An Introduction to Biomechanics

Solids and Fluids, Analysis and Design

Springer Science+Business Media LLC

Jay D. Humphrey Sherry L. O'Rourke

An Introduction to Biomechanics

Solids and Fluids, Analysis and Design

Second Edition

Springer

Jay D. Humphrey
Yale University
New Haven, CT, USA

Sherry L. O'Rourke
Medtronic, plc
Mounds View, MN, USA

ISBN 978-1-4939-2622-0 ISBN 978-1-4939-2623-7 (eBook)
DOI 10.1007/978-1-4939-2623-7

Library of Congress Control Number: 2015938917

Springer New York Heidelberg Dordrecht London
© Springer Science+Business Media New York 2015
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer Science+Business Media LLC New York is part of Springer Science+Business Media (www.springer.com)

◆

To my daughters,
Kaitlyn Leigh and Sarah Elizabeth,
I am so proud of you.
J.D. Humphrey

◆

To my loving parents, *Stephen and Mary O'Rourke*,
and sister, *Shelly O'Rourke,*
for giving meaning and purpose to my life.
S.L. O'Rourke

◆

Preface to the Second Edition

We have very much appreciated the overwhelmingly positive response to this book by professors and students alike at universities across the USA and abroad. The goals and approach of this Second Edition thus remain as originally presented: to motivate the need for continuum biomechanics across diverse areas of study, to present a consistent introductory approach to the biomechanics of solids and fluids and their interactions, and to illustrate this general approach via numerous Examples and Exercises. This Second Edition has allowed us, however, to add new "Observations" that highlight further implications of mechanics within biology and medicine, to add a new Appendix, to update the references, to include additional Exercises, and to correct some typographical errors. Perhaps most importantly, this Second Edition has allowed us to add at the end of each chapter a "Chapter Summary" to help emphasize general points of importance as well as to reinforce the consistency of the big picture ideas across chapters.

It was just over a decade ago that we wrote this book with great excitement and we are very pleased to report that research and training in biomechanics continues to be universally recognized as both exciting and vitally important. Traditional areas of research within biomechanics continue to contribute to basic science as well as translational research and development whereas new areas continue to emerge with great promise. For example, the US National Committee on Biomechanics (*USNCB*) recently sponsored three Frontiers Meetings that highlighted special opportunities for biomechanics in areas ranging from developmental biology to the fight against cancer and infections. Since the first printing of this book, the National Science Foundation has added another funding program entitled, Biomechanics and Mechanobiology, and a Federal Interagency Modeling and Analysis Group (IMAG), led by Grace Peng, Ph.D., has been established to emphasize across many funding agencies the importance of multiscale mathematical modeling in biology and medicine, which prominently includes biomechanics. Hence, in areas new and old alike, we continue to see the importance of the fundamentals of biomechanics in the

formulation and solution of diverse biological and medical problems of importance. We thus continue to encourage the reader to focus on learning the fundamentals well and, of course, to enjoy the journey.

Jay D. Humphrey
New Haven, CT, USA

Preface to the First Edition

Biomechanics aims to explain the mechanics of life and living. From molecules to organisms, everything must obey the laws of mechanics.

—Y.C. Fung (1990)

It is purported that Leonardo da Vinci once said, "by means of this [mechanics], all animated bodies that have movement perform all their actions." Although this assertion is obviously overstated, it serves to remind us that scientists have long thought that mechanics plays an extremely important role in governing biological as well as physical actions. Indeed, perhaps one of the most exciting recent discoveries in cell biology is that of mechanotransduction. It is now known that many cell types express different genes (i.e., perform different functions) in response to even small changes in their mechanical environment. Because cells are the fundamental structural and functional units of all living things, the importance of mechanics in biology and medicine is thereby far reaching! One goal of this book is to serve as an introduction to a few of the many, many applications of biomechanics—one of the cornerstones of biomedical engineering. Before proceeding, however, a few words on the scope and philosophy of approach.

There are five general areas of mechanics: discrete, continuum, statistical, quantum, and relativistic. Each is important, but this text focuses on biomechanics from a continuum perspective, which we will see embraces many aspects of biomedical engineering at various length and time scales.

Introductory textbooks on mechanics sometimes give the wrong impression that the subject is primarily a collection of solutions to individual problems—nothing could be further from the truth. As a branch of classical physics, continuum mechanics is a deductive science founded upon a few basic postulates and concepts through which all problems must be formulated and then solved. Mechanics should be recognized, therefore, as a consistent, focused approach to the solution of classes of problems rather than as a collection of special results. Another goal of this textbook is to introduce the student to biosolid and biofluid mechanics such that it is the underlying, consistent

approach that is learned and reinforced throughout. Indeed, the ultimate goal here is to enable the reader to formulate and solve real-life problems, many of which have yet to be identified. In other words, the primary goal of a student should not be to learn how to solve the specific problems (illustrative examples and exercises) in this text; we, as a community, already know their solution. Textbook problems should be used simply as a means to practice the underlying *approach of mechanics,* to gain confidence in formulating and solving problems, and to develop intuition.

Although this philosophy of learning the fundamentals is as old as mechanics itself, it has at no time in our history been more important. With continued advances in computer technology and engineering software, the biomechanicist will have increasingly remarkable experimental, computational, and design tools at his/her disposal to address the incredibly complex real-life problems of biomechanics. The only way to ensure that these tools are used well, rather than misused, is to understand the underlying general approach as well as the specific assumptions (with associated limitations) within a given formulation. For example, a finite element program should not be treated as a black box capable of finding any solution of interest; rather, it should be used cautiously as a tool only by one who understands how the program actually works. Toward this end, note one caveat. It has been appropriately said that undergraduates should be told the truth, nothing but the truth, but *not* the whole truth. Why not the whole truth? From a purist perspective, we do not know the whole truth, scientific knowledge being relative to current advances. From a practical perspective, however, continuum biomechanics has tremendous breadth and depth and it is impossible in an introductory course to scratch the surface of the whole truth. Therefore the interested student is strongly encouraged to pursue intermediate and advanced study in biomechanics, which will successively reveal more and more of the beauty and, indeed, the power of biomechanics. Biomechanics is a lifelong pursuit, one with many rewards.

Whereas graduate courses on biomechanics are often best taught using a problem-based paradigm, we suggest that an introductory course on biomechanics should be taught using a traditional discipline-based paradigm; that is, graduate courses are often best taught by focusing on a particular tissue, organ, or system, or, alternatively, on a specific disease or treatment modality, and then by bringing to bear all tools (experimental, computational, theoretical, biological) that aid in the solution of that class of problems. An introduction to biomechanics should be different, however. To see the overall approach used in mechanics, it is best to introduce all of the general tools (e.g., concept of stress, strain, and equilibrium) and then to illustrate their use via multiple similar problems that build in complexity but continually reinforce the same approach. Hence, this book is divided according to approach (e.g., via chapters on beam theory and Navier–Stokes solutions), not according to areas of research such as cardiovascular, musculoskeletal, pulmonary, or cell

mechanics. Therefore, we employ illustrative problems from various fields of study, often within single chapters.

As a first course in biomechanics, the primary prerequisites are the sequence of courses on calculus for engineers (including vectors, which are reviewed in Chap. 7) and some basic biology (reviewed in Chap. 1); many students will likely have had a course in engineering statics (briefly reviewed in Chap. 1), which will help but it is not necessary. Although a course on differential equations would also be helpful (briefly reviewed in Chap. 8), related methods needed herein will be reviewed at the appropriate time. Given the availability of personal computers and useful software packages, the student will be asked to obtain numerical solutions to many exercises.

I would like to close with a quote from the 1998 Bioengineering Consortium (BECON) Report of the National Institutes of Health:

> The success of reductionist and molecular approaches in modern medical science has led to an explosion of information, but progress in integrating information has lagged ... Mathematical models provide a rational approach for integrating this ocean of data, as well as providing deep insight into biological processes.

Biomechanics provides us with a means to model mathematically many biological behaviors and processes; thus biomechanics will continue to play a central role in both basic and applied research. The key, therefore, is to learn well the basic approaches.

Jay D. Humphrey
College Station, TX, USA

Comments from a Student to a Student

Although one tends to teach the way he or she was taught, this textbook is designed to be different. For example, rather than introduce biomechanics through a sequence of increasingly more involved and detailed problems, with each illustrating new foundational concepts, we choose to introduce the basic concepts first and then to illustrate and reinforce the use of these concepts through the consideration of increasing more complex problems. In addition, rather than have two professors coauthor the book from their two perspectives, we chose to have a professor and student coauthor the book from their two perspectives—teacher and learner. Ms. O'Rourke completed a sequence of three biomechanics courses at Texas A&M University (BMEN 302 Biosolid Mechanics, BMEN 421 Biofluid Mechanics, and BMEN 689 Cardiovascular Mechanics) offered by Professor Humphrey. The goal of this joint effort, therefore, is to present the material in a way that a professor feels is most beneficial and yet in a way that a student feels is most easily assimilated. Here, therefore, consider comments from a student to a student.

When taking my first course from Professor Humphrey "Biosolid Mechanics," he explained mechanics in such a way that made sense to me. He introduced the idea of a continuum and that classes of problems that fall within the realm of continuum mechanics, whether it be solid or fluid mechanics, are governed by the same fundamental relations. The backbone of mechanics was revealed and the basic/fundamental equations were derived from a single perspective. This introductory text introduces these basic concepts, which are essential to all problems in biomechanics. It presents a unified approach that helps the student to understand and learn the basic concepts and allows one to build upon these concepts to formulate and solve problems of increasing difficulty.

The concepts introduced in Part II of this text are reinforced in Part III, as we apply the same governing equations to different classes of problems, again deriving necessary equations as we move along. As a student, I found the course in "Biofluid Mechanics" to be easier than "Biosolid Mechanics" partly because

I had already learned the general approach to formulating and solving problems. In these ways, I feel that this book reflects the positive aspects from my learning experiences at Texas A&M.

All in all, as a student, I appreciated having a unified problem-solving process presented to me and reinforced throughout each course as well as knowing from where the basic/fundamental equations were derived. I also appreciated reference to real-life problems for motivation, and derivations that skipped very few steps. This book reflects these ideas, upon which one can build.

Sherry L. O'Rourke
College Station, TX, USA

Acknowledgments

First Edition

We especially thank two Texas A&M University undergraduate students, Anne Price and Elizabeth Wang, who worked diligently for many months to help type much of the manuscript, compose the figures, and proofread the text. Some of the figures are reprinted from an earlier book, *Cardiovascular Solid Mechanics*, and were drawn by another undergraduate student, William Rogers. Numerous graduate students (G. David, C. Farley, R. Gleason, M. Heistand, J. Hu, S. Na, and P. Wells) proofread portions of the text, which is gratefully acknowledged, and Professor Larry A. Taber from Washington University read portions of the text and provided valuable input. JDH would also like to thank daughters Kaitlyn and Sarah, and wife Rita, for continued understanding and support. We also thank Dr. John H. Linehan and The Whitaker Foundation for providing partial financial support to complete this first edition through their Teaching Materials Program. The American Heart Association, Army Research Office, National Institutes of Health, National Science Foundation, Texas Advanced Research Program, and The Whitaker Foundation funded numerous research projects from which some of the material was taken. Finally, we thank Paula M. Callaghan, Senior Life Sciences Editor at Springer New York, for wonderful encouragement, suggestions, and support throughout.

Second Edition

We wish to thank some of the current graduate students and post-doctoral fellows in the Continuum Biomechanics Laboratory at Yale University for proof-reading portions of this Second Edition. They are: Chiara Bellini, Matthew Bersi, Paolo Di Achille, Ramak Khosravi, Arina Korneva, and Manuel Rausch. We especially thank Paolo Di Achille for providing computational results that have been used for the cover illustration. JDH would like to thank Rita, his wife, for continued support in all things, including help with

proof-reading this Second Edition. Portions of the first author's work that contributed to this book have been funded by grants from the National Institutes of Health (NHLBI and NIBIB), National Science Foundation, and National Marfan Foundation, which are gratefully acknowledged. Finally, we especially thank Merry Stuber, Editor, Biomedical Engineering at Springer, NY, for bringing this Second Edition to publication.

Contents

Preface to the Second Edition vii
Preface to the First Edition ix
Comments from a Student to a Student xiii
Acknowledgments xv
Contents xvii
Summary of Observations xxv

Part I Background 1

1. Introduction 3
1.1 Point of Departure 3
1.2 Health Care Applications 5
1.3 What Is Continuum Mechanics? 9
1.4 A Brief on Cell Biology 11
1.5 The Extracellular Matrix 17
1.6 Mechanotransduction in Cells 23
1.7 General Method of Approach 24
Chapter Summary 27
Appendix 1: Engineering Statics 28
Exercises 41

Part II Biosolid Mechanics 45

2. Stress, Strain, and Constitutive Relations 47
2.1 Introduction 47
2.2 Concept of Stress 49
2.3 Stress Transformations 58

2.4 Principal Stresses and Maximum Shear 63
2.5 Concept of Strain 68
2.6 Constitutive Behavior 83
2.6.1 Illustrative Characteristic Behaviors 88
2.6.2 Hookean LEHI Behavior 89
2.6.3 Hooke's Law for Transverse Isotropy 93
2.6.4 Hooke's Law for Orthotropy 94
2.6.5 Other Coordinate Systems 94
2.7 Mechanical Properties of Bone 97
Chapter Summary 99
Appendix 2: Material Properties 100
Exercises 101

3. Equilibrium, Universal Solutions, and Inflation 109
3.1 General Equilibrium Equations 109
3.2 Navier–Space Equilibrium Equations 113
3.3 Axially Loaded Rods 116
3.3.1 Biological Motivation 117
3.3.2 Mathematical Formulation 119
3.4 Pressurization and Extension of a Thin-Walled Tube 125
3.4.1 Biological Motivation 126
3.4.2 Mathematical Formulation 127
3.5 Pressurization of a Thin Spherical Structure 138
3.5.1 Biological Motivation 138
3.5.2 Mathematical Formulation 140
3.6 Thick-Walled Cylinders 142
Chapter Summary 149
Appendix 3: First Moments of Area 151
Exercises 156

4. Extension and Torsion 163
4.1 Deformations Due to Extension 164
4.1.1 Biological Motivation 164
4.1.2 Theoretical Framework 169
4.1.3 Clinical Application 175
4.2 Shear Stress Due to Torsion 178
4.2.1 Introduction 178
4.2.2 Biological Motivation 179
4.2.3 Mathematical Formulation 179
4.3 Principal Stresses and Strains in Torsion 186
4.4 Angle of Twist Due to Torque 191
4.4.1 Basic Derivation 192
4.4.2 Statically Indeterminate Problems 194

4.5 Experimental Design: Bone Properties 198
4.6 Experimental Design: Papillary Muscles 199
4.6.1 Biological Motivation 199
4.6.2 Experimental Design 201
4.7 Inflation, Extension, and Twist 203
Chapter Summary 204
Appendix 4: Second Moments of Area 206
Exercises 208

5. Beam Bending and Column Buckling 213
5.1 Shear Forces and Bending Moments 214
5.2 Stresses in Beams 223
5.2.1 Biological Motivation 223
5.2.2 Theoretical Framework 224
5.2.3 Illustrative Examples 234
5.3 Deformation in Beams 241
5.3.1 Biological Motivation 241
5.3.2 Theoretical Framework 241
5.3.3 Illustrative Examples 244
5.4 Transducer Design: The AFM 250
5.4.1 Introduction 250
5.4.2 The Atomic Force Microscope 251
5.4.3 Illustrative Example 252
5.5 Principle of Superposition 255
5.6 Column Buckling 262
5.6.1 Concept of Stability 262
5.6.2 Buckling of a Cantilevered Column 264
Chapter Summary 271
Appendix 5: Parallel Axis Theorem and Composite Sections 273
Exercises 278

6. Some Nonlinear Problems 289
6.1 Kinematics 289
6.2 Pseudoelastic Constitutive Relations 295
6.3 Design of Biaxial Tests on Planar Membranes 304
6.3.1 Biological Motivation 304
6.3.2 Theoretical Framework 305
6.4 Stability of Elastomeric Balloons 311
6.4.1 Biological Motivation 311
6.4.2 Theoretical Framework 312
6.4.3 Inflation of a Neuroangioplasty Balloon 315

6.5 Residual Stress and Arteries . . . 321
6.5.1 Biological Motivation . . . 321
6.5.2 Theoretical Framework . . . 324
6.5.3 Illustrative Results . . . 331
6.6 A Role of Vascular Smooth Muscle . . . 334
6.6.1 Muscle Basics . . . 334
6.6.2 Quantification . . . 335
Chapter Summary . . . 338
Appendix 6: Matrices . . . 341
Exercises . . . 344

Part III Biofluid Mechanics . . . 351

7. Stress, Motion, and Constitutive Relations . . . 353
7.1 Introduction . . . 353
7.2 Stress and Pressure . . . 354
7.3 Kinematics: The Study of Motion . . . 356
7.3.1 Velocity and Acceleration . . . 357
7.3.2 Fluid Rotation . . . 365
7.3.3 Rate of Deformation . . . 368
7.4 Constitutive Behavior . . . 373
7.4.1 Newtonian Behavior . . . 374
7.4.2 Non-Newtonian Behavior . . . 379
7.5 Blood Characteristics . . . 383
7.5.1 Plasma . . . 383
7.5.2 Blood Cells . . . 384
7.5.3 Additional Rheological Considerations . . . 385
7.6 Cone-and-Plate Viscometry . . . 388
Chapter Summary . . . 392
Appendix 7: Vector Calculus Review . . . 393
Exercises . . . 400

8. Fundamental Balance Relations . . . 405
8.1 Balance of Mass . . . 406
8.2 Balance of Linear Momentum . . . 409
8.3 Navier–Stokes Equations . . . 413
8.4 The Euler Equation . . . 419
8.5 The Bernoulli Equation . . . 423
8.5.1 Bernoulli Equation for Flow Along a Streamline . . . 423
8.5.2 Bernoulli Equation for Irrotational Flow . . . 425
8.5.3 Further Restrictions for the Bernoulli Equation . . . 431

8.6 Measurement of Pressure and Flow 439
8.6.1 Pressure 439
8.6.2 Flow 441
8.7 Navier–Stokes Worksheets 444
Chapter Summary 447
Appendix 8: Differential Equations 448
Exercises 451

9. Some Exact Solutions 455
9.1 Flow Between Parallel Flat Plates 456
9.1.1 Biological Motivation 456
9.1.2 Mathematical Formulation 457
9.2 Steady Flow in Circular Tubes 474
9.2.1 Biological Motivation 474
9.2.2 Mathematical Formulation 477
9.3 Circumferential Flow Between Concentric Cylinders 483
9.3.1 Bioreactor Application 483
9.3.2 Mathematical Formulation 484
9.3.3 Viscometer Application 488
9.4 Steady Flow in an Elliptical Cross Section 492
9.4.1 Biological Motivation 492
9.4.2 Mathematical Formulation 494
9.5 Pulsatile Flow 496
9.5.1 Some Biological Motivation 496
9.5.2 Mathematical Formulation 497
9.6 Non-Newtonian Flow in a Circular Tube 504
9.6.1 Motivation 504
9.6.2 Mathematical Formulation 505
Chapter Summary 508
Appendix 9: Biological Parameters 509
Exercises 511

10. Control Volume and Semi-empirical Methods 525
10.1 Fundamental Equations 525
10.1.1 Theoretical Framework 528
10.1.2 Special Cases for Mass and Momentum 532
10.1.3 The Energy Equation 533
10.2 Control Volume Analyses in Rigid Conduits 534
10.2.1 Clinical Motivation 534
10.2.2 Illustrative Examples 535

10.3 Control Volume Analyses in Deforming Containers 546
10.3.1 Clinical Motivation 546
10.3.2 Mathematical Formulation 546
10.4 Murray's Law and Optimal Design 549
10.4.1 Straight Segment 550
10.4.2 Bifurcation Areas 551
10.4.3 Bifurcation Patterns 553
10.5 Buckingham Pi and Experimental Design 555
10.5.1 Motivation 555
10.5.2 Recipe 556
10.6 Pipe Flow 566
10.7 Conclusion 583
Chapter Summary 584
Appendix 10: Thermodynamics 585
Exercises 588

Part IV Closure 599

11. Coupled Solid–Fluid Problems 601
11.1 Vein Mechanobiology 602
11.1.1 Biological Motivation 602
11.1.2 Theoretical Framework 602
11.2 Diffusion Through a Membrane 603
11.2.1 Biological Motivation 603
11.2.2 Theoretical Basis 604
11.2.3 Illustration 608
11.3 Dynamics of a Saccular Aneurysm 619
11.3.1 Biological Motivation 619
11.3.2 Mathematical Framework 620
11.4 Viscoelasticity: QLV and Beyond 630
11.4.1 Linearized Viscoelasticity 632
11.4.2 Quasilinear Viscoelasticity 645
11.4.3 Need for Nonlinear Theories 645
11.5 Lubrication of Articulating Joints 646
11.5.1 Biological Motivation 646
11.5.2 Hydrodynamic Lubrication 646
11.5.3 Need for a Mixture Theory 651
11.6 Thermomechanics, Electromechanics, and Chemomechanics 654
Chapter Summary 657
Appendix 11: Wave Equations 658
Exercises 660

12. Epilogue 667
12.1 Future Needs in Biomechanics 668
12.2 Need for Lifelong Learning 672
12.3 Conclusion 673

References 675
About the Authors 681
Index 683

Summary of Observations

In each chapter, we attempt (1) to motivate further the need for biomechanics via the identification of illustrative biological and clinical problems, (2) to provide a consistent approach to the formulation and solution of diverse biomechanical problems that is easily remembered, and (3) to illustrate such needs and approaches via numerous Examples and Exercises. In addition, we intersperse throughout Chaps. 2–11 various "Observations" to encourage the reader to pause and think about further topics within mechanics and biology that are not addressed in detail but are nevertheless important to biomechanics. Indeed, we hope that the reader develops a habit of pausing at other times to consider additional areas where biomechanics can and should be applied or other aspects of the mechanics that can and should be developed or extended to address better the diversity of problems that arise in biology and medicine. Toward this end, note the following *Observations*:

2.1 Interpolation-Based Strain Analysis 76
2.2 Digital Image Correlation and Inverse Methods 87
2.3 Delamination and Experimental Approaches 98

3.1 Mechanochemistry and MMPs 116
3.2 Molecular-Level Biomechanical Tests 122
3.3 Functional Tissue Engineering 136

4.1 Mechanobiology of Skeletal Development 163
4.2 Strains and Microdamage in Bone 190
4.3 Fracture Toughness and Bone Cement 197

5.1 Stress Concentrations 239
5.2 Micro-Cantilevered Beams for Studying Cells 253
5.3 Fatigue Strength and Failure 260

6.1 Probing Mechanical Properties of Cells 293
6.2 Mechanics of the Eye 319
6.3 Uniqueness of Vascular Elastin 333

7.1 Mathematical Concept of a Field 363
7.2 Pulse Wave Velocity and Arterial Stiffness 382
7.3 Synovial Fluid and Joint Mechanics 387

8.1 Incompressibility and Flow 409
8.2 Buoyancy and Archimedes 418
8.3 Irrotationality and Flow 423

9.1 Flow Effects on Cell Monolayers 465
9.2 Microgravity Effects on Free-Floating Cells 488
9.3 Unsteady Flows and Womersley 503

10.1 Gas Exchange in the Lungs 525
10.2 Pressure Drops in Stenoses 573
10.3 Non-dimensionalization of Governing Equations 581

11.1 Nonlinear Dynamical Biological Systems 614
11.2 Glycosaminoglycans and Donnan Swelling 651
11.3 Cancer Biomechanics 653