Engineering Electromagnetics

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This book is lovingly dedicated to Vera, my wife and partner in life.

Preface

You can because you ought.

—Imanuel Kant

One of the main difficulties in teaching electromagnetic fields is the perception on the part of many students that electromagnetics is essentially a supportive topic. They are told that they need to study electromagnetics early in the curriculum because they will need it later to understand other topics in the electrical engineering curriculum, such as electric machines, microwaves, or communication. This, with the prevailing perception of the topic being difficult, esoteric, or boring, creates a heavy atmosphere around the subject. More often than not, this leads to self-fulfilling prophecies, and as a result, even those students who perform well do not get the full benefit of the experience such an exciting topic can impart. This is particularly sad, because electromagnetics motivates many students to enter electrical engineering. They are familiar with electromagnetic waves, electric motors, magnetic recording, and data storage, and have been exposed to hundreds of electromagnetic devices. Yet few make the connection between these and the electromagnetics they are taught.

The answer is to study electromagnetics for what it is rather than in preparation for something that will happen in the future. The study of electromagnetic fields is not more difficult than any other topic in the electrical engineering curriculum and, in many ways, is more interesting and more applied. The number of applications is so vast that any attempt to summarize will easily fill a good-sized book. One can only guess the total share of electromagnetics to the industrial output. Huge turbo generators for generation of electricity, power transmission lines, electric motors, actuators, relays, radio, TV and microwave transmission and reception, magnetic storage, and even the mundane little magnet used to hold a paper note on the refrigerator are all electromagnetic in nature. It is indeed uncommon to find a device that works without relying on any electromagnetic principle or effect. One only has to ask oneself who is going to design these systems and what are the tools necessary to do so, and the answer to why one should study electromagnetics becomes self-evident.

This text attempts to present electromagnetics as a topic in itself with specific objectives and specific applications. The fact that it is used as a prerequisite for other subjects is merely a consequence that those other topics are based on electromagnetics. A good theoretical understanding of the electromagnetic field equations is required for electromagnetic design. The text fulfills this need by a rigorous treatment of the theoretical aspects of electromagnetics. In addition, it treats a large number of electromagnetic applications that the student will find interesting and useful.

The text assumes the student has the necessary background in calculus. Other required topics, including vector algebra and vector calculus, are fully covered in the text. In addition, all mathematical relations (such as integrals, derivatives, series, and others) are listed as needed in the text. In this sense, the book is fully self-contained. An effort has been made to use only quantities that have been defined previously, even if this requires, for example, change of units in mid-chapter. There will be a few exceptions to this rule, and when this happens, the reasons for doing so are also given. The reasons for this purist approach are many, but the most important is the fact that the book assumes no prior knowledge of any field quantity.

In style, the text relies on simple physical explanations, in plain language and based on known phenomena, to simplify understanding. There are many detailed examples, exercising every significant relation and topic in the book. Many of the examples rely on important applications, with particular emphasis on sensing and actuation, and contain complete step-by-step solutions and derivations as necessary. There is almost no use of acronyms. These are only used when an acronym is better known than what it represents, such as TV and FM. The presentation often relies on repetition of relations and explanations. This serves to reinforce understanding and avoids convoluted referencing to equations and text. In most cases, referencing is only done for completeness purposes, and the required equation is repeated when needed. Important or often-used relations are boxed and are always accompanied by their associated units. The notation used in the book is standard and should be familiar to students from physics and mathematics. The most important change in this respect is the use of unit vectors. Unit vectors always precede the scalar component. For example, $\mathrm{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$ is a vector with scalar components A_x in the x direction, A_y in the y direction, and A_z in the z direction. $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the corresponding unit vectors.

The structure of the book is unique in another way; most topics are discussed in two or three separate chapters. The first chapter introduces the subject and discusses the basic quantities and relations. The second chapter complements and expands on the first and introduces additional topics related to the main subject. In certain cases, a third chapter discusses additional topics or a new topic related to the first two. For example, **Chapter 3** introduces the electric field and the postulates governing it; **Chapter 4** continues with Gauss's law, effects of and on materials, capacitance, and other quantities associated with the electric field; **Chapter 5** then continues with analytical methods of solution of electrostatic problems. This pairing includes **Chapters 1** and **2** (vector algebra followed by vector calculus), **Chapters 3**, **4**, and **5** (electric field, electric potential, and boundary value problems), **Chapters 8** and **9** (the static magnetic field and magnetic materials and properties), **Chapters 12** and **13** (electromagnetic waves and propagation and reflection and transmission of plane waves), and **Chapters 14**, **15**, and **16** (theory of transmission lines, the Smith chart and transmission line circuits, and transients on transmission lines). The purpose of this grouping of chapters is twofold. First, it divides the material into more coherent, easier to follow, shorter units. Second, it provides intermediate breaking points at which both students and teachers can assess the situation and decide on the next steps. It also allows selection of topics without the need for skipping sections within chapters. For example, whereas a chapter on time-dependent fields normally includes all material associated with Faraday's law, Maxwell's equations, and wave propagation, I have chosen to divide this material into three chapters. One is on Faraday's law and includes all phenomena associated with induction (**Chapter 10**). The second discusses Maxwell's equations with associated material, including the continuity equation and interface conditions (**Chapter 11**). The third discusses wave propagation as a consequence of displacement currents (**Chapter 12**). The three chapters discuss different aspects, using various approaches.

Chapters 1 and **2** discuss vector algebra and vector calculus, and are rather different from the rest of the book in that the student will find no reference to electromagnetics in these chapters. This serves two purposes. First, it indicates that at this stage the student has little formal knowledge of electromagnetic field quantities but, paradoxically, he or she is aware of the properties of electromagnetic fields through knowledge acquired in other areas of physics or everyday experience. Second, it shows that the same methods and the same mathematical tools are used in other disciplines and for other applications. This approach should alleviate some of the anxiety associated with the study of electromagnetics while still acquiring all vector algebra and calculus tools needed for the study of electromagnetics. More importantly, the approach lends itself to self-study. If the student or the instructor feels that **Chapters 1** and **2** are not necessary, they may be skipped without affecting subsequent topics.

The method of presentation of the material distinguishes between basic field relations and mathematical tools. The latter are introduced in **Chapters 1** and **2**, but wherever they are needed, they are repeated to reinforce understanding of the tools and to avoid having to refer back to **Chapter 1** or **2**. Similarly, other relations, like trigonometric functions, derivatives, and integrals, are given as needed, and as close as possible to where they are used. This should help students with reviewing material they learned previously, but do not recall or are not certain of.

A summary is provided at the end of each chapter except **Chapters 1** and **2** that are themselves summaries of vector algebra and vector calculus, respectively. The purpose of these summaries is to collect the important relations in each chapter, to reemphasize these, and to serve as a means of first reviewing the material and second referencing back to the material in the chapter. It is hoped that by doing so, the student will spend less time searching topics especially when it becomes necessary to refer back to material in a chapter that was studied earlier. For this reason, the equations listed in the summary retain the original equation numbers when appropriate. On the other hand, tables and figures are not included but, rather, referred to when necessary. These summaries, coming as they are at the end of a chapter, also provide a retrospective view of the material, something that may help solidify understanding. However, summaries cannot replace the chapter. They are short, lack most explanations, and only the most common form of the relations is usually given. The relations in the chapters have alternative forms, some of which are useful in specific situations. The student should view these summaries simply as reminders and as an indication of the content and importance of subjects.

Each chapter includes an extensive set of problems. The problems are of two types. Some are exercises, used to ensure that the student has a chance to review the field relations and to use them in the way they were intended to be used. The second type is more involved and often based on a physical application or, in some cases, on a simplified version of a physical structure. These problems are designed to present some of the many applications in electromagnetics, in addition to their value as exercise problems. It is hoped that this will bring the student closer to the idea of design than exercise problems can.

Most chapters contain a section on applications and a section on experiments. The section on applications is intended to expand on material in the chapter and to expose the student to some of the myriad applications in electromagnetics or, in some cases, to physical phenomena that depend on electromagnetism. Naturally, only a very small selection of applications is given. The description is short but complete.

The section on experiments presents a few simple experiments that can be used to demonstrate the principles discussed in the chapter. These experiments are designed to be short and simple, and to require a minimum of materials and equipment. They are qualitative experiments: no measurements are taken, and no exact relations are expected to be satisfied. The instructor may choose to use these as an introduction to a particular topic or as a means to stimulate interest. The student may view the experiments as demonstrations of possible applications. Many of the experiments can be repeated by students if they wish to do so. However, none of the experiments require laboratory facilities. The main purpose is to take electromagnetic fields off the pedestal and down to earth. I found these simple experiments particularly useful as a way of introducing a new subject. It wakes the students up, gets them to ask questions, and creates anticipation toward the subject. The simplicity of the principles involved intrigues them, and they are more inclined to look at the mathematics involved as a means rather than the goal. Invariably, in student evaluations, the experiments are mentioned in very positive terms. I would even venture to say that the students tend to exaggerate their importance. Either way, there is value in showing the students that a discarded magnet and an old coil can demonstrate the principle of the AC generator. Even if no demonstrations are performed, it is recommended to read them as part of the study of the chapter—the student will find some of the explanations useful. Both the applications and experiments sections are excellent candidates for self-study.

This textbook was written specifically for a two-semester sequence of courses but can be used equally well for a one-semester course. In a two-semester sequence, the topics in **Chapters 3–10** are expected to be covered in the first semester. If necessary, **Chapters 1** and **2** may be reviewed at the beginning of the semester, or if the students' background in vector algebra and calculus is sufficiently strong, these two chapters may be skipped or assigned for self-study. **Chapter 6** is self-standing and, depending on the instructor's preference, may or may not be covered. The second semester should ideally cover **Chapters 11–18** or at least **Chapters 11–16**. **Chapters 17** and **18** are rather extensive discussions on waveguides and antennas, respectively, and as such introduce mostly new applications and derived relations rather than fundamental, new ideas. These may form the basis of more advanced elective courses on these subjects. In a one-semester course, there are two approaches that may be followed. In the first, **Chapters 3–5** and **7–12** are covered. This should give students a solid basis in electromagnetic fields and a short introduction to electromagnetic waves. The second approach is to include topics from **Chapters 3–5** and **7–16**. It is also possible to define a program that emphasizes wave propagation by utilizing **Chapters 11–18** and excluding all topics in static electric and magnetic fields.

There is a variety of methods for the solution of boundary value problems. The classical methods of separation of variables or the image methods are presented as methods of solving particular problems. However, one of the most frustrating aspects of fields is that there are no systematic, simple ways of solving problems with any degree of generality. Too often, we rely on a canned solution to idealized geometries, such as infinite structures. The introduction of numerical methods at this stage is intended to reassure students that solutions indeed exist and that the numerical methods needed to do so are not necessarily complicated. Some methods can be introduced very early in the course of study. Finite differences and the method of moments are of this type. Finite-element methods are equally simple, at least at their basic levels. These methods are introduced in **Chapter 6** and are applied to simple yet useful electrostatic configurations.

The history associated with electromagnetics is long and rich. Many of the people involved in its development had unique personalities. While information on history of the science is not in itself necessary for understanding of the material, I feel it has a value in its own right. It creates a more intimate association with the subject and often places things in perspective. A student can appreciate the fact that the great people in electromagnetics had to struggle with the concepts the students try to understand or that Maxwell's equations, the way we know them today, were not written by Maxwell but by Heaviside, almost twenty years after Maxwell's death. Or perhaps it is of some interest to realize that Lord Kelvin did not believe Maxwell's theory well after it was proven experimentally by Hertz. Many will enjoy the eccentric characters of Heaviside and Tesla, or the unlikely background of Coulomb. Still others were involved in extracurricular activities that had nothing to do with the sciences. Benjamin Franklin was what we might call a special envoy to England and France, and Gilbert was personal physician to Queen Victoria. All these people contributed in their own way to the development of the theory of fields, and their story is the story of electromagnetics. Historical notes are given throughout the book, primarily as footnotes.

To aid in understanding, and to facilitate some of the more complex calculations, a number of computer programs (written in Matlab) are available for download. These are of four types. The first type is demonstrative—programs in this group are intended to display a concept such as vector addition or reflection of electromagnetic waves. Wherever appropriate in the text, the relevant program is indicated and the student, as well as the instructor, is welcome to use these to emphasize the various concepts. An explanation file is available to explain the various inputs and outputs and the use of the programs. The second group includes simple programs used to solve a particular example or to compute values in end-of-chapter problems. Some of the programs are rather specific but others are more general. The third group of programs includes auxiliary charts and computational tools including a full implementation of the Smith chart (**Chapter 15**). The fourth type of programs relates to **Chapter 6** exclusively. They address the use of the finite difference method, the finite-element method, and the method of moments and allow the student first to duplicate the results in the various examples and then to apply these numerical techniques to other calculations including those in the problems section.

The programs are intended to be run as scripts and are mostly interactive. Needless to say, the student is more than welcome to modify these programs for use in other examples, problems, etc. To get the full benefit of these tools, the student should download the files available and read carefully the explanation files before using the various programs. All programs are available for download at http://extras.springer.com/2014/978-3-319-07806-9

Finally, I wish to thank those who were associated with the writing of this text. In particular, Frank Lewis (class of '96), Dana Adkins (class of '97), Shi Ming (class of '94), and Paul Stager (class of '94) have solved some of the examples and end-of-chapter problems and provided valuable input into the writing of the first edition of this text. Professor J.P.A. Bastos (Federal University of Santa Catarina, Florianopolis, Brazil) contributed a number of examples and problems. Dr. Charles Borges de Lima (Federal University of Santa Catarina, Florianopolis, Brazil) wrote the demonstration software as well as the Smith chart program included with this text while a postdoctoral student at The University of Akron in 2005–2006. Prof. Guido Bassotti (National University of San Juan, Argentina) has brought to my attention a number of errors and needed modifications to the second edition. I particularly wish to thank Prof. Richard E. Denton from Dartmouth College for providing a thorough critique, raising questions as to content and method and suggesting corrections and modifications. Many of the changes incorporated in the present edition, including chapter summaries, are based on his recommendations. I thank them all and invite comments from all who venture to study from this book.

Akron, OH, USA Nathan Ida
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Introduction to Electromagnetics

A Simple View of Electromagnetics

Charge is the fundamental electric quantity in nature. The same charge that will cause a spark when shuffling your shoes on a carpet and then touching a door knob or the charge that causes your clothes to stick together. The effects that constitute electromagnetics are directly linked to charge and to the behavior of charge. Charge exists in two forms. One is the charge of the electron and is negative. The second is that of a proton and is positive (and equal in magnitude to that of the electron). Experiment has shown that like charges reject each other whereas opposite charges attract. Electrons and protons occur in atoms, usually in pairs and thus materials are usually charge neutral. When a material acquires excess electrons it becomes negatively charged. Excess protons (deficiency of electrons) cause the material to be positively charged. Thus, when shuffling your shoes on a carpet, electrons are stripped off the carpet causing your body to have an excess of electrons (to become negatively charged) whereas the carpet becomes positively charged. Charges can be stationary or can move. Since charges can exert forces and can be affected by them, there is also an energy associated with charge and with its interactions. The charge itself and the way it moves define the electric, magnetic, or electromagnetic phenomena we observe. There are three possibilities that we consider and these three possibilities correspond to the range of phenomena that constitute electromagnetics.

(a) **Stationary charges**. A stationary charge produces an electric field intensity around itself. We shall see shortly that the electric field intensity is in fact force per unit charge and a field implies a distribution of this force in space and in materials. In a way we can say that the charge produces a force field around itself. This gives rise to an electric energy that can be useful. Examples of this energy are the forces that cause the toner to stick to a page in a copier or dust to accumulate on the TV screen, lightning, sparks in an internal combustion engine, a stunt gun, a camera flash, and the like. Since the charge is static, the phenomenon is called electrostatics (sometimes called static electricity). The electrostatic field is useful in a variety of applications including printing, copying, pollution control in power plants, painting, production of sand paper, and many others that we will discuss in **Chapters 3–6**.

(b) **Charges moving at a constant velocity.** Electrons and protons moving at constant velocity cause three effects:

1. An electric field intensity just as in **(a)**. The force field moves with the electron.
2. An electric current due to this motion since any moving charge constitutes an electric current.
3. A magnetic field intensity, produced by the current.

We say that a moving electron produces a static magnetic field since that field is constant in time as long as currents are constant (due to the constant velocity of charges). Since both an electric and a magnetic field exist, we can also say that moving charges generate an electromagnetic field. The relative strengths of the electric and magnetic fields dictate which of the effects dominates and hence the applications associated with moving charges. The magnetic field acts on currents with a magnetic force that can be very high. Again, we may say that a current generates a force field around itself. Many of the applications commonly in use belong here. These include lighting, electric machines, heating, power distribution, etc. These are mostly DC applications but also low-frequency AC applications including power generation and distribution systems. These and other applications will be disused in **Chapters 7–9**.

(c) Charges moving with a time-dependent velocity. Accelerating charge produces all the effects in (b) plus radiation of energy. Electrons moving with a time-dependent velocity necessarily generate time-dependent currents. Energy and hence power, radiate away from the time-dependent current. One can say that the electromagnetic field now propagates in space and in materials, at a specific velocity, carrying the energy associated with the electromagnetic field in a way that can best be described as a wave. The most common manifestation of this form is transmission and reception of signals to affect communication but also in applications such as microwave heating or X-ray imaging. The relations, phenomena, and applications resulting from this unique form of electromagnetic fields will be discussed in detail in **Chapters 11–18**.

Naturally, the transition between charges moving at constant velocity and charges that accelerate and decelerate is gradual and linked to alternating currents. That is, an AC current necessarily means charges move at a nonconstant velocity and, hence, any AC current will produce radiation. But, depending on the rate of acceleration of charges, the radiation effect may be dominant or may be negligible. As a rule, low-frequency currents radiate very little and behave more like DC currents. Because of that, the domain of low-frequency currents, and in particular at power frequencies, is often called ***quasi-static***, and the fields are said to be ***quasi-static fields***, meaning that the electric and magnetic fields behave more like those of charges moving at constant velocity than those due to accelerating charges. This is fortunate since it allows use of DC methods of analysis, such as circuit theory, to extend to low-frequency AC applications. This aspect of electromagnetic fields will be introduced in **Chapter 10** and, to an extent, in **Chapter 11** and serves as a transition to electromagnetic waves. At higher frequencies the radiation effects dominate, the observed behavior is totally different, and the tools necessary for analysis must also change.

It is important to emphasize that the electromagnetic effects have been "discovered" experimentally and their proof is based entirely on experimental observation. All laws of electromagnetics were obtained by careful measurements, which were then cast in the forms of mathematical relations. In the learning process, we will make considerable use of the mathematical tools outlined in **Chapters 1** and **2**. It is easy to forget that the end purpose is physical design; however, every relation and every equation implies some physical quantity or property of the fields involved. It is very important to remember that however involved the mathematics may seem, electromagnetics deals with practical physical phenomena, and when studying electromagnetics, we study the effects and implications of quantities that can be measured and, more importantly, that can and are being put to practical use. There are two reasons why it is important to emphasize electromagnetics as an applied science. First, it shows that it is a useful science, and its study leads to understanding of nature and, perhaps most significantly from the engineering point of view, to understanding of the application of electromagnetics to practical and useful designs.

Units

The system of units adopted throughout this book is the Système Internationale (SI). The SI units are defined by the International Committee for Weights and Measures and include seven base units as shown in **Table 1**. The base units are as follows:

Table 1 The base SI units

Physical quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	degree kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Length The **meter** (m) is the distance traveled by light in a vacuum during a time interval equal to 1/299,792,458 s.

Mass The **kilogram** (kg) is the prototype kilogram, a body made of a platinum-iridium compound and preserved in a vault in Sèvres, France.

Time The **second** (s) is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

Electric current The **ampère** (A) is the constant current that, if maintained in two straight conductors of infinite length and of negligible circular cross section, placed 1 m apart in a vacuum, produces between the conductors a force of 2×10^{-7} newton per meter (N/m).

Temperature The **kelvin** (K) unit of thermodynamic temperature is 1/273.16 of the thermodynamic temperature of the triple point of water (the temperature and pressure at which ice, water, and water vapor are in thermodynamic equilibrium). The triple point of water is 273.16 K at a vapor pressure of 611.73 Pa.

Luminous intensity The **candela** (cd) is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and has a radiation intensity in that direction of 1/683 watts per steradian (W/sr) (see the definition of steradian below).

Amount of substance The **mole** (mol) is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. (The entities may be atoms, molecules, ions, electrons, or any other particles.) The accepted number of entities (i.e., molecules) is known as Avogadro's number and equals approximately 6.0221×10^{23}.

Derived Units

All other metric units in common use are derived from the base SI units. These have been defined for convenience based on some physical law, even though they can be expressed directly in the base units. For example, the unit of force is the newton [N]. This is derived from Newton's law of force as $F=ma$. The unit of mass is the kilogram and the unit of acceleration is meters per second squared (m/s^2). Thus, the newton is in fact kilogram meters per second squared ($kg \cdot m/s^2$). Many of the common electrical units (such as the volt [V], the watt [W], and the ohm [Ω]) are derived units sanctioned by the SI standard. There are however some commonly used units that are discouraged (good examples are the watt-hour [W·h] and the electron-volt [e·V]). We will make considerable use of derived units throughout the book and these will be introduced and discussed as they become needed in the discussion.

Supplementary Units

The system of units includes so-called derived nondimensional units, also termed "supplementary units." These are the unit for the plane angle, the radian [rad], and the unit for the solid angle, the steradian [sr]. The radian is defined as the planar angle at the center of a circle of radius R subtended by an arc of length R. The steradian is defined as the solid angle at the center of a sphere of radius R subtended by a section of its surface, whose area equals R^2.

Customary Units

In addition to the base SI units and derived units sanctioned by the International Committee for Weights and Measures, there are many other units, some current, some obsolete, and some nonmetric. These are usually referred to as "customary units." They include commonly used units such as the calorie [cal] or the kilowatt-hour [kW • h] and less common units (except in the United States) such as the foot, mile, gallon, psi (pounds per square inch), and many others. Some units are associated almost exclusively with particular disciplines. The units may be SI, metric (current or obsolete), or customary. These have been defined for convenience and, as with any other unit, they represent a basic quantity that is meaningful in that discipline. For example, in astronomy one finds the astronomical unit [AU], which is equal to the average distance between earth and the sun (1 AU = 149,597,870.7 km). In physics, the angstrom [Å] represents atomic dimensions (1 Å= 0.1 nm). Similarly utilitarian units are the electron volt [e • V] for energy (1 e • V = 1.602×10^{-19} joule [J]), the atmosphere [atm] for pressure (1 atm = 101,325 pascal [Pa], 1Pa = 1 N/m^2), ppm (parts per million) for chemical quantities, the sievert [sv] (1 sv = 1 J/kg) for dose equivalents in radiation exposure, and so on. Although we will stick almost exclusively to SI units, it is important to remember that should the need arise to use customary units, conversion values to and from SI units can be substituted as necessary.

Prefixes

In conjunction with units, the SI system also defines the proper prefixes that provide standard notation of very small or very large quantities. The prefixes allow one to express large and small numbers in a compact and universal fashion and are summarized in **Table 2**. Again, this is mostly a convenience, but since their use is common, it is important to use the proper notation to avoid mistakes and confusion. Some of the prefixes are commonly used, others are rare, and still others are used in specialized areas. Atto, femto, peta, and exa are rarely used, whereas prefixes such as deca, deci, and hecto are more commonly used with liquids (but see below the usage of deci in decibel).

Other Units and Measures

Units of Information

There are a few other measures that are in common use in designating specific quantities. Since digital systems use base 2, base 8, or base 16 counting and mathematics, the decimal system is not particularly convenient as a measure. Therefore special prefixes have been devised for digital systems. The basic unit of information is the bit (a 0 or a 1). Bits are grouped into bytes, where 1 byte contains 8 bits, sometimes also called a "word." A kilobyte (kbyte or kb) is 2^{10} = 1024 bytes or 8192 bits. Similarly a megabyte [Mb] is 2^{20} (or 1024^2) bytes or 1,048,576 bytes (or 8,388,608 bits). Although these prefixes are confusing enough, their common usage is even more confusing, as it is common to mix digital and decimal prefixes. As an example, it is common to rate a storage device or memory board as containing,

Table 2 Common prefixes used in conjunction with the SI system of units

Prefix	Symbol	Multiplier	Examples
atto	a	10^{-18}	as (attosecond)
femto	f	10^{-15}	fs (femtosecond)
pico	p	10^{-12}	pF (picofarad)
nano	n	10^{-9}	nH (nanohenry)
micro	μ	10^{-6}	μm (micrometer)
milli	m	10^{-3}	mm (millimeter)
centi	c	10^{-2}	cl (centiliter)
deci	d	10^{-1}	dm (decimeter)
deca	da	10^{1}	dag (decagram)
hecto	h	10^{2}	hl (hectoliter)
kilo	k	10^{3}	kg (kilogram)
mega	M	10^{6}	MHz (megahertz)
giga	G	10^{9}	GW (gigawatt)
tera	T	10^{12}	Tb (terabit)
peta	P	10^{15}	PHz (petahertz)
Exa	E	10^{18}	EHz (exahertz)

say, 100 Gb. The digital prefix should mean that the device contains 2^{30} or 1024^3 bytes, or approximately 107.4×10^9 bytes. Rather, the device contains 100×10^9 bytes. In digital notation, the device actually contains only 91.13 Gb.

The Decibel (dB) and Its Use

There are instances in which the use of the common prefixes is inconvenient at the very least. In particular, when a physical quantity spans a very large range of numbers, it is difficult to properly grasp the magnitude of the quantity. Often, too, a quantity only has meaning with respect to a reference value. Take, for example, a voltage amplifier. It may be a unity amplifier or may amplify by a factor of 10^6 or more with a reference at 1. Another example is the human eye. It can see in luminance from about 10^{-6} cd/m^2 to 10^6 cd/m^2. This is a vast range and the natural reference value is the lowest luminance the eye can detect.

The use of normal scientific notation for such vast scales is inconvenient and is not particularly telling for a number of reasons. Using again the example of our eyes response to light, it is not linear, but rather logarithmic. That is, for an object to appear twice as bright, the illumination needs to be about 10 times higher. The same applies to sound and to many other quantities. In such instances, the quantities in question are described as ratios on a logarithmic scale using the notation of decibel (dB). The basic ideas in the use of the decibel are as follows:

1. Given a quantity, divide it by the reference value for that quantity. That may be a "natural" value, such as the threshold of vision, or it may be a constant, agreed upon value such as 1 or 10^{-6}.
2. Take the base 10 logarithm of the ratio.
3. If the quantities involved are power related (power, power density, energy, etc.), multiply by 10:

$$p = 10\log_{10}\left(\frac{P}{P_0}\right) \qquad [\text{dB}]$$

4. If the quantities involved are field quantities (voltage, current, force, pressure, etc.), multiply by 20:

$$v = 20\log_{10}\left(\frac{V}{V_0}\right) \qquad [\mathrm{dB}]$$

In the amplifier described above, a voltage amplification of 10^6 corresponds to 20log10(10^6/1) = 120 dB. In the case of vision, the reference value is 10^{-6} cd/m^2. A luminance of 10^{-6} cd/m^2 is therefore 0 dB. A luminance of 10^3 cd/m^2 is 10log10($10^3/10^{-6}$) = 90 dB. One can say that the human eye has a span of 120 dB.

When dealing with quantities of a specific range, the reference value can be selected to accommodate that range. For example, if one wishes to describe quantities that are typically in milliwatts [mW], the reference value is taken as 1 mW and power values are indicated in decibel milliwatts (dBm). Similarly, if one needs to deal with voltages in the microvolt [μV] range, the reference value is taken as 1 μV and the result is given in decibel microvolts [dBμV]. The use of a specific reference value simply places the 0 dB point at that value. As an example on the dBm scale, 0 dBm means 1 mW. On the normal scale, 0 dB means 1 W. It is therefore extremely important to indicate the scale used or confusion may occur. There are many different scales, each clearly denoted to make sure the reference value is known.

As a final note it should be remembered that unit analysis can facilitate understanding of the material and prevent errors in computation.

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