Springer Undergraduate Mathematics Series

More information about this series at http://www.springer.com/series/3423

Jörg Liesen · Volker Mehrmann

Linear Algebra

Springer

Jörg Liesen
Institute of Mathematics
Technical University of Berlin
Berlin
Germany

Volker Mehrmann
Institute of Mathematics
Technical University of Berlin
Berlin
Germany

ISSN 1615-2085 ISSN 2197-4144 (electronic)
Springer Undergraduate Mathematics Series
ISBN 978-3-319-24344-3 ISBN 978-3-319-24346-7 (eBook)
DOI 10.1007/978-3-319-24346-7

Library of Congress Control Number: 2015950442

Mathematics Subject Classification (2010): 15-01

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Preface

This is a translation of the (slightly revised) second German edition of our book "Lineare Algebra", published by Springer Spektrum in 2015. Our general view of the field of Linear Algebra and the approach to it that we have chosen in this book were already described in our Preface to the First German Edition, published by Vieweg+Teubner in 2012. In a nutshell, our exposition is matrix-oriented, and we aim at presenting a rather complete theory (including all details and proofs), while keeping an eye on the applicability of the results. Many of them, though appearing very theoretical at first sight, are of an immediate practical relevance. In our experience, the matrix-oriented approach to Linear Algebra leads to a better intuition and a deeper understanding of the abstract concepts, and therefore simplifies their use in real-world applications.

Starting from basic mathematical concepts and algebraic structures we develop the classical theory of matrices, vectors spaces, and linear maps, culminating in the proof of the Jordan canonical form. In addition to the characterization of important special classes of matrices or endomorphisms, the last chapters of the book are devoted to special topics: Matrix functions and systems of differential equations, the singular value decomposition, the Kronecker product, and linear matrix equations. These chapters can be used as starting points of more advanced courses or seminars in Applied Linear Algebra.

Many people helped us with the first two German editions and this English edition of the book. In addition to those mentioned in the Preface to the First German Edition, we would like to particularly thank Olivier Sète, who carefully worked through the entire draft of the second edition and gave numerous comments, as well as Leonhard Batzke, Carl De Boor, Sadegh Jokar, Robert Luce, Christian Mehl, Helia Niroomand Rad, Jan Peter Schäfermeier, Daniel Wachsmuth, and Gisbert

Wüstholz. Thanks also to the staff of Springer Spektrum, Heidelberg, and Springer-Verlag, London, for their support and assistance with editorial aspects of this English edition.

Berlin
July 2015

Jörg Liesen
Volker Mehrmann

Preface to the First German Edition

> Mathematics is the instrument that links theory and practice, thinking and observing; it establishes the connecting bridge and builds it stronger and stronger. This is why our entire culture these days, as long as it is concerned with understanding and harnessing nature, has Mathematics as its foundation.[1]

This assessment of the famous mathematician David Hilbert (1862–1943) is even more true today. Mathematics is found not only throughout the classical natural sciences, Biology, Chemistry and Physics, its methods have become indispensable in Engineering, Economics, Medicine, and many other areas of life. This continuing mathematization of the world is possible because of the *transversal strength* of Mathematics. The abstract objects and operations developed in Mathematics can be used for the description and solution of problems in numerous different situations.

While the high level of abstraction of modern Mathematics continuously increases its potential for applications, it represents a challenge for students. This is particularly true in the first years, when they have to become familiar with a lot of new and complicated terminology. In order to get students excited about mathematics and capture their imagination, it is important for us teachers of basic courses such as Linear Algebra to present Mathematics as a living science in its global context. The short historical notes in the text and the list of some historical papers at the end of this book show that Linear Algebra is the result of a human endeavor.

An important guideline of the book is to demonstrate the *immediate practical relevance* of the developed theory. Right in the beginning we illustrate several concepts of Linear Algebra in everyday life situations. We discuss mathematical basics of the search engine Google and of the premium rate calculations of car

[1] "Das Instrument, welches die Vermittlung bewirkt zwischen Theorie und Praxis, zwischen Denken und Beobachten, ist die Mathematik; sie baut die verbindende Brücke und gestaltet sie immer tragfähiger. Daher kommt es, dass unsere ganze gegenwärtige Kultur, soweit sie auf der geistigen Durchdringung und Dienstbarmachung der Natur beruht, ihre Grundlage in der Mathematik findet."

insurances. These and other applications will be investigated in later chapters using theoretical results. Here the goal is not to study the concrete examples or their solutions, but the presentation of the transversal strength of mathematical methods in the Linear Algebra context.

The central object for our approach to Linear Algebra is the *matrix*, which we introduce early on, immediately after discussing some of the basic mathematical foundations. Several chapters deal with some of their most important properties, before we finally make the big step to abstract vector spaces and homomorphisms. In our experience the matrix-oriented approach to Linear Algebra leads to a better intuition and a deeper understanding of the abstract concepts.

The same goal should be reached by the MATLAB-Minutes[2] that are scattered throughout the text and that allow readers to comprehend the concepts and results via computer experiments. The required basics for these short exercises are introduced in the Appendix. Besides the MATLAB-Minutes there are a large number of classical exercises, which just require a pencil and paper.

Another advantage of the matrix-oriented approach to Linear Algebra is given by the simplifications when transferring theoretical results into practical algorithms. Matrices show up wherever data are systematically ordered and processed, which happens in almost all future job areas of bachelor students in the mathematical sciences. This has also motivated the topics in the last chapters of this book: matrix functions, the singular value decomposition, and the Kronecker product.

Despite many comments on algorithmic and numerical aspects, the focus in this book is on the theory of Linear Algebra. The German physicist Gustav Robert Kirchhoff (1824–1887) is attributed to have said:

> A good theory is the most practical thing there is.[3]

This is exactly how we view our approach to the field.

This book is based on our lectures at TU Chemnitz and TU Berlin. We would like to thank all students, co-workers, and colleagues who helped in preparing and proofreading the manuscript, in the formulation of exercises, and with the content of lectures. Our special thanks go to André Gaul, Florian Goßler, Daniel Kreßner, Robert Luce, Christian Mehl, Matthias Pester, Robert Polzin, Timo Reis, Olivier Sète, Tatjana Stykel, Elif Topcu, Wolfgang Wülling, and Andreas Zeiser.

We also thank the staff of the Vieweg+Teubner Verlag and, in particular, Ulrike Schmickler-Hirzebruch, who strongly supported this endeavor.

Berlin
July 2011

Jörg Liesen
Volker Mehrmann

[2] MATLAB® trademark of The MathWorks Inc.

[3] "Eine gute Theorie ist das Praktischste, was es gibt."

Contents