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Graduate Texts in Physics

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Introduction to Mathematica[®] for Physicists

 Springer

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Preface

Computer algebra systems are widely used in pure and applied mathematics, physics, and other natural sciences, engineering, economics, as well as in higher and secondary education (see, e.g., [1–5]). For example, many important calculations in theoretical physics could never be done by hand, without wide use of computer algebra. Polynomial or trigonometric manipulations using paper and pen are becoming as obsolete as school long division in the era of calculators.

There are several powerful general-purpose computer algebra systems. The system *Mathematica* is most popular. It contains a huge amount of mathematical knowledge in its libraries. The fundamental book on this system [6] has more than 1,200 pages. Fortunately, the same information (more up-to-date than in a printed book) is available in the help system and hence is always at the fingertips of any user. Many books about *Mathematica* and its application in various areas have been published; see, for example, the series [7–10] of four books (each more than 1,000 pages long) or [11]. The present book does not try to replace these manuals. Its first part is a short systematic introduction to computer algebra and *Mathematica*; it can (and should) be read sequentially. The second part is a set of unrelated examples from physics and mathematics which can be studied selectively and in any order. Having understood the statement of a problem, try to solve it yourself. Have a look at the book to get a hint only when you get stuck. Explanations in this part are quite short.

This book¹ is a result of teaching at the physics department of Novosibirsk State University. Starting from 2004, the course “Symbolic and numeric computations in physics applications” is given to students preparing for M.Sc., and an introduction to *Mathematica* is the first part of this course (the second part is mainly devoted to Monte Carlo methods). Practical computer classes form a required (and most important) part of the course. Most students have no problems with mastering the basics of *Mathematica* and applying it to problems in their own areas of interest.

The book describes *Mathematica* 9. Most of the material is applicable to other versions too. The *Mathematica* Book (fifth edition) [6], as well as, e.g., the book

¹ Work partially supported by the Russian Ministry of Education and Science.

series [7–10], describes *Mathematica* 5. The main source of up-to-date information is the *Mathematica* Help system.

The whole book (except Lecture 1 and Problems for students) consists of *Mathematica* notebooks. They can be found at

<http://www.inp.nsk.su/~grozin/mma/mma.zip>

The zip file is password protected. The password is the last sentence of Lecture 7 (case-sensitive, including the trailing period). The reader is encouraged to experiment with these notebook files. In the printed version of the book, plots use different curve styles (dashed, dotted, etc.) instead of colors.

The book will be useful for students, Ph.D. students, and researchers in the area of physics (and other natural sciences) and mathematics.

Novosibirsk, Russia

Andrey Grozin

Contents

Preface	v
Part I Lectures	
1 Computer Algebra Systems	3
2 Overview of <i>Mathematica</i>	9
2.1 Symbols	9
2.2 Numbers	10
2.3 Polynomials and Rational Functions	10
2.4 Elementary Functions	12
2.5 Calculus	13
2.6 Lists	14
2.7 Plots	15
2.8 Substitutions	17
2.9 Equations	18
3 Expressions	21
3.1 Atoms	21
3.2 Composite Expressions	22
3.3 Queries	24
3.4 Forms of an Expression	25
4 Patterns and Substitutions	27
4.1 Simple Patterns	27
4.2 One-Shot and Repeated Substitutions	28
4.3 Products	29
4.4 Sums	31
4.5 Conditions	32
4.6 Variable Number of Arguments	33

5	Functions	35
	5.1 Immediate and Delayed Assignment	35
	5.2 Functions	36
	5.3 Functions Remembering Their Values	36
	5.4 Fibonacci Numbers	37
	5.5 Functions from Expressions	38
	5.6 Antisymmetric Functions	39
	5.7 Functions with Options	40
	5.8 Attributes	40
	5.9 Upvalues	41
6	<i>Mathematica</i> as a Programming Language	43
	6.1 Compound Expressions	43
	6.2 Conditional Expressions	44
	6.3 Loops	46
	6.4 Functions	47
	6.5 Local Variables	49
	6.6 Table	50
	6.7 Parallelization	50
	6.8 Functions with an Index	51
	6.9 Hold and Evaluate	51
7	Gröbner Bases	55
	7.1 Statement of the Problem	55
	7.2 Monomial Orders	55
	7.3 Reduction of Polynomials	56
	7.4 S-Polynomials	57
	7.5 Buchberger Algorithm	58
	7.6 Is the System Compatible?	59
	7.7 Gröbner Bases with Respect to Lexicographic Order	60
	7.8 Is the Number of Solutions Finite?	61
8	Calculus	63
	8.1 Series	63
	8.2 Differentiation	67
	8.3 Integration	68
	8.4 Summation	70
	8.5 Differential Equations	70
9	Numerical Calculations	73
	9.1 Approximate Numbers in <i>Mathematica</i>	73
	9.2 Solving Equations	76
	9.3 Numerical Integration and Summation	77
	9.4 Differential Equations	78

10 Risch Algorithm	79
10.1 Rational Functions	79
10.2 Logarithmic Extension	80
10.3 Exponential Extension	85
10.4 Elementary Functions	89
11 Linear Algebra	91
11.1 Constructing Matrices	91
11.2 Parts of a Matrix	92
11.3 Queries	93
11.4 Operations with Matrices and Vectors	93
11.5 Eigenvalues and Eigenvectors	95
11.6 Jordan Form	96
11.7 Symbolic Vectors, Matrices, and Tensors	97
12 Input–Output and Strings	99
12.1 Reading and Writing .m Files	99
12.2 Output	101
12.3 C, Fortran, and TeX Forms	102
12.4 Strings	102
13 Packages	105
13.1 Contexts	105
13.2 Packages	106
13.3 Writing Your Own Package	106
Part II Computer Classes	
14 Plots	111
14.1 2D Plots	111
14.2 3D Plots	120
15 Trigonometric Functions	125
16 Quantum Oscillator	127
16.1 Lowering and Raising Operators	127
16.2 Ground State	129
16.3 Excited States	129
16.4 Some Properties	131
17 Spherical Harmonics	133
17.1 Angular Momentum in Quantum Mechanics	133
17.2 $Y_{l l}(\theta, \varphi)$	134
17.3 $Y_{l m}(\theta, \varphi)$	135

18	Adding Angular Momenta in Quantum Mechanics	139
19	Classical Nonlinear Oscillator	145
	19.1 Statement of the Problem	145
	19.2 The First Correction	146
	19.3 The Second Correction	147
	19.4 The n th Correction	149
20	Quantum Nonlinear Oscillator	153
	20.1 Perturbation Theory	153
	20.2 Nonlinear Oscillator	154
	20.3 Energy Levels	154
	20.4 Correspondence Principle	157
	20.5 States	159
21	Riemann Curvature Tensor	161
22	Multi-ζ Functions	165
	22.1 Definition	165
	22.2 Stuffing Relations	165
	22.3 Integral Representation	167
	22.4 Shuffling Relations	168
	22.5 Duality Relations	170
	22.6 Weight 4	170
	22.7 Weight 5	171
23	Rainbow	173
	23.1 Statement of the Problem	173
	23.2 0 Ray Segments Inside the Drop	174
	23.3 1 Ray Segment Inside the Drop	179
	23.4 2 Ray Segments Inside the Drop	181
	23.5 L Ray Segments Inside the Drop	188
24	Cyclohexane	193
	24.1 Statement of the Problem	193
	24.2 First Steps	194
	24.3 Equations	196
	24.4 Projection onto the x, y Plane	198
	24.5 Complete Analysis of the Solutions	205
	24.6 Shape of the Molecule	206
25	Problems for Students	209
	References	213
	Index	215