

Notation Index

$\mathbb{1}_A$	Indicator function of the set A
2^Ω	Set of all subsets of Ω
$\#A$	Cardinality of the set A
A^c	Complement $\Omega \setminus A$ of the set $A \subset \Omega$
$A \cap B$	Intersection of the sets A and B
$A \cup B$	Union of the sets A and B
$A \uplus B$	Disjoint union of A and B
$A \subset B$	A is a (not necessarily strict) subset of B
$A \setminus B$	Difference set
$A \triangle B$	Symmetric difference of A and B , 28
$A \times B$	Cartesian product of A and B
\mathcal{A}	Subset of 2^Ω , usually a σ -algebra
$\mathcal{A} _B$	Trace of the class \mathcal{A} on B , 10
$\mathcal{A} \otimes \mathcal{A}'$	Product of the σ -algebras \mathcal{A} and \mathcal{A}' , 274
$\mathcal{B}(E)$	Borel σ -algebra on E , 8
Ber_p	Bernoulli distribution, 42
$\beta_{r,s}$	Beta distribution with parameters r and s , 45
$b_{n,p}$	Binomial distribution, 42, 303
$b_{r,p}^-$	Negative binomial distribution, 43, 303
$C(E), C_b(E), C_c(E)$	Space of continuous (bounded) functions, and with compact support, respectively, 247
\mathcal{C}_{qv}	Functions with continuous square variation, 499
\mathbb{C}	Set of complex numbers
Cau_a	Cauchy distribution, 303
$\text{Cov}[X, Y]$	Covariance of the random variables X and Y , 102
CPoi_ν	Compound Poisson distribution, 333
δ_x	Dirac distribution, 11
$\mathbf{E}[X]$	Expectation (or mean) of the random variable X , 101
$\mathbf{E}[X; A]$	$= \mathbf{E}[X \mathbb{1}_A]$, 171
$\mathbf{E}[X \mathcal{F}]$	Conditional expectation, 173
\exp_θ	Exponential distribution, 44, 303

$\mathbb{F} = (\mathcal{F}_t)_{t \in I}$	Filtration, 191
a.s., a.e.	Almost surely and almost everywhere, 30
$G(x, y)$	Green function of a Markov chain, 369
$\Gamma_{\theta, r}$	Gamma distribution with scale parameter $\theta > 0$ and shape parameter $r > 0$, 45, 303
$\gamma_p = b_{1, p}^-$	Geometric distribution with parameter p , 42
$\gcd(M)$	Greatest common divisor of all $m \in M \subset \mathbb{N}$, 390
$H \cdot X$	Discrete stochastic integral of H with respect to X , 198
\mathcal{I}	Set of invariant distributions of a Markov chain, 378
iff	If and only if
i.i.d.	Independent and identically distributed, 55
$\text{Im}(z)$	Imaginary part of $z \in \mathbb{C}$, 295
λ, λ^n	Lebesgue measure, n -dimensional, 24
$\text{Lip}(E)$	Space of Lipschitz continuous functions on E , 249
\mathcal{L}^p, L^p	Lebesgue spaces of integrable functions, 91, 145, 146
$\mathcal{L}(X)$	Distribution of the random variable X
$\mathcal{M}(E), \mathcal{M}_f(E), \mathcal{M}_{\leq 1}, \mathcal{M}_1(E)$	Set of measures on E , finite measures on E , (sub-)probability measures on E , respectively, 16, 247
$\mathcal{M}_{\text{loc}, c}$	Space of continuous local martingales, 502
$\mu \otimes \nu$	Product of the measures μ and ν , 26, 277
$\mu * \nu$	Convolution of the measures μ and ν , 60, 279
$\mu^{\otimes n}$	n th power of a measure μ , 277
μ^{*n}	n th convolution power of a measure μ , 60
$\mu \ll \nu$	μ is absolutely continuous with respect to ν , 157
$\mu \perp \nu$	μ and ν are mutually singular, 157
$\mu \approx \nu$	μ and ν are equivalent, 157
$\mu \leq_{\text{st}} \nu$	μ is stochastically smaller than (or equal to) ν , 385
\mathbb{N}, \mathbb{N}_0	$\mathbb{N} = \{1, 2, 3, \dots\}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$
$\mathcal{N}_{\mu, \sigma^2}$	Normal distribution, 44, 303
$d\mu/d\nu$	Radon–Nikodym derivative, 158
Ω	Space of elementary events on which \mathbf{P} is defined
\mathbf{P}	Generic probability measure
$\mathbf{P}[A B], \mathbf{P}[A \mathcal{F}]$	Conditional probabilities, 170, 173
$\mathbf{P}_X = \mathbf{P} \circ X^{-1}$	Distribution of the random variable X , 41
Poi_λ	Poisson distribution with parameter $\lambda \geq 0$, 43, 303
$p^n(x, y) = p^{(n)}(x, y)$	n -step transition probability of a Markov chain, 358
$\mathcal{P}_{S, T}^n, \mathcal{P}_T^n$	See page 499
φ_X	Characteristic function of the random variable X , 303
ψ_X	Generating function of the random variable X , 77
\mathbb{Q}	Set of rational numbers
\mathbb{R}	Set of real numbers
$\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$	Two point compactification of the real numbers
Rad_p	$= p\delta_1 + (1 - p)\delta_{-1}$ Rademacher distribution, 42
$\text{Re}(z)$	Real part of $z \in \mathbb{C}$, 295
$\text{sign}(x)$	$= \mathbb{1}_{(0, \infty)}(x) - \mathbb{1}_{(-\infty, 0)}(x)$, sign of $x \in \mathbb{R}$, 36

$\sigma(\cdot)$	σ -algebra or filtration generated by \cdot , 6, 33, 191
τ_x^k	Time of the k th visit of a Markov chain at x , 367
$\mathcal{T}(\cdot)$	Tail σ -algebra, 62
\mathcal{U}_A	Uniform distribution on A , 12, 31, 303
$V^1(G), V^2(G)$	Variation and square variation of G , 498, 499
$\text{Var}[X]$	Variance of the random variable X , 101
v-lim	Vague limit, 252
w-lim	Weak limit, 252
X^τ	Stopped process, 210
$\langle X \rangle$	Square variation process of X , 206, 499, 503, 507
$f(t) \sim g(t), t \rightarrow a$	$:\iff \lim_{t \rightarrow a} f(t)/g(t) = 1$
$X \sim \mu$	The random variable X has distribution μ , 41
$x \vee y, x \wedge y, x^+, x^-$	Maximum, minimum, positive part, negative part of real numbers, 36
$\lfloor x \rfloor, \lceil x \rceil$	Floor and ceiling of x , 35
\bar{z}	Complex conjugate of $z \in \mathbb{C}$, 295
\mathbb{Z}	Set of integers
$\stackrel{\mathcal{D}}{=}$	Equal in distribution, 41
$\xrightarrow[n \rightarrow \infty]{\mathcal{D}}, \xrightarrow[n \rightarrow \infty]{\implies}$	Convergence of distributions, 255
$\xrightarrow[n \rightarrow \infty]{\implies}, \xrightarrow[n \rightarrow \infty]{\text{fdd}}$	Convergence of finite-dimensional distributions, 484
$\xrightarrow[\text{meas}]{\implies}, \xrightarrow[\text{a.s.}]{\implies}, \xrightarrow[\text{a.e.}]{\implies}$	Convergence in measure, almost surely, and almost everywhere, 132

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