Undergraduate Texts in Mathematics

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Undergraduate Texts in Mathematics are generally aimed at third- and fourth-year undergraduate mathematics students at North American universities. These texts strive to provide students and teachers with new perspectives and novel approaches. The books include motivation that guides the reader to an appreciation of interrelations among different aspects of the subject. They feature examples that illustrate key concepts as well as exercises that strengthen understanding.

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Elementary Analysis

The Theory of Calculus

Second Edition

In collaboration with Jorge M. López, University of
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Springer

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Preface

Preface to the First Edition A study of this book, and especially the exercises, should give the reader a thorough understanding of a few basic concepts in analysis such as continuity, convergence of sequences and series of numbers, and convergence of sequences and series of functions. An ability to read and write proofs will be stressed. A precise knowledge of definitions is essential. The beginner should memorize them; such memorization will help lead to understanding.

Chapter 1 sets the scene and, except for the completeness axiom, should be more or less familiar. Accordingly, readers and instructors are urged to move quickly through this chapter and refer back to it when necessary. The most critical sections in the book are §§7–12 in Chap. 2. If these sections are thoroughly digested and understood, the remainder of the book should be smooth sailing.

The first four chapters form a unit for a short course on analysis. I cover these four chapters (except for the enrichment sections and §20) in about 38 class periods; this includes time for quizzes and examinations. For such a short course, my philosophy is that the students are relatively comfortable with derivatives and integrals but do not really understand sequences and series, much less sequences and series of functions, so Chaps. 1–4 focus on these topics. On two or three occasions, I draw on the Fundamental Theorem of Calculus or the Mean Value Theorem, which appears later in the book, but of course these important theorems are at least discussed in a standard calculus class.

In the early sections, especially in Chap. 2, the proofs are very detailed with careful references for even the most elementary facts. Most sophisticated readers find excessive details and references a hindrance (they break the flow of the proof and tend to obscure the main ideas) and would prefer to check the items mentally as they proceed. Accordingly, in later chapters, the proofs will be somewhat less detailed, and references for the simplest facts will often be omitted. This should help prepare the reader for more advanced books which frequently give very brief arguments.

Mastery of the basic concepts in this book should make the analysis in such areas as complex variables, differential equations, numerical analysis, and statistics more meaningful. The book can also serve as a foundation for an in-depth study of real analysis given in books such as [4, 33, 34, 53, 62, 65] listed in the bibliography.

Readers planning to teach calculus will also benefit from a careful study of analysis. Even after studying this book (or writing it), it will not be easy to handle questions such as "What is a number?" but at least this book should help give a clearer picture of the subtleties to which such questions lead.

The enrichment sections contain discussions of some topics that I think are important or interesting. Sometimes the topic is dealt with lightly, and suggestions for further reading are given. Though these sections are not particularly designed for classroom use, I hope that some readers will use them to broaden their horizons and see how this material fits in the general scheme of things.

I have benefitted from numerous helpful suggestions from my colleagues Robert Freeman, William Kantor, Richard Koch, and John Leahy and from Timothy Hall, Gimli Khazad, and Jorge López. I have also had helpful conversations with my wife Lynn concerning grammar and taste. Of course, remaining errors in grammar and mathematics are the responsibility of the author.

Several users have supplied me with corrections and suggestions that I've incorporated in subsequent printings. I thank them all,

including Robert Messer of Albion College, who caught a subtle error in the proof of Theorem 12.1.

Preface to the Second Edition After 32 years, it seemed time to revise this book. Since the first edition was so successful, I have retained the format and material from the first edition. The numbering of theorems, examples, and exercises in each section will be the same, and new material will be added to some of the sections. Every rule has an exception, and this rule is no exception. In §11, a theorem (Theorem 11.2) has been added, which allows the simplification of four almost-identical proofs in the section: Examples 3 and 4, Theorem 11.7 (formerly Corollary 11.4), and Theorem 11.8 (formerly Theorem 11.7).

Where appropriate, the presentation has been improved. See especially the proof of the Chain Rule 28.4, the shorter proof of Abel's Theorem 26.6, and the shorter treatment of decimal expansions in §16. Also, a few examples have been added, a few exercises have been modified or added, and a couple of exercises have been deleted.

Here are the main additions to this revision. The proof of the irrationality of e in §16 is now accompanied by an elegant proof that π is also irrational. Even though this is an "enrichment" section, it is especially recommended for those who teach or will teach precollege mathematics. The Baire Category Theorem and interesting consequences have been added to the enrichment §21. Section 31, on Taylor's Theorem, has been overhauled. It now includes a discussion of Newton's method for approximating zeros of functions, as well as its cousin, the secant method. Proofs are provided for theorems that guarantee when these approximation methods work. Section 35 on Riemann-Stieltjes integrals has been improved and expanded. A new section, §38, contains an example of a continuous nowhere-differentiable function and a theorem that shows "most" continuous functions are nowhere differentiable. Also, each of §§22, 32, and 33 has been modestly enhanced.

It is a pleasure to thank many people who have helped over the years since the first edition appeared in 1980. This includes David M. Bloom, Robert B. Burckel, Kai Lai Chung, Mark Dalthorp (grandson), M. K. Das (India), Richard Dowds, Ray Hoobler, Richard M. Koch, Lisa J. Madsen, Pablo V. Negrón Marrero (Puerto Rico), Rajiv Monsurate (India), Theodore W. Palmer, Jürg Rätz (Switzerland), Peter Renz, Karl Stromberg, and Jesús Sueiras (Puerto Rico).

Special thanks go to my collaborator, Jorge M. López, who provided a huge amount of help and support with the revision. Working with him was also a lot of fun. My plan to revise the book was supported from the beginning by my wife, Ruth Madsen Ross. Finally, I thank my editor at Springer, Kaitlin Leach, who was attentive to my needs whenever they arose.

Especially for the Student: Don't be dismayed if you run into material that doesn't make sense, for whatever reason. It happens to all of us. Just tentatively accept the result as true, set it aside as something to return to, and forge ahead. Also, don't forget to use the Index or Symbols Index if some terminology or notation is puzzling.

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