Appendix A
A Short Introduction to MATLAB

MATLAB[1] is an interactive software system for numerical computations, simulations and visualizations. It contains a large number of predefined functions and allows users to implement their programs in so-called *m-files*.

The name MATLAB originates from *MATrix LABoratory*, which indicates the matrix orientation of the software. Indeed, matrices are the major objects in MATLAB. Due to the simple and intuitive use of matrices, we consider MATLAB well suited for teaching in the field of Linear Algebra.

In this short introduction we explain the most important ways to enter and operate with matrices in MATLAB. One can learn the essential matrix operations as well as important algorithms and concepts in the context of matrices (and Linear Algebra in general) by actively using the *MATLAB-Minutes* in this book. These only use predefined functions.

A matrix in MATLAB can be entered in form of a list of entries enclosed by square brackets. The entries in the list are ordered by rows in the natural order of the indices, i.e., from "top to bottom" and "left to right"). A new row starts after every semicolon. For example, the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$ is entered in MATLAB by typing `A=[1 2 3;4 5 6;7 8 9];`

A semicolon after the matrix `A` suppresses the output in MATLAB. If it is omitted then MATLAB writes out all the entered or computed quantities. For example, after entering

```
A=[1 2 3;4 5 6;7 8 9]
```

[1]MATLAB® is a registered trademark of The MathWorks Inc.

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MATLAB gives the output

```
A =
     1   2   3
     4   5   6
     7   8   9
```

One can access parts of matrices by the corresponding indices. The list of indices from k to m is abbreviated by

`k:m`.

A colon : means all rows for given column indices, or all columns for given row indices. If `A` is as above, then for example

`A(2,1)`	is the matrix	$[4]$,
`A(3,1:2)`	is the matrix	$[7\ \ 8]$,
`A(:,2:3)`	is the matrix	$\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 8 & 9 \end{bmatrix}$.

There are several predefined functions that produce matrices. In particular, for given positive integers `n` and `m`,

`eye(n)`	the identity matrix I_n,
`zeros(n,m)`	an $n \times m$ matrix with all zeros,
`ones(n,m)`	an $n \times m$ matrix with all ones,
`rand(n,m)`	an $n \times m$ "random matrix".

Several matrices (of appropriate sizes) be combined to a new matrix. For example, the commands

```
A=eye(2);   B=[4;3];   C=[2  -1];   D=[-5]; E=[A  B;C  D]
```

lead to

```
E =
     1    0    4
     0    1    3
     2   -1   -5
```

The help function in MATLAB is started with the command `help`. In order to get information about specific functions one adds the name of the function. For example:

Input:	Information on:
`help ops`	operations and operators in MATLAB (in particular addition, multiplication, transposition)
`help matfun`	MATLAB functions that operate with matrices
`help gallery`	collection of example matrices
`help det`	determinant
`help expm`	matrix exponential function

Selected Historical Works on Linear Algebra

(We describe the content of these works using modern terms.)

- A. L. CAUCHY, *Sur l'équation à l'aide de laquelle on détermine les inégalités séculaires des mouvements des planètes*, Exercises de Mathématiques, 4 (1829).
  Proves that real symmetric matrices have real eigenvalues.
- H. GRASSMANN, *Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik*, Otto Wiegand, Leipzig, 1844.
  Contains the first development of abstract vector spaces and linear independence, including the dimension formula for subspaces.
- J. J. SYLVESTER, *Additions to the articles in the September Number of this Journal, "On a new Class of Theorems," and on Pascal's Theorem*, Philosophical Magazine, 37 (1850), pp. 363–370.
  Introduces the terms matrix and minor.
- J. J. SYLVESTER, *A demonstration of the theorem that every homogeneous quadratic polynomial is reducible by real orthogonal substitutions to the form of a sum of positive and negative squares*, Philosophical Magazine, 4 (1852), pp. 138–142.
  Proof of Sylvester's law of inertia.
- A. CAYLEY, *A memoir on the theory of matrices*, Proc. Royal Soc. of London, 148 (1858), pp. 17–37.
  First presentation of matrices as independent algebraic objects, including the basic matrix operations, the Cayley-Hamilton theorem (without a general proof) and the idea of a matrix square root.
- K. WEIERSTRASS, *Zur Theorie der bilinearen und quadratischen Formen*, Monatsber. Königl. Preußischen Akad. Wiss. Berlin, (1868), pp. 311–338.
  Proof of the Weierstrass normal form, which implies the Jordan normal form.
- C. JORDAN, *Traité des substitutions et des équations algébriques*, Paris, 1870.
  Contains the proof of the Jordan normal form independent of Weierstrass' work.
- G. FROBENIUS, *Ueber lineare Substitutionen und bilineare Formen*, J. reine angew. Math., 84 (1878), pp. 1–63.
  Contains the concept of the minimal polynomial, the (arguably) first complete proof of the Cayley-Hamilton theorem, and results on equivalence, similarity and congruence of matrices (or bilinear forms).
- G. PEANO, *Calcolo Geometrico secondo l'Ausdehnungslehre di H. Grassmann preceduto dalle operazioni della logica deduttiva*, Fratelli Bocca, Torino, 1888.
  Contains the first axiomatic definition of vector spaces, which Peano called "sistemi lineari", and studies properties of linear maps, including the (matrix) exponential function and the solution of differential equation systems.

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- I. SCHUR, *Über die charakteristischen Wurzeln einer linearen Substitution mit einer Anwendung auf die Theorie der Integralgleichungen*, Math. Annalen, 66 (1909), pp. 488–510.
  Proof of the Schur form of complex matrices.
- O. TOEPLITZ, *Das algebraische Analogon zu einem Satze von Fejér*, Math. Zeitschrift, 2 (1918), pp. 187–197.
  Introduces the concept of a normal bilinear form and proves the equivalence of normality and unitary diagonalizability.
- F. D. MURNAGHAN AND A. WINTNER, *A canonical form for real matrices under orthogonal transformations*, Proc. Natl. Acad. Sci. U.S.A., 17 (1931), pp. 417–420.
  Proof of the real Schur form.
- C. ECKART AND G. YOUNG, *A principal axis transformation for non-hermitian matrices*, Bull. Amer. Math. Soc., 45 (1939), pp. 118–121.
  Proof of the modern form of the singular value decomposition of a general complex matrix.

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