

Statically Indeterminate Structures

Statically indeterminate structures are over-restrained in the sense that there are more force unknowns than available equilibrium equations. This situation arises when there are more supports than needed to prevent rigid body motion. Multi-span continuous beams and two-hinged frames are examples of this case. Indeterminacy may also result when there is an excess of members, such as a truss with multiple diagonals. Two dominant methods of analysis are used for indeterminate structures.

The traditional approach for analyzing statically indeterminate structures is based on the assumption that the structure behaves in a linear elastic manner, and therefore displacement patterns corresponding to different systems of forces can be superimposed to achieve a desired displacement pattern. One replaces the displacement constraints with unknown forces, determines the deflected shapes for each unit force, and then combines and scales these shapes to obtain a final deflected shape that satisfies the constraints. Since one works with force unknowns, this approach is called the “Force Method.” It is also called the “Flexibility Method.” Engineers find the method appealing since the process of superimposing the different deflected shapes can be easily visualized and the computational details, which are suited for hand computation, provide insight into the deflection behavior.

A second method is based on solving a set of equilibrium equations expressed in terms of certain displacement measures that define the loaded configuration. It views the structure as an assemblage of members and uses a set of member end force–end displacement relations called the slope deflection equations. In general, the number of displacement unknowns is larger than the number of force unknowns, but the method is readily programmed and numerous software packages now exist. We refer to this approach as the “Displacement Method.” It is also called the “Stiffness Method” since the equations involve stiffness coefficients.

In what follows, we discuss both methods. We also describe some approximate hand calculation-based methods that are suitable for rapidly estimating the response due to gravity and lateral loads. Finally, we describe the underlying theory for the Displacement Method and illustrate how to apply the method using computer software.