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Anders Malthe-Sørenssen

# Elementary Mechanics Using Python

A Modern Course Combining Analytical  
and Numerical Techniques

 Springer

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*To Mina, Aurora and Olav.*

# Preface

This book was developed as a textbook for use in the course “Introduction to Mechanics” in the Department of Physics at the University of Oslo starting 2007. In this course we aimed at providing a seamless integration of analytical and numerical methods when solving physics problems, thereby allowing us to solve more advanced and applied problems in mechanics, and providing examples that are perceived as more relevant for the students. We could address not only the very special cases that have analytical solutions, but could instead focus on choosing problems that would initiate discussions and provide the students with physical insights.

Through the processes of introducing and developing advanced problems, it also became clear that this approach brought the students closer to the way physics is discovered and applied. In addition, it introduced the students to a more exploratory way of understanding phenomena and of developing their physical concepts. Well-developed examples that also include elements of numerical computations gave the students a feeling of discovering physical processes while also understanding how they are results of the underlying simple physical laws. In many cases, the advanced examples and exercises spawned interesting and rewarding discussions about the underlying physical processes, and also forced the students to understand the various forms of representation used to illustrate physical processes, such as motion diagrams and energy diagrams, and use these diagrams to reason about physical processes.

As the course, examples, and exercises were developed it also became clear that the introduction of numerical methods in an introductory course in physics also helped build the notion that numerical methods are no different from analytical methods—they are part of the theoretical toolbox that any physicist is supposed to master. Our aim became to make it as natural for our students to solve their problems by developing a small program and discussing the results, as it was to use a calculator.

It has been particularly rewarding to observe the way that many of the examples and exercises trigger discussions when students discover unexpected results, in the form of unexpected resonances in a simple model for friction or in the case of

Greenwood gaps in the distribution of asteroids in the solar system. The insight that the simple laws of mechanics that they learned actually had observable consequences and explanatory power was often an important insight as well as an important reinforcer for the students. We also believe that this helps the student build a more realistic image of how science actually is done.

In order to get most of the numerical parts of this text it is advantageous for the students to have some prior knowledge of scientific programming, preferably with a scripting type language such as Matlab or Python, but this is not absolutely necessary. We encourage readers who are not familiar with scripting type programming first to study Chap. 2. However, in our experience students who read the book, study the examples, and do the exercises will already be developing programmers by the end of the course.

This book grew out of a larger, collaborative effort at the University of Oslo. I would like to thank Morten Hjorth-Jensen and Arnt Inge Vistnes for including me in the physics part of the Computers in Science Education program. I also thank Hans Petter Langtangen and Knut Mørken at the Department of Informatics for their dedication, support, and inspiration for introducing numerical approaches in the basic curriculum. I thank the Faculty for Mathematics and Natural Sciences for their support used to develop exercises and examples used in this text. I would also like to thank Arnt Inge Vistnes, Jonas van den Brinck, and Sigve Bøe Skattum for developing some of the exercises that have been included in this book as examples or exercises. Sigve Bøe Skattum has also provided many of the illustrations.

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Anders Malthe-Sørenssen

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