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Introduction to Time Series and Forecasting

Third Edition

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To Pam and Patti

Preface

This book is aimed at the reader who wishes to gain a working knowledge of time series and forecasting methods as applied in economics, engineering, and the natural and social sciences. Unlike our more advanced book, *Time Series: Theory and Methods*, Brockwell and Davis (1991), this one requires only a knowledge of basic calculus, matrix algebra and elementary statistics at the level, for example, of Mendenhall et al. (1990). It is intended for upper-level undergraduate students and beginning graduate students.

The emphasis is on methods and the analysis of data sets. The professional version of the time series package ITSM2000, for Windows-based PC, enables the reader to reproduce most of the calculations in the text (and to analyze further data sets of the reader's own choosing). It is available for download, together with most of the data sets used in the book, from <http://extras.springer.com>. Appendix E contains a detailed introduction to the package.

Very little prior familiarity with computing is required in order to use the computer package. The book can also be used in conjunction with other computer packages for handling time series. Chapter 14 of the book by Venables and Ripley (2003) describes how to perform many of the calculations using S and R. The package ITSMR of Weigt (2015) can be used in R to reproduce many of the features of ITSM2000. The package Yuima, also for R, can be used for simulation and estimation of the Lévy-driven CARMA processes discussed in Section 11.5 (see Iacus and Mercuri (2015)). Both of these packages can be downloaded from <https://cran.rproject.org/web/packages>.

There are numerous problems at the end of each chapter, many of which involve use of the programs to study the data sets provided.

To make the underlying theory accessible to a wider audience, we have stated some of the key mathematical results without proof, but have attempted to ensure that the logical structure of the development is otherwise complete. (References to proofs are provided for the interested reader.)

There is sufficient material here for a full-year introduction to univariate and multivariate time series and forecasting. Chapters 1 through 6 have been used for several years in introductory one-semester courses in univariate time series at Columbia University, Colorado State University, and Royal Melbourne Institute of Technology. The chapter on spectral analysis can be excluded without loss of continuity by readers who are so inclined.

In view of the explosion of interest in financial time series in recent decades, the third edition includes a new chapter (Chapter 7) specifically devoted to this topic. Some of the basic tools required for an understanding of continuous-time financial time series models (Brownian motion, Lévy processes, and Itô calculus) have also been added as

Appendix D, and a new Section 11.5 provides an introduction to continuous parameter ARMA (or CARMA) processes.

The diskette containing the student version of the package ITSM2000 is no longer included with the book since the professional version (which places no limit on the length of the series to be studied) can now be downloaded from <http://extras.springer.com> as indicated above. A tutorial for the use of the package is provided as Appendix E and a searchable file, ITSM_HELP.pdf, giving more detailed instructions, is included with the package.

We are greatly indebted to the readers of the first and second editions of the book and especially to Matthew Calder, coauthor of the computer package ITSM2000 and to Anthony Brockwell, both of whom made many valuable comments and suggestions. We also wish to thank Colorado State University, Columbia University, the National Science Foundation, Springer-Verlag, and our families for their continuing support during the preparation of this third edition.

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