Statics and Mechanics of Structures

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Preface

The theory of statics of structures has developed from intuition via gradual refinement to its current state, where the basic principles are put into a systematic framework that enables precise analysis. Although the basic laws governing statics of structures have been known for several centuries, the methods of analysis have developed considerably over the last decades. At the current state of this development an introductory book on statics should aim at the dual goal of providing sufficient background for developing an intuitive understanding of structures, and at the same time lay a solid foundation for modern analysis, typically made by computational techniques. In this vein the present book makes extensive use of simple but realistic examples to develop familiarity and understanding of how structures carry and distribute the loads through the structural members to the supports. This is then supplemented by a few simple computer programs that illustrate, how the theories for trusses and frames are implemented, and open up to a more general approach to computational mechanics as a natural extension of the present book.

The book is organized as follows. The first five chapters build up a basic understanding of the statics of structures. It starts with force systems and reactions in Chapter 1, then proceeding to the intuitively very accessible theory of trusses, first analyzed by hand calculation procedures and then reformulated as a small systematic finite element program MiniTruss in Chapter 2. Chapter 3 develops the statics of beams and introduces the concept of internal forces. The internal forces are then related to deformation mechanisms of curvature, shear and extension in Chapter 4, and the principle of virtual work is developed in a concise form and used for calculation of specific displacements. The introductory part is rounded off in Chapter 5 on the analysis of columns, describing instability as a bifurcation problem, solved by eigenvalue analysis, and design principles based on the existence of a characteristic imperfection. This part of the book covers material suitable for an introductory one-semester course on basic statics of structures.

The remaining six chapters treat various extensions, that are typically included in one form or another in a second semester course. The Chapters 6 and 7 deal with analysis of statically indeterminate frame structures. The

first of these chapters gives a systematic development of the force method and describes how simple structures can conveniently be analyzed by hand. The following chapter then develops the deformation method in which the displacements of individual nodes play the key role. This then serves to introduce the idea of the finite element formulation of frame structures. This development is supported by the small program MiniFrame for internal forces and displacements, and an extension MiniFrameS for linearized stability analysis. The Chapters 8 and 9 introduce three-dimensional states of stress and strain, and present the theory of linear elasticity and some common failure conditions. This material provides the background for the Chapters 10 and 11, in which the simple two-dimensional beam theory used in the previous chapters is extended to flexure and torsion of non-symmetric beams, and the associated shear stress distributions.

The three small computer programs are coded in Matlab. The syntax and input structure are described in connection with the corresponding theory in the text, and the code is available from the authors via e-mail.

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