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Stellar Structure and Evolution

Second Edition

Springer

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To Our Wives

Preface to the First Edition

The attempt to understand the physics of the structure of stars and their change in time – their evolution – has been bothering many physicists and astronomers ever since the last century. This long chain of successful research is well documented not only by numerous papers in the corresponding journals but also by a series of books. Some of them are so excellently written that despite their age they can still be recommended and not only as documents of the state of the art at that time. A few outstanding examples are the books of Emden (1907), Eddington (1926), Chandrasekhar (1939), and Schwarzschild (1958). But our science has rapidly expanded in the last few decades, and new aspects have emerged which could not even be anticipated, say, 30 years ago and which today have to be carefully explored.

This does not mean, however, that our ambition is to present a complete account of the latest and most refined numerical results. This can well be left to the large and growing number of excellent review articles. This book is intended rather to be a textbook that will help students and teachers to understand these results as far as possible and present them in a simple and clear manner. We know how difficult this is since we ourselves have tried for the largest part of our scientific career to understand "how the stars work" – and then to make others believe it. In these attempts we have found that often enough a simplified analytical example can be more helpful than the discussion of an exceptionally beautiful numerical solution. Therefore we do not hesitate to include many simple considerations and estimates, if necessary, even at the expense of rigour and the latest results. The reader should also note that the list of references given in this book is not intended to represent a table of honour for the (known and unknown) heroes of the theory of stellar structure; it is merely designed to help the beginner to find a few first paths in the literature jungle and presents those papers from which we have more or less randomly chosen the numbers for figures and numerical examples (There are others of at least the same quality!).

The choice of topics for a book such as this is difficult and certainly subject to personal preferences. Completeness is neither possible nor desirable. Still, one may wonder why we did not include, for example, binary stars, although we are obviously interested in their evolution. The reason is that here one would have had

to include the physics of essentially non-spherical objects (such as disks), while we concentrate mainly on spherical configurations; even in the brief description of rotation the emphasis is on small deviations from spherical symmetry.

This book would never have been completed without the kind and competent help of many friends and colleagues. We mention particularly Wolfgang Duschl and Peter Schneider who read critically through the whole manuscript; Norman Baker, Gerhard Börner, Mounib El Eid, Wolfgang Hillebrandt, Helmuth Kahler, Ewald Müller, Henk Spruit, Joachim Wambsganß, and many others read through particular chapters and gave us their valuable advice. In fact it would probably be simpler to give a complete list of those of our colleagues who have *not* contributed than of those who helped us.

In addition we have to thank many secretaries at our institutes; several have left their jobs (for other reasons!) during the five years in which we kept them busy. Most of this work was done by Cornelia Rickl and Petra Berkemeyer in Munich and Christa Leppien and Heinke Heise in Hamburg, while Gisela Wimmersberger prepared all the graphs. We are grateful to them all.

Finally we wish to thank Springer-Verlag for their enthusiastic cooperation.

Munich and Hamburg — Rudolf Kippenhahn
December 1989 — Alfred Weigert

Preface to the Second Edition

Twenty years after its first publication, this textbook is still a major reference for scientists and students interested in or working on problems of stellar structure and evolution. But with the incredible growth of computational power, the computation of stellar models has to large extent become a standard tool for astrophysics. While the early computations were restricted to single choices for mass, compositions and possibly evolutionary stage, by now models for the whole parameter space exist. The first edition of this book was restricted to a few examples for low- and intermediate-mass star evolution and lacked the broader view now being possible. There are even semi-automatic stellar evolution codes that may be used remotely via the Internet.

However, stellar evolution programs should not be used without a thorough understanding of the stellar physics. Therefore, a textbook concentrating on the foundations of the theory and explaining in detail specific phases and events in the life of a star is very much needed to allow scientifically solid modelling of stars. This is the reason why this book deserved a second edition.

Much to our regret, A. Weigert passed away two years after publication of the first edition. He left a gap that cannot be filled. Given the above mentioned need for a second edition and the requirement to add up-to-date stellar models, it was decided to have A. Weiss join R. Kippenhahn in preparing the new edition.

The two authors of this book came to discriminate between the *eternal truth* and the *mutable* parts. The latter ones refer to the current state of modelling and knowledge obtained from numerical models and their comparison to observations. Such chapters were updated, extended, or added. As far as possible, the stellar models shown were specifically calculated for this purpose, with the present, much evolved version of the original code by Kippenhahn, Weigert, and Hofmeister. The numerical results are therefore much more homogeneous and consistent than in the first edition.

The *eternal truth* concerns the aforementioned basic physics and their understanding. These parts of the book have been left almost untouched, since the authors (and those readers who were consulted) did not see any reason to change them.

The authors are indebted to many friends and colleagues who gave their advice or comments, with respect to both necessary changes and the new text passages.

The support of Santi Cassisi, Jørgen Christensen-Dalsgaard, Wolfgang Hillebrandt, Thomas Janka, Ralf Klessen, Ewald Müller, Hans Ritter, Maurizio Salaris, and Helmut Schlattl was essential for us.

We are also very grateful to all those colleagues who very generously provided their own data to help filling gaps that we could not fill with our own models. They were (again in alphabetical order) Leandro Althaus, Isabelle Baraffe, Raphael Hirschi, Marco Limongi, Marcelo Miller Bertolami, Aldo Serenelli, and Lionel Siess. Needless to say, their data also came with much wanted and helpful advice and sometimes fruitful scientific discussions about details of the models.

Norbert Grüner's help in the difficult task of generating a useful index is acknowledged, too.

Last, but not least, we thank Mrs. Rosmarie Mayr-Ihbe, who designed, corrected, and improved the many figures that we added to this second edition.

Garching Achim Weiss
February 2012

Contents

Part I The Basic Equations

1 Coordinates, Mass Distribution, and Gravitational Field in Spherical Stars 3
1.1 Eulerian Description 3
1.2 Lagrangian Description 4
1.3 The Gravitational Field 6

2 Conservation of Momentum 9
2.1 Hydrostatic Equilibrium 9
2.2 The Role of Density and Simple Solutions 10
2.3 Simple Estimates of Central Values P_c, T_c 12
2.4 The Equation of Motion for Spherical Symmetry 13
2.5 The Non-spherical Case 15
2.6 Hydrostatic Equilibrium in General Relativity 15
2.7 The Piston Model 17

3 The Virial Theorem 19
3.1 Stars in Hydrostatic Equilibrium 19
3.2 The Virial Theorem of the Piston Model 21
3.3 The Kelvin–Helmholtz Timescale 22
3.4 The Virial Theorem for Non-vanishing Surface Pressure 23

4 Conservation of Energy 25
4.1 Thermodynamic Relations 25
4.2 The Perfect Gas and the Mean Molecular Weight 28
4.3 Thermodynamic Quantities for the Perfect, Monatomic Gas 30
4.4 Energy Conservation in Stars 31
4.5 Global and Local Energy Conservation 33
4.6 Timescales 35

5 Transport of Energy by Radiation and Conduction 37
5.1 Radiative Transport of Energy 37
5.1.1 Basic Estimates 37
5.1.2 Diffusion of Radiative Energy 38
5.1.3 The Rosseland Mean for κ_ν 40
5.2 Conductive Transport of Energy 42
5.3 The Thermal Adjustment Time of a Star 43
5.4 Thermal Properties of the Piston Model 45

6 Stability Against Local, Non-spherical Perturbations 47
6.1 Dynamical Instability 47
6.2 Oscillation of a Displaced Element 52
6.3 Vibrational Stability 54
6.4 The Thermal Adjustment Time 55
6.5 Secular Instability 56
6.6 The Stability of the Piston Model 58

7 Transport of Energy by Convection 61
7.1 The Basic Picture 62
7.2 Dimensionless Equations 65
7.3 Limiting Cases, Solutions, Discussion 66
7.4 Extensions of the Mixing-Length Theory 70

8 The Chemical Composition 73
8.1 Relative Mass Abundances 73
8.2 Variation of Composition with Time 74
8.2.1 Radiative Regions 74
8.2.2 Diffusion 76
8.2.3 Convective Regions 80

9 Mass Loss 83

Part II The Overall Problem

10 The Differential Equations of Stellar Evolution 89
10.1 The Full Set of Equations 89
10.2 Timescales and Simplifications 91

11 Boundary Conditions 93
11.1 Central Conditions 93
11.2 Surface Conditions 95
11.3 Influence of the Surface Conditions and Properties of Envelope Solutions 98
11.3.1 Radiative Envelopes 98
11.3.2 Convective Envelopes 101
11.3.3 Summary 102
11.3.4 The $T-r$ Stratification 102

12 Numerical Procedure 105
12.1 The Shooting Method 105
12.2 The Henyey Method 106
12.3 Treatment of the First- and Second-Order Time Derivatives 113
12.4 Treatment of the Diffusion Equation 115
12.5 Treatment of Mass Loss 117
12.6 Existence and Uniqueness 118

Part III Properties of Stellar Matter

13 The Perfect Gas with Radiation 123
13.1 Radiation Pressure 123
13.2 Thermodynamic Quantities 124

14 Ionization 127
14.1 The Boltzmann and Saha Formulae 127
14.2 Ionization of Hydrogen 130
14.3 Thermodynamical Quantities for a Pure Hydrogen Gas 132
14.4 Hydrogen–Helium Mixtures 133
14.5 The General Case 135
14.6 Limitation of the Saha Formula 137

15 The Degenerate Electron Gas 139
15.1 Consequences of the Pauli Principle 139
15.2 The Completely Degenerate Electron Gas 140
15.3 Limiting Cases 144
15.4 Partial Degeneracy of the Electron Gas 145

16 The Equation of State of Stellar Matter 151
16.1 The Ion Gas 151
16.2 The Equation of State 152
16.3 Thermodynamic Quantities 154
16.4 Crystallization 157
16.5 Neutronization 158
16.6 Real Gas Effects 159

17 Opacity 163
17.1 Electron Scattering 163
17.2 Absorption Due to Free–Free Transitions 164
17.3 Bound–Free Transitions 165
17.4 Bound–Bound Transitions 166
17.5 The Negative Hydrogen Ion 168
17.6 Conduction 169
17.7 Molecular Opacities 170
17.8 Opacity Tables 172

18 Nuclear Energy Production 175
18.1 Basic Considerations 175
18.2 Nuclear Cross Sections 179
18.3 Thermonuclear Reaction Rates 182
18.4 Electron Shielding 188
18.5 The Major Nuclear Burning Stages 192
18.5.1 Hydrogen Burning 193
18.5.2 Helium Burning 197
18.5.3 Carbon Burning and Beyond 199
18.6 Neutron-Capture Nucleosynthesis 201
18.7 Neutrinos 205

Part IV Simple Stellar Models

19 Polytropic Gaseous Spheres 213
19.1 Polytropic Relations 213
19.2 Polytropic Stellar Models 215
19.3 Properties of the Solutions 216
19.4 Application to Stars 218
19.5 Radiation Pressure and the Polytrope $n = 3$ 219
19.6 Polytropic Stellar Models with Fixed K 220
19.7 Chandrasekhar's Limiting Mass 221
19.8 Isothermal Spheres of an Ideal Gas 222
19.9 Gravitational and Total Energy for Polytropes 224
19.10 Supermassive Stars 226
19.11 A Collapsing Polytrope 227

20 Homology Relations 233
20.1 Definitions and Basic Relations 233
20.2 Applications to Simple Material Functions 237
20.2.1 The Case $\delta = 0$ 237
20.2.2 The Case $\alpha = \delta = \varphi = 1, a = b = 0$ 237
20.2.3 The Role of the Equation of State 239
20.3 Homologous Contraction 241

21 Simple Models in the U–V Plane 243
21.1 The U–V Plane 243
21.2 Radiative Envelope Solutions 246
21.3 Fitting of a Convective Core 248
21.4 Fitting of an Isothermal Core 250

22 The Zero-Age Main Sequence 251
22.1 Surface Values 251
22.2 Interior Solutions 254
22.3 Convective Regions 258
22.4 Extreme Values of M 260
22.5 The Eddington Luminosity 261

23 Other Main Sequences 263
23.1 The Helium Main Sequence 263
23.2 The Carbon Main Sequence 266
23.3 Generalized Main Sequences 267

24 The Hayashi Line 271
24.1 Luminosity of Fully Convective Models 272
24.2 A Simple Description of the Hayashi Line 273
24.3 The Neighbourhood of the Hayashi Line and the Forbidden Region 276
24.4 Numerical Results 279
24.5 Limitations for Fully Convective Models 281

25 Stability Considerations 283
25.1 General Remarks 283
25.2 Stability of the Piston Model 285
25.2.1 Dynamical Stability 285
25.2.2 Inclusion of Non-adiabatic Effects 286
25.3 Stellar Stability 288
25.3.1 Perturbation Equations 289
25.3.2 Dynamical Stability 290
25.3.3 Non-adiabatic Effects 292
25.3.4 The Gravothermal Specific Heat 293
25.3.5 Secular Stability Behaviour of Nuclear Burning 294

Part V Early Stellar Evolution

26 The Onset of Star Formation 299
26.1 The Jeans Criterion 299
26.1.1 An Infinite Homogeneous Medium 299
26.1.2 A Plane-Parallel Layer in Hydrostatic Equilibrium 302
26.2 Instability in the Spherical Case 303
26.3 Fragmentation 307

27 The Formation of Protostars 311
27.1 Free-Fall Collapse of a Homogeneous Sphere 311
27.2 Collapse onto a Condensed Object 313
27.3 A Collapse Calculation 314
27.4 The Optically Thin Phase and the Formation of a Hydrostatic Core 315
27.5 Core Collapse 317
27.6 Evolution in the Hertzsprung–Russell Diagram 320

28 Pre-Main-Sequence Contraction 323
28.1 Homologous Contraction of a Gaseous Sphere 323
28.2 Approach to the Zero-Age Main Sequence 326

29 From the Initial to the Present Sun 329
29.1 Known Solar Data 329
29.2 Choosing the Initial Model 331
29.3 A Standard Solar Model 333
29.4 Results of Helioseismology 336
29.5 Solar Neutrinos 338

30 Evolution on the Main Sequence 343
30.1 Change in the Hydrogen Content 343
30.2 Evolution in the Hertzsprung–Russell Diagram 346
30.3 Timescales for Central Hydrogen Burning 347
30.4 Complications Connected with Convection 348
30.4.1 Convective Overshooting 349
30.4.2 Semiconvection 354
30.5 The Schönberg–Chandrasekhar Limit 356
30.5.1 A Simple Approach: The Virial Theorem and Homology 358
30.5.2 Integrations for Core and Envelope 360
30.5.3 Complete Solutions for Stars with Isothermal Cores 361

Part VI Post-Main-Sequence Evolution

31 Evolution Through Helium Burning: Intermediate-Mass Stars 367
31.1 Crossing the Hertzsprung Gap 367
31.2 Central Helium Burning 371
31.3 The Cepheid Phase 375
31.4 To Loop or Not to Loop 378
31.5 After Central Helium Burning 384

32 Evolution Through Helium Burning: Massive Stars 385
32.1 Semiconvection 385
32.2 Overshooting 387
32.3 Mass Loss 389

33 Evolution Through Helium Burning: Low-Mass Stars 391
33.1 Post-Main-Sequence Evolution 391
33.2 Shell-Source Homology 392
33.3 Evolution Along the Red Giant Branch 397
33.4 The Helium Flash 401
33.5 Numerical Results for the Helium Flash 402
33.6 Evolution After the Helium Flash 407
33.7 Evolution from the Zero-Age Horizontal Branch 410

Part VII Late Phases of Stellar Evolution

34 Evolution on the Asymptotic Giant Branch 417
34.1 Nuclear Shells on the Asymptotic Giant Branch 417
34.2 Shell Sources and Their Stability 419

34.3 Thermal Pulses of a Shell Source 422
34.4 The Core-Mass-Luminosity Relation for Large Core Masses 424
34.5 Nucleosynthesis on the AGB 426
34.6 Mass Loss on the AGB 430
34.7 A Sample AGB Evolution 433
34.8 Super-AGB Stars 436
34.9 Post-AGB Evolution 438

35 Later Phases of Core Evolution 439
35.1 Nuclear Cycles 439
35.2 Evolution of the Central Region 441

36 Final Explosions and Collapse 449
36.1 The Evolution of the CO-Core 450
36.2 Carbon Ignition in Degenerate Cores 454
36.2.1 The Carbon Flash 454
36.2.2 Nuclear Statistical Equilibrium 455
36.2.3 Hydrostatic and Convective Adjustment 458
36.2.4 Combustion Fronts 459
36.2.5 Carbon Burning in Accreting White Dwarfs 461
36.3 Collapse of Cores of Massive Stars 461
36.3.1 Simple Collapse Solutions 462
36.3.2 The Reflection of the Infall 465
36.3.3 Effects of Neutrinos 466
36.3.4 Electron-Capture Supernovae 469
36.3.5 Pair-Creation Instability 469
36.4 The Supernova-Gamma-Ray-Burst Connection 471

Part VIII Compact Objects

37 White Dwarfs 475
37.1 Chandrasekhar's Theory 475
37.2 The Corrected Mechanical Structure 479
37.2.1 Crystallization 480
37.2.2 Pycnonuclear Reactions 482
37.2.3 Inverse β Decays 483
37.2.4 Nuclear Equilibrium 483
37.3 Thermal Properties and Evolution of White Dwarfs 487

38 Neutron Stars 497
38.1 Cold Matter Beyond Neutron Drip 497
38.2 Models of Neutron Stars 501

39 Black Holes 509

Part IX Pulsating Stars

40 Adiabatic Spherical Pulsations 519
40.1 The Eigenvalue Problem 519
40.2 The Homogeneous Sphere 523
40.3 Pulsating Polytropes 525

41 Non-adiabatic Spherical Pulsations 529
41.1 Vibrational Instability of the Piston Model 529
41.2 The Quasi-adiabatic Approximation 531
41.3 The Energy Integral 532
41.3.1 The κ Mechanism 534
41.3.2 The ε Mechanism 534
41.4 Stars Driven by the κ Mechanism: The Instability Strip 535
41.5 Stars Driven by the ε Mechanism 541

42 Non-radial Stellar Oscillations 543
42.1 Perturbations of the Equilibrium Model 543
42.2 Normal Modes and Dimensionless Variables 545
42.3 The Eigenspectra 548
42.4 Stars Showing Non-radial Oscillations 552

Part X Stellar Rotation

43 The Mechanics of Rotating Stellar Models 557
43.1 Uniformly Rotating Liquid Bodies 557
43.2 The Roche Model 560
43.3 Slowly Rotating Polytropes 562

44 The Thermodynamics of Rotating Stellar Models 565
44.1 Conservative Rotation 565
44.2 Von Zeipel's Theorem 566
44.3 Meridional Circulation 567
44.4 The Non-conservative Case 569
44.5 The Eddington–Sweet Timescale 570
44.6 Meridional Circulation in Inhomogeneous Stars 573

45 The Angular-Velocity Distribution in Stars 575
45.1 Viscosity 575
45.2 Dynamical Stability 577
45.3 Secular Stability 582

References 587

Index 595