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Linear and Nonlinear Programming

Fourth Edition

 Springer

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*To Susan, Robert, Jill, and Jenna;
Daisun, Fei, Tim, and Kaylee*

Preface

This book is intended as a text covering the central concepts of practical optimization techniques. It is designed for either self-study by professionals or classroom work at the undergraduate or graduate level for students who have a technical background in engineering, mathematics, or science. Like the field of optimization itself, which involves many classical disciplines, the book should be useful to system analysts, operations researchers, numerical analysts, management scientists, and other specialists from the host of disciplines from which practical optimization applications are drawn. The prerequisites for convenient use of the book are relatively modest; the prime requirement being some familiarity with introductory elements of linear algebra. Certain sections and developments do assume some knowledge of more advanced concepts of linear algebra, such as eigenvector analysis, or some background in sets of real numbers, but the text is structured so that the mainstream of the development can be faithfully pursued without reliance on this more advanced background material.

Although the book covers primarily material that is now fairly standard, this edition emphasizes methods that are both state-of-the-art and popular. One major insight is the connection between the purely analytical character of an optimization problem, expressed perhaps by properties of the necessary conditions, and the behavior of algorithms used to solve a problem. This was a major theme of the first edition of this book and the fourth edition expands and further illustrates this relationship.

As in the earlier editions, the material in this fourth edition is organized into three separate parts. Part I is a self-contained introduction to linear programming, a key component of optimization theory. The presentation in this part is fairly conventional, covering the main elements of the underlying theory of linear programming, many of the most effective numerical algorithms, and many of its important special applications. Part II, which is independent of Part I, covers the theory of unconstrained optimization, including both derivations of the appropriate optimality conditions and an introduction to basic algorithms. This part of the book explores the general properties of algorithms and defines various notions of convergence. Part III

extends the concepts developed in the second part to constrained optimization problems. Except for a few isolated sections, this part is also independent of Part I. It is possible to go directly into Parts II and III omitting Part I, and, in fact, the book has been used in this way in many universities. Each part of the book contains enough material to form the basis of a one-quarter course. In either classroom use or for self-study, it is important not to overlook the suggested exercises at the end of each chapter. The selections generally include exercises of a computational variety designed to test one's understanding of a particular algorithm, a theoretical variety designed to test one's understanding of a given theoretical development, or of the variety that extends the presentation of the chapter to new applications or theoretical areas. One should attempt at least four or five exercises from each chapter. In progressing through the book it would be unusual to read straight through from cover to cover. Generally, one will wish to skip around. In order to facilitate this mode, we have indicated sections of a specialized or digressive nature with an asterisk*.

New to this edition is a special Chap. 6 devoted to Conic Linear Programming, a powerful generalization of Linear Programming. While the constraint set in a normal linear program is defined by a finite number of linear inequalities of finite-dimensional vector variables, the constraint set in conic linear programming may be defined, for example, as a linear combination of symmetric positive semi-definite matrices of a given dimension. Indeed, many conic structures are possible and useful in a variety of applications. It must be recognized, however, that conic linear programming is an advanced topic, requiring special study.

Another important topic is an accelerated steepest descent method that exhibits superior convergence properties, and for this reason, has become quite popular. The proof of the convergence property for both standard and accelerated steepest descent methods are presented in Chap. 8.

As the field of optimization advances, addressing greater complexity, treating problems with ever more variables (as in Big Data situations), ranging over diverse applications. The field responds to these challenges, developing new algorithms, building effective software, and expanding overall theory. An example of a valuable new development is the work on big data problems. Surprisingly, coordinate descent, with randomly selected coordinates at each step, is quite effective as explained in Chap. 8. As another example some problems are formulated so that the unknowns can be split into two sub groups, there are linear constraints and the objective function is separable with respect to the two groups of variables. The augmented Lagrangian can be computed and it is natural to use an alternating series method. We discuss the alternating direction method with multipliers as a dual method in Chap. 14. Interestingly, this method is convergent for when the number of partition groups is two, but not for finer partitions.

We wish to thank the many students and researchers who over the years have given us comments concerning the book and those who encouraged us to carry out this revision.

Stanford, CA, USA
Stanford, CA, USA
January 2015

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Y. Ye

Contents

1	Introduction	1
1.1	Optimization	1
1.2	Types of Problems	2
1.3	Size of Problems	5
1.4	Iterative Algorithms and Convergence	6

Part I Linear Programming

2	Basic Properties of Linear Programs	11
2.1	Introduction	11
2.2	Examples of Linear Programming Problems	14
2.3	Basic Solutions	19
2.4	The Fundamental Theorem of Linear Programming	20
2.5	Relations to Convexity	23
2.6	Exercises	27
3	The Simplex Method	33
3.1	Pivots	33
3.2	Adjacent Extreme Points	38
3.3	Determining a Minimum Feasible Solution	42
3.4	Computational Procedure: Simplex Method	45
3.5	Finding a Basic Feasible Solution	49
3.6	Matrix Form of the Simplex Method	54
3.7	Simplex Method for Transportation Problems	56
3.8	Decomposition	68
3.9	Summary	72
3.10	Exercises	73
4	Duality and Complementarity	83
4.1	Dual Linear Programs	83
4.2	The Duality Theorem	86

4.3	Relations to the Simplex Procedure	88
4.4	Sensitivity and Complementary Slackness	92
4.5	Max Flow–Min Cut Theorem	94
4.6	The Dual Simplex Method	100
4.7	*The Primal-Dual Algorithm	102
4.8	Summary	106
4.9	Exercises	107
5	Interior-Point Methods	115
5.1	Elements of Complexity Theory	117
5.2	*The Simplex Method Is Not Polynomial-Time	118
5.3	*The Ellipsoid Method	119
5.4	The Analytic Center	123
5.5	The Central Path	125
5.6	Solution Strategies	130
5.7	Termination and Initialization	137
5.8	Summary	142
5.9	Exercises	143
6	Conic Linear Programming	149
6.1	Convex Cones	149
6.2	Conic Linear Programming Problem	150
6.3	Farkas' Lemma for Conic Linear Programming	154
6.4	Conic Linear Programming Duality	158
6.5	Complementarity and Solution Rank of SDP	166
6.6	Interior-Point Algorithms for Conic Linear Programming	170
6.7	Summary	173
6.8	Exercises	174
Part II Unconstrained Problems		
7	Basic Properties of Solutions and Algorithms	179
7.1	First-Order Necessary Conditions	180
7.2	Examples of Unconstrained Problems	182
7.3	Second-Order Conditions	185
7.4	Convex and Concave Functions	188
7.5	Minimization and Maximization of Convex Functions	192
7.6	*Zero-Order Conditions	194
7.7	Global Convergence of Descent Algorithms	196
7.8	Speed of Convergence	204
7.9	Summary	209
7.10	Exercises	209

- 8 Basic Descent Methods** 213
 - 8.1 Line Search Algorithms 214
 - 8.2 The Method of Steepest Descent 229
 - 8.3 Applications of the Convergence Theory 239
 - 8.4 Accelerated Steepest Descent 243
 - 8.5 Newton’s Method 245
 - 8.6 Coordinate Descent Methods 252
 - 8.7 Summary 257
 - 8.8 Exercises 258

- 9 Conjugate Direction Methods** 263
 - 9.1 Conjugate Directions 263
 - 9.2 Descent Properties of the Conjugate Direction Method 266
 - 9.3 The Conjugate Gradient Method 268
 - 9.4 The C–G Method as an Optimal Process 270
 - 9.5 The Partial Conjugate Gradient Method 273
 - 9.6 Extension to Nonquadratic Problems 276
 - 9.7 *Parallel Tangents 279
 - 9.8 Exercises 281

- 10 Quasi-Newton Methods** 285
 - 10.1 Modified Newton Method 286
 - 10.2 Construction of the Inverse 288
 - 10.3 Davidon-Fletcher-Powell Method 290
 - 10.4 The Broyden Family 293
 - 10.5 Convergence Properties 296
 - 10.6 Scaling 300
 - 10.7 Memoryless Quasi-Newton Methods 304
 - 10.8 *Combination of Steepest Descent and Newton’s Method 306
 - 10.9 Summary 312
 - 10.10 Exercises 313

Part III Constrained Minimization

- 11 Constrained Minimization Conditions** 321
 - 11.1 Constraints 321
 - 11.2 Tangent Plane 323
 - 11.3 First-Order Necessary Conditions (Equality Constraints) 326
 - 11.4 Examples 327
 - 11.5 Second-Order Conditions 333
 - 11.6 Eigenvalues in Tangent Subspace 335
 - 11.7 Sensitivity 338
 - 11.8 Inequality Constraints 340
 - 11.9 Zero-Order Conditions and Lagrangian Relaxation 344
 - 11.10 Summary 351
 - 11.11 Exercises 352

12	Primal Methods	357
12.1	Advantage of Primal Methods	357
12.2	Feasible Direction Methods	358
12.3	Active Set Methods	360
12.4	The Gradient Projection Method	364
12.5	Convergence Rate of the Gradient Projection Method	370
12.6	The Reduced Gradient Method	378
12.7	Convergence Rate of the Reduced Gradient Method	383
12.8	*Variations	390
12.9	Summary	392
12.10	Exercises	392
13	Penalty and Barrier Methods	397
13.1	Penalty Methods	398
13.2	Barrier Methods	401
13.3	Properties of Penalty and Barrier Functions	403
13.4	Newton's Method and Penalty Functions	412
13.5	Conjugate Gradients and Penalty Methods	413
13.6	Normalization of Penalty Functions	415
13.7	Penalty Functions and Gradient Projection	417
13.8	*Exact Penalty Functions	421
13.9	Summary	423
13.10	Exercises	425
14	Duality and Dual Methods	429
14.1	Global Duality	430
14.2	Local Duality	435
14.3	Canonical Convergence Rate of Dual Steepest Ascent	440
14.4	Separable Problems and Their Duals	441
14.5	Augmented Lagrangian	445
14.6	The Method of Multipliers	449
14.7	The Alternating Direction Method of Multipliers	454
14.8	*Cutting Plane Methods	458
14.9	Exercises	464
15	Primal-Dual Methods	467
15.1	The Standard Problem	467
15.2	A Simple Merit Function	470
15.3	Basic Primal-Dual Methods	471
15.4	Modified Newton Methods	477
15.5	Descent Properties	478
15.6	*Rate of Convergence	483
15.7	Primal-Dual Interior Point Methods	485
15.8	Summary	488
15.9	Exercises	489

- A Mathematical Review** 495
 - A.1 Sets 495
 - A.2 Matrix Notation 496
 - A.3 Spaces 497
 - A.4 Eigenvalues and Quadratic Forms 498
 - A.5 Topological Concepts 499
 - A.6 Functions 500

- B Convex Sets** 505
 - B.1 Basic Definitions 505
 - B.2 Hyperplanes and Polytopes 507
 - B.3 Separating and Supporting Hyperplanes 509
 - B.4 Extreme Points 511

- C Gaussian Elimination** 513

- D Basic Network Concepts** 517
 - D.1 Flows in Networks 519
 - D.2 Tree Procedure 519
 - D.3 Capacitated Networks 521

- Bibliography** 523

- Index** 539