

Graduate Texts in Physics

Series editors

Kurt H. Becker, Polytechnic School of Engineering, Brooklyn, USA

Sadri Hassani, Illinois State University, Normal, USA

Bill Munro, NTT Basic Research Laboratories, Atsugi, Japan

Richard Needs, University of Cambridge, Cambridge, UK

Jean-Marc Di Meglio, Université Paris Diderot, Paris, France

William T. Rhodes, Florida Atlantic University, Boca Raton, USA

Susan Scott, Australian National University, Acton, Australia

H. Eugene Stanley, Boston University, Boston, USA

Martin Stutzmann, TU München, Garching, Germany

Andreas Wipf, Friedrich-Schiller-Univ Jena, Jena, Germany

Graduate Texts in Physics

Graduate Texts in Physics publishes core learning/teaching material for graduate- and advanced-level undergraduate courses on topics of current and emerging fields within physics, both pure and applied. These textbooks serve students at the MS- or PhD-level and their instructors as comprehensive sources of principles, definitions, derivations, experiments and applications (as relevant) for their mastery and teaching, respectively. International in scope and relevance, the textbooks correspond to course syllabi sufficiently to serve as required reading. Their didactic style, comprehensiveness and coverage of fundamental material also make them suitable as introductions or references for scientists entering, or requiring timely knowledge of, a research field.

More information about this series at <http://www.springer.com/series/8431>

Rainer Dick

Advanced Quantum Mechanics

Materials and Photons

Second Edition

 Springer

Rainer Dick
Department of Physics and Engineering Physics
University of Saskatchewan
Saskatoon, Saskatchewan
Canada

ISSN 1868-4513

Graduate Texts in Physics

ISBN 978-3-319-25674-0

DOI 10.1007/978-3-319-25675-7

ISSN 1868-4521 (electronic)

ISBN 978-3-319-25675-7 (eBook)

Library of Congress Control Number: 2016932403

© Springer International Publishing Switzerland 2012, 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

Preface to the Second Edition

The second edition features 62 additional end of chapter problems and many sections were edited for clarity and improvement of presentation. Furthermore, the chapter on Klein-Gordon and Dirac fields has been expanded and split into Chapter 21 on relativistic quantum fields and Chapter 22 on applications of quantum electrodynamics. This was motivated by the renewed interest in the notions and techniques of relativistic quantum theory due to their increasing relevance for materials research. Of course, relativistic quantum theory has always been an important tool in subatomic physics and in quantum optics since the dynamics of photons or high energy particles is expressed in terms of relativistic quantum fields. Furthermore, relativistic quantum mechanics has also always been important for chemistry and condensed matter physics through the impact of relativistic corrections to the Schrödinger equation, primarily through the Pauli term and through spin-orbit couplings. These terms usually dominate couplings to magnetic fields and relativistic corrections to energy levels in materials, and spin-orbit couplings became even more prominent due to their role in manipulating spins in materials through electric fields. Relativistic quantum mechanics has therefore always played an important foundational role throughout the physical sciences and engineering.

However, we have even seen discussions of fully quasirelativistic wave equations in materials research in recent years. This development is driven by discoveries of materials like Graphene or Dirac semimetals, which exhibit low energy effective Lorentz symmetries in sectors of momentum space. In these cases c and m become effective low energy parameters which parametrize quasirelativistic cones or hyperboloids in regions of (E, \mathbf{k}) space. As a consequence, materials researchers now do not only deal with Pauli and spin-orbit terms, but with representations of γ matrices and solutions of Dirac equations in various dimensions.

To prepare graduate students in the physical sciences and engineering better for the increasing number of applications of (quasi-)relativistic quantum physics, Section 21.5 on the non-relativistic limit of the Dirac equation now also contains a detailed discussion of the Foldy-Wouthuysen transformation including a derivation of the general spin-orbit coupling term and a discussion of the origin of Rashba

terms, and the Section 21.6 on quantization of the Maxwell field in Lorentz gauge has been added. The discussion of applications of quantum electrodynamics now also includes the new Section 22.2 on electron-nucleus scattering. Finally, the new Appendix I discusses the transformation properties of scalars, spinors and gauge fields under parity or time reversal.

Saskatoon, SK, Canada

Rainer Dick

Preface to the First Edition

Quantum mechanics was invented in an era of intense and seminal scientific research between 1900 and 1928 (and in many regards continues to be developed and expanded) because neither the properties of atoms and electrons, nor the spectrum of radiation from heat sources could be explained by the classical theories of mechanics, electrodynamics and thermodynamics. It was a major intellectual achievement and a breakthrough of curiosity driven fundamental research which formed quantum theory into one of the pillars of our present understanding of the fundamental laws of nature. The properties and behavior of every elementary particle is governed by the laws of quantum theory. However, the rule of quantum mechanics is not limited to atomic and subatomic scales, but also affects macroscopic systems in a direct and profound manner. The electric and thermal conductivity properties of materials are determined by quantum effects, and the electromagnetic spectrum emitted by a star is primarily determined by the quantum properties of photons. It is therefore not surprising that quantum mechanics permeates all areas of research in advanced modern physics and materials science, and training in quantum mechanics plays a prominent role in the curriculum of every major physics or chemistry department.

The ubiquity of quantum effects in materials implies that quantum mechanics also evolved into a major tool for advanced technological research. The construction of the first nuclear reactor in Chicago in 1942 and the development of nuclear technology could not have happened without a proper understanding of the quantum properties of particles and nuclei. However, the real breakthrough for a wide recognition of the relevance of quantum effects in technology occurred with the invention of the transistor in 1948 and the ensuing rapid development of semiconductor electronics. This proved once and for all the importance of quantum mechanics for the applied sciences and engineering, only 22 years after publication of the Schrödinger equation! Electronic devices like transistors rely heavily on the quantum mechanical emergence of energy bands in materials, which can be considered as a consequence of combination of many atomic orbitals or as a consequence of delocalized electron states probing a lattice structure. Today the rapid developments of spintronics, photonics and nanotechnology provide continuing testimony to the technological relevance of quantum mechanics.

As a consequence, every physicist, chemist and electrical engineer nowadays has to learn aspects of quantum mechanics, and we are witnessing a time when also mechanical and aerospace engineers are advised to take at least a 2nd year course, due to the importance of quantum mechanics for elasticity and stability properties of materials. Furthermore, quantum information appears to become increasingly relevant for computer science and information technology, and a whole new area of quantum technology will likely follow in the wake of this development. Therefore it seems safe to posit that within the next two generations, 2nd and 3rd year quantum mechanics courses will become as abundant and important in the curricula of science and engineering colleges as first and second year calculus courses.

Quantum mechanics continues to play a dominant role in particle physics and atomic physics – after all, the Standard Model of particle physics is a quantum theory, and the spectra and stability of atoms cannot be explained without quantum mechanics. However, most scientists and engineers use quantum mechanics in advanced materials research. Furthermore, the dominant interaction mechanisms in materials (beyond the nuclear level) are electromagnetic, and many experimental techniques in materials science are based on photon probes. The introduction to quantum mechanics in the present book takes this into account by including aspects of condensed matter theory and the theory of photons at earlier stages and to a larger extent than other quantum mechanics texts. Quantum properties of materials provide neat and very interesting illustrations of time-independent and time-dependent perturbation theory, and many students are better motivated to master the concepts of quantum mechanics when they are aware of the direct relevance for modern technology. A focus on the quantum mechanics of photons and materials is also perfectly suited to prepare students for future developments in quantum information technology, where entanglement of photons or spins, decoherence, and time evolution operators will be key concepts.

Other novel features of the discussion of quantum mechanics in this book concern attention to relevant mathematical aspects which otherwise can only be found in journal articles or mathematical monographs. Special appendices include a mathematically rigorous discussion of the completeness of Sturm-Liouville eigenfunctions in one spatial dimension, an evaluation of the Baker-Campbell-Hausdorff formula to higher orders, and a discussion of logarithms of matrices. Quantum mechanics has an extremely rich and beautiful mathematical structure. The growing prominence of quantum mechanics in the applied sciences and engineering has already reinvigorated increased research efforts on its mathematical aspects. Both students who study quantum mechanics for the sake of its numerous applications, as well as mathematically inclined students with a primary interest in the formal structure of the theory should therefore find this book interesting.

This book emerged from a quantum mechanics course which I had introduced at the University of Saskatchewan in 2001. It should be suitable both for advanced undergraduate and introductory graduate courses on the subject. To make advanced quantum mechanics accessible to wider audiences which might not have been exposed to standard second and third year courses on atomic physics, analytical mechanics, and electrodynamics, important aspects of these topics are briefly, but

concisely introduced in special chapters and appendices. The success and relevance of quantum mechanics has reached far beyond the realms of physics research, and physicists have a duty to disseminate the knowledge of quantum mechanics as widely as possible.

Saskatoon, SK, Canada

Rainer Dick

To the Students

Congratulations! You have reached a stage in your studies where the topics of your inquiry become ever more interesting and more relevant for modern research in basic science and technology.

Together with your professors, I will have the privilege to accompany you along the exciting road of your own discovery of the bizarre and beautiful world of quantum mechanics. I will aspire to share my own excitement that I continue to feel for the subject and for science in general.

You will be introduced to many analytical and technical skills that are used in everyday applications of quantum mechanics. These skills are essential in virtually every aspect of modern research. A proper understanding of a materials science measurement at a synchrotron requires a proper understanding of photons and quantum mechanical scattering, just like manipulation of qubits in quantum information research requires a proper understanding of spin and photons and entangled quantum states. Quantum mechanics is ubiquitous in modern research. It governs the formation of microfractures in materials, the conversion of light into chemical energy in chlorophyll or into electric impulses in our eyes, and the creation of particles at the Large Hadron Collider.

Technical mastery of the subject is of utmost importance for understanding quantum mechanics. Trying to decipher or apply quantum mechanics without knowing how it really works in the calculation of wave functions, energy levels, and cross sections is just idle talk, and always prone for misconceptions. Therefore we will go through a great many technicalities and calculations, because you and I (and your professor!) have a common goal: You should become an expert in quantum mechanics.

However, there is also another message in this book. The apparently exotic world of quantum mechanics is *our* world. Our bodies and all the world around us is built on quantum effects and ruled by quantum mechanics. It is not apparent and only visible to the *cognoscenti*. Therefore we have developed a mode of thought and explanation of the world that is based on classical pictures – mostly waves and particles in mechanical interaction. This mode of thought was amended by the notions of gravitational and electromagnetic forces, thus culminating in a powerful tool called classical physics. However, by 1900 those who were paying attention had caught enough glimpses of the underlying non-classical world to embark on the exciting journey of discovering quantum mechanics. Indeed, every single atom in your body is ruled by the laws of quantum mechanics, and could not even exist as a classical particle. The electrons that provide the light for your long nights of studying generate this light in stochastic quantum leaps from a state of a single electron to a state of an electron and a photon. And maybe the most striking example of all: There is *absolutely nothing classical* in the sunlight that provides the energy for all life on Earth.

Quantum theory is not a young theory any more. The scientific foundations of the subject were developed over half a century between 1900 and 1949, and

many of the mathematical foundations were even developed in the 19th century. The steepest ascent in the development of quantum theory appeared between 1924 and 1928, when matrix mechanics, Schrödinger's equation, the Dirac equation and field quantization were invented. I have included numerous references to original papers from this period, not to ask you to read all those papers – after all, the primary purpose of a textbook is to put major achievements into context, provide an introductory overview at an appropriate level, and replace often indirect and circuitous original derivations with simpler explanations – but to honour the people who brought the then nascent theory to maturity. Quantum theory is an extremely well established and developed theory now, which has proven itself on numerous occasions. However, we still continue to improve our collective understanding of the theory and its wide ranging applications, and we test its predictions and its probabilistic interpretation with ever increasing accuracy. The implications and applications of quantum mechanics are limitless, and we are witnessing a time when many technologies have reached their “quantum limit”, which is a misnomer for the fact that any methods of classical physics are just useless in trying to describe or predict the behavior of atomic scale devices. It is a “limit” for those who do not want to learn quantum physics. For you, it holds the promise of excitement and opportunity if you are prepared to work hard and if you can understand the calculations.

Quantum mechanics combines power and beauty in a way that even supersedes advanced analytical mechanics and electrodynamics. Quantum mechanics is universal and therefore incredibly versatile, and if you have a sense for mathematical beauty: The structure of quantum mechanics is breathtaking, indeed.

I sincerely hope that reading this book will be an enjoyable and exciting experience for you.

To the Instructor

Dear Colleague,

As professors of quantum mechanics courses, we enjoy the privilege of teaching one of the most exciting subjects in the world. However, we often have to do this with fewer lecture hours than were available for the subject in the past, when at the same time we should include more material to prepare students for research or modern applications of quantum mechanics. Furthermore, students have become more mobile between universities (which is good) and between academic programs (which can have positive and negative implications). Therefore we are facing the task to teach an advanced subject to an increasingly heterogeneous student body with very different levels of preparation. Nowadays the audience in a fourth year undergraduate or beginning graduate course often includes students who have not gone through a course on Lagrangian mechanics, or have not seen the covariant formulation of electrodynamics in their electromagnetism courses. I deal with this

problem by including one special lecture on each topic in my quantum mechanics course, and this is what Appendices A and B are for. I have also tried to be as inclusive as possible without sacrificing content or level of understanding by starting at a level that would correspond to an advanced second year Modern Physics or Quantum Chemistry course and then follow a steeply ascending route that takes the students all the way from Planck's law to the photon scattering tensor.

The selection and arrangement of topics in this book is determined by the desire to develop an advanced undergraduate and introductory graduate level course that is useful to as many students as possible, in the sense of giving them a head start into major current research areas or modern applications of quantum mechanics without neglecting the necessary foundational training.

There is a core of knowledge that every student is expected to know by heart after having taken a course in quantum mechanics. Students must know the Schrödinger equation. They must know how to solve the harmonic oscillator and the Coulomb problem, and they must know how to extract information from the wave function. They should also be able to apply basic perturbation theory, and they should understand that a wave function $\langle x|\psi(t)\rangle$ is only one particular representation of a quantum state $|\psi(t)\rangle$.

In a North American physics program, students would traditionally learn all these subjects in a 300-level Quantum Mechanics course. Here these subjects are discussed in Chapters 1–7 and 9. This allows the instructor to use this book also in 300-level courses or introduce those chapters in a 400-level or graduate course if needed. Depending on their specialization, there will be an increasing number of students from many different science and engineering programs who will have to learn these subjects at M.Sc. or beginning Ph.D. level before they can learn about photon scattering or quantum effects in materials, and catering to these students will also become an increasingly important part of the mandate of physics departments. Including chapters 1–7 and 9 with the book is part of the philosophy of being as inclusive as possible to disseminate knowledge in advanced quantum mechanics as widely as possible.

Additional training in quantum mechanics in the past traditionally focused on atomic and nuclear physics applications, and these are still very important topics in fundamental and applied science. However, a vast number of our current students in quantum mechanics will apply the subject in materials science in a broad sense encompassing condensed matter physics, chemistry and engineering. For these students it is beneficial to see Bloch's theorem, Wannier states, and basics of the theory of covalent bonding embedded with their quantum mechanics course. Another important topic for these students is quantization of the Schrödinger field. Indeed, it is also useful for students in nuclear and particle physics to learn quantization of the Schrödinger field because it makes quantization of gauge fields and relativistic matter fields so much easier if they know quantum field theory in the non-relativistic setting.

Furthermore, many of our current students will use or manipulate photon probes in their future graduate and professional work. A proper discussion of photon-matter interactions is therefore also important for a modern quantum mechanics course.

This should include minimal coupling, quantization of the Maxwell field, and applications of time-dependent perturbation theory for photon absorption, emission and scattering.

Students should also know the Klein-Gordon and Dirac equations after completion of their course, not only to understand that Schrödinger's equation is not the final answer in terms of wave equations for matter particles, but to understand the nature of relativistic corrections like the Pauli term or spin-orbit coupling.

The scattering matrix is introduced as early as possible in terms of matrix elements of the time evolution operator on states in the interaction picture, $S_f(t, t') = \langle f | U_D(t, t') | i \rangle$, cf. equation (13.26). This representation of the scattering matrix appears so naturally in ordinary time-dependent perturbation theory that it makes no sense to defer the notion of an S-matrix to the discussion of scattering in quantum field theory with two or more particles in the initial state. It actually mystifies the scattering matrix to defer its discussion until field quantization has been introduced. On the other hand, introducing the scattering matrix even earlier in the framework of scattering off static potentials is counterproductive, because its natural and useful definition as matrix elements of a time evolution operator cannot properly be introduced at that level, and the notion of the scattering matrix does not really help with the calculation of cross sections for scattering off static potentials.

I have also emphasized the discussion of the various roles of transition matrix elements depending on whether the initial or final states are discrete or continuous. It helps students to understand transition probabilities, decay rates, absorption cross sections and scattering cross sections if the discussion of these concepts is integrated in one chapter, cf. Chapter 13. Furthermore, I have put an emphasis on canonical field quantization. Path integrals provide a very elegant description for free-free scattering, but bound states and energy levels, and basic many-particle quantum phenomena like exchange holes are very efficiently described in the canonical formalism. Feynman rules also appear more intuitive in the canonical formalism of explicit particle creation and annihilation.

The core advanced topics in quantum mechanics that an instructor might want to cover in a traditional 400-level or introductory graduate course are included with Chapters 8, 11–13, 15–18, and 21. However, instructors of a more inclusive course for general science and engineering students should include materials from Chapters 1–7 and 9, as appropriate.

The direct integration of training in quantum mechanics with the foundations of condensed matter physics, field quantization, and quantum optics is very important for the advancement of science and technology. I hope that this book will help to achieve that goal. I would greatly appreciate your comments and criticism. Please send them to rainer.dick@usask.ca.

Contents

1	The Need for Quantum Mechanics	1
1.1	Electromagnetic spectra and evidence for discrete energy levels .	1
1.2	Blackbody radiation and Planck's law	3
1.3	Blackbody spectra and photon fluxes	7
1.4	The photoelectric effect	15
1.5	Wave-particle duality.....	16
1.6	Why Schrödinger's equation?	17
1.7	Interpretation of Schrödinger's wave function.....	19
1.8	Problems	23
2	Self-adjoint Operators and Eigenfunction Expansions	25
2.1	The δ function and Fourier transforms	25
2.2	Self-adjoint operators and completeness of eigenstates	30
2.3	Problems	34
3	Simple Model Systems	37
3.1	Barriers in quantum mechanics.....	37
3.2	Box approximations for quantum wells, quantum wires and quantum dots	44
3.3	The attractive δ function potential.....	47
3.4	Evolution of free Schrödinger wave packets	51
3.5	Problems	57
4	Notions from Linear Algebra and Bra-Ket Notation	63
4.1	Notions from linear algebra.....	64
4.2	Bra-ket notation in quantum mechanics.....	73
4.3	The adjoint Schrödinger equation and the virial theorem	78
4.4	Problems	81
5	Formal Developments	85
5.1	Uncertainty relations	85
5.2	Frequency representation of states	90
5.3	Dimensions of states	92

5.4	Gradients and Laplace operators in general coordinate systems ..	94
5.5	Separation of differential equations	97
5.6	Problems	100
6	Harmonic Oscillators and Coherent States	103
6.1	Basic aspects of harmonic oscillators	103
6.2	Solution of the harmonic oscillator by the operator method	104
6.3	Construction of the states in the x -representation	107
6.4	Lemmata for exponentials of operators	109
6.5	Coherent states	112
6.6	Problems	119
7	Central Forces in Quantum Mechanics	121
7.1	Separation of center of mass motion and relative motion	121
7.2	The concept of symmetry groups	124
7.3	Operators for kinetic energy and angular momentum	125
7.4	Matrix representations of the rotation group	127
7.5	Construction of the spherical harmonic functions	132
7.6	Basic features of motion in central potentials	136
7.7	Free spherical waves: The free particle with sharp M_z, M^2	137
7.8	Bound energy eigenstates of the hydrogen atom	139
7.9	Spherical Coulomb waves	147
7.10	Problems	152
8	Spin and Addition of Angular Momentum Type Operators	157
8.1	Spin and magnetic dipole interactions	158
8.2	Transformation of scalar, spinor, and vector wave functions under rotations	160
8.3	Addition of angular momentum like quantities	163
8.4	Problems	168
9	Stationary Perturbations in Quantum Mechanics	171
9.1	Time-independent perturbation theory without degeneracies	171
9.2	Time-independent perturbation theory with degenerate energy levels	176
9.3	Problems	181
10	Quantum Aspects of Materials I	185
10.1	Bloch's theorem	185
10.2	Wannier states	189
10.3	Time-dependent Wannier states	192
10.4	The Kronig-Penney model	193
10.5	kp perturbation theory and effective mass	198
10.6	Problems	199
11	Scattering Off Potentials	207
11.1	The free energy-dependent Green's function	209
11.2	Potential scattering in the Born approximation	212

11.3	Scattering off a hard sphere	216
11.4	Rutherford scattering	220
11.5	Problems	224
12	The Density of States	227
12.1	Counting of oscillation modes	228
12.2	The continuum limit	230
12.3	The density of states in the energy scale	233
12.4	Density of states for free non-relativistic particles and for radiation	234
12.5	The density of states for other quantum systems	235
12.6	Problems	236
13	Time-dependent Perturbations in Quantum Mechanics	241
13.1	Pictures of quantum dynamics	242
13.2	The Dirac picture	247
13.3	Transitions between discrete states	251
13.4	Transitions from discrete states into continuous states: Ionization or decay rates	256
13.5	Transitions from continuous states into discrete states: Capture cross sections	265
13.6	Transitions between continuous states: Scattering	268
13.7	Expansion of the scattering matrix to higher orders	273
13.8	Energy-time uncertainty	275
13.9	Problems	276
14	Path Integrals in Quantum Mechanics	283
14.1	Correlation and Green's functions for free particles	284
14.2	Time evolution in the path integral formulation	287
14.3	Path integrals in scattering theory	293
14.4	Problems	299
15	Coupling to Electromagnetic Fields	301
15.1	Electromagnetic couplings	301
15.2	Stark effect and static polarizability tensors	309
15.3	Dynamical polarizability tensors	311
15.4	Problems	318
16	Principles of Lagrangian Field Theory	321
16.1	Lagrangian field theory	321
16.2	Symmetries and conservation laws	324
16.3	Applications to Schrödinger field theory	328
16.4	Problems	330
17	Non-relativistic Quantum Field Theory	333
17.1	Quantization of the Schrödinger field	334
17.2	Time evolution for time-dependent Hamiltonians	342
17.3	The connection between first and second quantized theory	344

17.4	The Dirac picture in quantum field theory	349
17.5	Inclusion of spin	353
17.6	Two-particle interaction potentials and equations of motion	360
17.7	Expectation values and exchange terms	365
17.8	From many particle theory to second quantization	368
17.9	Problems	370
18	Quantization of the Maxwell Field: Photons	383
18.1	Lagrange density and mode expansion for the Maxwell field	383
18.2	Photons	390
18.3	Coherent states of the electromagnetic field	392
18.4	Photon coupling to relative motion	394
18.5	Energy-momentum densities and time evolution	396
18.6	Photon emission rates	400
18.7	Photon absorption	409
18.8	Stimulated emission of photons	414
18.9	Photon scattering	416
18.10	Problems	425
19	Quantum Aspects of Materials II	431
19.1	The Born-Oppenheimer approximation	432
19.2	Covalent bonding: The dihydrogen cation	436
19.3	Bloch and Wannier operators	445
19.4	The Hubbard model	449
19.5	Vibrations in molecules and lattices	451
19.6	Quantized lattice vibrations: Phonons	463
19.7	Electron-phonon interactions	468
19.8	Problems	472
20	Dimensional Effects in Low-dimensional Systems	477
20.1	Quantum mechanics in d dimensions	477
20.2	Inter-dimensional effects in interfaces and thin layers	483
20.3	Problems	489
21	Relativistic Quantum Fields	495
21.1	The Klein-Gordon equation	495
21.2	Klein's paradox	503
21.3	The Dirac equation	507
21.4	Energy-momentum tensor for quantum electrodynamics	515
21.5	The non-relativistic limit of the Dirac equation	520
21.6	Covariant quantization of the Maxwell field	529
21.7	Problems	532
22	Applications of Spinor QED	545
22.1	Two-particle scattering cross sections	545
22.2	Electron scattering off an atomic nucleus	550

22.3	Photon scattering by free electrons	555
22.4	Møller scattering	565
22.5	Problems	575
Appendix A: Lagrangian Mechanics		577
Appendix B: The Covariant Formulation of Electrodynamics		587
Appendix C: Completeness of Sturm-Liouville Eigenfunctions		605
Appendix D: Properties of Hermite Polynomials		621
Appendix E: The Baker-Campbell-Hausdorff Formula		625
Appendix F: The Logarithm of a Matrix		629
Appendix G: Dirac γ matrices		633
Appendix H: Spinor representations of the Lorentz group		645
Appendix I: Transformation of fields under reflections		655
Appendix J: Green's functions in d dimensions		659
Bibliography		685
Index		687