Undergraduate Texts in Mathematics

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Real Analysis

Foundations and Functions of One Variable

First English Edition

Springer

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Preface

Analysis forms an essential basis of mathematics as a whole, as well as of the natural sciences, and more and more of the social sciences too. The theory of analysis (differentiation and integration) was created—after Galileo's insight—for the purposes of describing the universe in the language of mathematics. Working out the precise theory took almost 300 years, with a large portion of this time devoted to definitions that encapsulate the essence of limits and continuity. Mastering these concepts can be a difficult process; this is one of the reasons why analysis is only barely present in most high-school curricula.

At the same time, in postsecondary education where mathematics is part of the program—including various branches of science and mathematics—analysis appears as a basic requirement. Our book is intended to be an introductory analysis textbook; we believe it would be useful in any areas where analysis is a part of the curriculum, in addition to the above, also in teacher education, engineering, or even some high schools. In writing this book, we used the experience we gained from our many decades of lectures at the Eotvos Lorand University, Budapest, Hungary.

We have placed strong emphasis on discussing the foundations of analysis: before we begin the actual topic of analysis, we summarize all that the theory builds upon (basics of logic, sets, real numbers), even though some of these concepts might be familiar from previous studies. We believe that a strong basis is needed not only for those who wish to master higher levels of analysis, but for everyone who wants to apply it, and especially for those who wish to teach analysis at any level.

The central concepts of analysis are limits, continuity, the derivative, and the integral. Our primary goal was to develop the precise concepts gradually, building on intuition and using many examples. We introduce and discuss applications of our topics as much as possible, while ensuring that understanding and mastering of this difficult material is advanced. This, among other reasons, is why we avoided a more abstract or general (topological or multiple variable) discussion.

We would like to emphasize that the—classical, mostly more than 100 year old—results discussed here still inspire active research in different areas. Due to the nature of this book, we cannot delve into this; we only mention a small handful of unsolved problems.

Mastering the material can be only achieved through solving many exercises of various difficulties. We have posed more than 500 exercises in our book, but few of these are routine questions—which can be found in many workbooks and exercise collections. However, we found it important to include questions that call for deeper understanding of results and methods. Of these, several more difficult questions, requiring novel ideas, are marked by $(*)$. A large number of exercises come with hints for solutions, while many others are provided with complete solutions. Exercises with hints and solutions are denoted by (H) and (S) respectively.

The book contains a much greater amount of material than what is necessary for most curricula. We trust that the organization of the book—namely the structure of subsections—makes the selection of self-contained curricula possible for several levels.

The book drew from Vera T. Sós' university textbook *Analízis*, which has been in print for over 30 years, as well as analysis lecture notes by Miklós Laczkovich. This book, which is the translation of the third edition of the Hungarian original, naturally expands on these sources and the previous editions in both content and presentation.

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