

Photo Credits

- page 1: Pierre Louis Dumesnil; 1884 copy by Nils Forsberg/Public domain image from *Wikimedia*.
- page 27: George M. Bergman/Archives of the Mathematisches Forschungsinstitut Oberwolfach.
- page 51: Gottlieb Biermann; photo by A. Wittmann/Public domain image from *Wikimedia*.
- page 117: Mostafa Azizi/Public domain image from *Wikimedia*.
- page 131: Hans-Peter Postel/Public domain image from *Wikimedia*.
- page 163: Public domain image from *Wikimedia*.
- page 203: Public domain image from *Wikimedia*. Original painting is in Tate Britain.
- page 224: Spiked Math.
- page 241: Public domain image from *Wikimedia*.
- page 275: Public domain image from *Wikimedia*. Original fresco is in the Vatican.
- page 295: Public domain image from *Wikimedia*.

Symbol Index

- A^{-1} , 296
 $A_{j,\cdot}$, 76
 $A_{j,k}$, 70
 $A_{\cdot,k}$, 76
 A^t , 109
- C**, 2
- deg, 31
 Δ , 179
det, 307, 314
dim, 44
 \oplus , 21
 D_k , 328
- $E(\lambda, T)$, 155
- F**, 4
 \mathbf{F}^∞ , 13
 $\mathbf{F}^{m,n}$, 73
 \mathbf{F}^n , 6
 \mathbf{F}^S , 14
- $G(\lambda, T)$, 245
- I , 52, 296
 \iff , 207
- Im, 118
 $-\infty$, 31
 $\int_\Omega f$, 327
- $\mathcal{L}(V)$, 86
 $\mathcal{L}(V, W)$, 52
- $\mathcal{M}(T)$, 70, 146
 $\mathcal{M}(v)$, 84
- perm, 311
- $\mathcal{P}(\mathbf{F})$, 30
 π , 97
 $\mathcal{P}_m(\mathbf{F})$, 31
 $p(T)$, 143
 P_U , 195
- R**, 2
Re, 118
- σ' , 327
 \subsetneq , 243
 \tilde{T} , 97
 \sqrt{T} , 233
 T' , 103
 T^* , 204
 T^{-1} , 80
 $T(\Omega)$, 324
 $T_{\mathbf{C}}$, 277
 T^m , 143
 $T|_U$, 132, 137
 T/U , 137
- U^\perp , 193
 U^0 , 104
 $\langle u, v \rangle$, 166
- V , 16
 $\|v\|$, 168
 V' , 101
 V/U , 95
 $-v$, 15
 $V_{\mathbf{C}}$, 276
 $v + U$, 94
- \bar{z} , 118
 $|z|$, 118

Index

- absolute value, 118
- addition
 - in quotient space, 96
 - of complex numbers, 2
 - of functions, 14
 - of linear maps, 55
 - of matrices, 72
 - of subspaces, 20
 - of vectors, 12
 - of vectors in \mathbf{F}^n , 7
- additive inverse
 - in \mathbf{C} , 3, 4
 - in \mathbf{F}^n , 9
 - in vector space, 12, 15
- additivity, 52
- adjoint of a linear map, 204
- affine subset, 94
- algebraic multiplicity, 255
- annihilator of a subspace, 104
- Apollonius's Identity, 179
- associativity, 3, 12, 56
- backward shift, 53, 59, 81, 86, 140
- basis, 39
 - of eigenvectors, 157, 218, 221, 224, 268
 - of generalized eigenvectors, 254
- Binet, Jacques, 317
- Blake, William, 203
- block diagonal matrix, 255
- box in \mathbf{R}^n , 323
- Cauchy, Augustin-Louis, 171, 317
- Cauchy–Schwarz Inequality, 172
- Cayley, Arthur, 262
- Cayley–Hamilton Theorem
 - on complex vector space, 261
 - on real vector space, 284
- change of basis, 298
- change of variables in integral, 328
- characteristic polynomial
 - on complex vector space, 261
 - on real vector space, 283
- characteristic value, 134
- Christina, Queen of Sweden, 1
- closed under addition, 18
- closed under scalar multiplication, 18
- column rank of a matrix, 111
- commutativity, 3, 7, 12, 25, 56, 75, 79, 144, 212
- complex conjugate, 118
- complex number, 2
- Complex Spectral Theorem, 218
- complex vector space, 13
- complexification
 - of a vector space, 276
 - of an operator, 277
- conjugate symmetry, 166

- conjugate transpose of a matrix, 207
- coordinate, 6
- cube root of an operator, 224
- cubic formula, 124
- degree of a polynomial, 31
- derivative, 327
- Descartes, René, 1
- determinant
 - of a matrix, 314
 - of an operator, 307
- diagonal matrix, 155
- diagonal of a square matrix, 147
- diagonalizable, 156
- differentiable, 327
- differentiation linear map, 53, 56, 59, 61, 62, 69, 72, 78, 144, 190, 248, 294
- dimension, 44
 - of a sum of subspaces, 47
- direct sum, 21, 42, 93
 - of a subspace and its orthogonal complement, 194
 - of null T^n and range T^n , 243
- distributive property, 3, 12, 16, 56, 79
- Division Algorithm for Polynomials, 121
- division of complex numbers, 4
- dot product, 164
- double dual space, 116
- dual
 - of a basis, 102
 - of a linear map, 103
 - of a vector space, 101
- eigenspace, 155
- eigenvalue of an operator, 134
- eigenvector, 134
- Euclid, 275
- Euclidean inner product, 166
- factor of a polynomial, 122
- Fibonacci, 131
- Fibonacci sequence, 161
- field, 10
- finite-dimensional vector space, 30
- Flatland*, 6
- Fundamental Theorem of Algebra, 124
- Fundamental Theorem of Linear Maps, 63
- Gauss, Carl Friedrich, 51
- generalized eigenspace, 245
- generalized eigenvector, 245
- geometric multiplicity, 255
- Gram, Jørgen, 182
- Gram–Schmidt Procedure, 182
- graph of a linear map, 98
- Halmos, Paul, 27
- Hamilton, William, 262
- harmonic function, 179
- Hermitian, 209
- homogeneity, 52
- homogeneous system of linear equations, 65, 90
- Hypatia, 241
- identity map, 52, 56
- identity matrix, 296
- image, 62
- imaginary part, 118
- infinite-dimensional vector space, 31
- inhomogeneous system of linear equations, 66, 90
- injective, 60
- inner product, 166

- inner product space, 167
- integral, 327
- invariant subspace, 132
- inverse
 - of a linear map, 80
 - of a matrix, 296
- invertible linear map, 80
- invertible matrix, 296
- isometry, 228, 292, 321
- isomorphic vector spaces, 82
- isomorphism, 82
- Jordan basis, 273
- Jordan Form, 273
- Jordan, Camille, 272
- kernel, 59
- Khayyám, Omar, 117
- Laplacian, 179
- length of list, 5
- Leonardo of Pisa, 131
- linear combination, 28
- Linear Dependence Lemma, 34
- linear functional, 101, 187
- linear map, 52
- linear span, 29
- linear subspace, 18
- linear transformation, 52
- linearly dependent, 33
- linearly independent, 32
- list, 5
 - of vectors, 28
- Lovelace, Ada, 295
- matrix, 70
 - multiplication, 75
 - of linear map, 70
 - of nilpotent operator, 249
 - of operator, 146
 - of product of linear maps, 75, 297
 - of T' , 110
 - of T^* , 208
 - of vector, 84
- minimal polynomial, 263, 279
- minimizing distance, 198
- monic polynomial, 262
- multiplication, *see* product
- multiplicity of an eigenvalue, 254
- Newton, Isaac, 203
- nilpotent operator, 248, 271
- nonsingular matrix, 296
- norm, 164, 168
- normal operator, 212, 287
- null space, 59
 - of powers of an operator, 242
 - of T' , 106
 - of T^* , 207
- one-to-one, 60
- onto, 62
- operator, 86
- orthogonal
 - complement, 193
 - operator, 229
 - projection, 195
 - vectors, 169
- orthonormal
 - basis, 181
 - list, 180
- parallel affine subsets, 94
- Parallelogram Equality, 174
- permutation, 311
- photo credits, 333
- point, 13
- polar coordinates, 329
- Polar Decomposition, 233
- polynomial, 30
- positive operator, 225
- positive semidefinite operator, 227

- product
 - of complex numbers, 2
 - of linear maps, 55
 - of matrices, 75
 - of polynomials, 144
 - of scalar and linear map, 55
 - of scalar and vector, 12
 - of scalar and vector in \mathbf{F}^n , 10
 - of vector spaces, 91
- Pythagorean Theorem, 170
- quotient
 - map, 97
 - operator, 137
 - space, 95
- range, 61
 - of powers of an operator, 251
 - of T' , 107
 - of T^* , 207
- rank of a matrix, 112
- Raphael, 275
- real part, 118
- Real Spectral Theorem, 221
- real vector space, 13
- restriction operator, 137
- Riesz Representation Theorem, 188
- Riesz, Frigyes, 187
- row rank of a matrix, 111
- scalar, 4
- scalar multiplication, 10, 12
 - in quotient space, 96
 - of linear maps, 55
 - of matrices, 73
- Schmidt, Erhard, 182
- School of Athens*, 275
- Schur's Theorem, 186
- Schur, Issai, 186
- Schwarz, Hermann, 171
- self-adjoint operator, 209
- sign of a permutation, 312
- signum, 313
- singular matrix, 296
- Singular Value Decomposition, 237
- singular values, 236
- span, 29
- spans, 30
- Spectral Theorem, 218, 221
- spherical coordinates, 330
- square root of an operator, 223, 225, 233, 259
- standard basis, 39
- subspace, 18
- subtraction of complex numbers, 4
- sum, *see* addition
- Supreme Court, 174
- surjective, 62
- trace
 - of a matrix, 300
 - of an operator, 299
- transpose of a matrix, 109, 207
- Triangle Inequality, 173
- tuple, 5
- unitary operator, 229
- upper-triangular matrix, 147, 256
- vector, 8, 13
- vector space, 12
- volume, 324
- zero of a polynomial, 122