two parts for simplicity. Thus, the material in Chap. 2 only uses continuous input functions, while in Chap. 6 we return to develop the theory of the Laplace transform for discontinuous functions, most notably, the step functions and functions with jump discontinuities that can be expressed in terms of step functions in a natural way. The Dirac delta function and differential equations that use the delta function are also developed here. The Laplace transform works very well as a tool for solving such differential equations. Sections 6.6–6.8 are a rather extensive treatment of periodic functions, their Laplace transform theory, and constant coefficient linear differential equations with periodic input function. These sections make for a good supplemental project for a motivated student.

Chapter 7 is an introduction to power series methods for linear differential equations. As a nice application of the Frobenius method, explicit Laplace inversion formulas involving rational functions with denominators that are powers of an irreducible quadratic are derived.

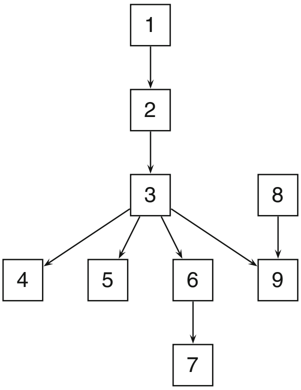
Chapter 8 is primarily included for completeness. It is a standard introduction to some matrix algebra that is needed for systems of linear differential equations. For those who have already had exposure to this basic algebra, it can be safely skipped or given as supplemental reading.

Chapter 9 is concerned with solving systems of linear differential equations. By the use of the Laplace transform, it is possible to give an explicit formula for the matrix exponential $e^{At} = \mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}$ that does not involve the use of eigenvectors or generalized eigenvectors. Moreover, we are then able to develop an efficient method for computing e^{At} known as Fulmer's method. Another thing which is somewhat unique is that we use the matrix exponential in order to solve a constant coefficient system $y' = Ay + f(t)$, $y(t_0) = y_0$ by means of an integrating factor. An immediate consequence of this is the existence and uniqueness theorem for higher order constant coefficient linear differential equations, a fact that is not commonly proved in texts at this level.

The text has numerous exercises, with answers to most odd-numbered exercises in the appendix. Additionally, a student solutions manual is available with solutions to most odd-numbered problems, and an instructors solution manual includes solutions to most exercises.

Chapter Dependence

The following diagram illustrates interdependence among the chapters.



Suggested Syllabi

The following table suggests two possible syllabi for one semester courses.

3-Hour Course	*4-Hour Course*	*Further Reading*
Sections 1.1–1.6	Sections 1.1–1.7	
Sections 2.1–2.8	Sections 2.1–2.8	
Sections 3.1–3.6	Sections 3.1–3.7	
Sections 4.1–4.3	Sections 4.1–4.4	Section 4.5
Sections 5.1–5.3, 5.6	Sections 5.1–5.6	
Sections 6.1–6.5	Sections 6.1–6.5	Sections 6.6–6.8
	Sections 7.1–7.3	Section 7.4
Sections 9.1–9.5	Sections 9.1–9.5, 9.7	Section 9.6
		Sections A.1, A.5

Chapter 8 is on matrix operations. It is not included in the syllabi given above since some of this material is sometimes covered by courses that precede differential equations. Instructors should decide what material needs to be covered for their students. The sections in the Further Reading column are written at a more advanced level. They may be used to challenge exceptional students.

We routinely provide a basic table of Laplace transforms, such as Tables 2.6 and 2.7, for use by students during exams.

Acknowledgments

We would like to express our gratitude to the many people who have helped to bring this text to its finish. We thank Frank Neubrander who suggested making the Laplace transform have a more central role in the development of the subject. We thank the many instructors who used preliminary versions of the text and gave valuable suggestions for its improvement. They include Yuri Antipov, Scott Baldridge, Blaise Bourdin, Guoli Ding, Charles Egedy, Hui Kuo, Robert Lipton, Michael Malisoff, Phuc Nguyen, Richard Oberlin, Gestur Olafsson, Boris Rubin, Li-Yeng Sung, Michael Tom, Terrie White, and Shijun Zheng. We thank Thomas Davidson for proofreading many of the solutions. Finally, we thank the many many students who patiently used versions of the text during its development.

Baton Rouge, Louisiana

William A. Adkins
Mark G. Davidson

Contents

List of Tables