

Springer Texts in Statistics

Series editors

R. DeVeaux

S. Fienberg

I. Olkin

More information about this series at <http://www.springer.com/series/417>

Richard A. Berk

Statistical Learning from a Regression Perspective

Second Edition

 Springer

Richard A. Berk
Department of Statistics
The Wharton School
University of Pennsylvania
Philadelphia, PA
USA

and

Department of Criminology
Schools of Arts and Sciences
University of Pennsylvania
Philadelphia, PA
USA

ISSN 1431-875X

Springer Texts in Statistics

ISBN 978-3-319-44047-7

DOI 10.1007/978-3-319-44048-4

ISSN 2197-4136 (electronic)

ISBN 978-3-319-44048-4 (eBook)

Library of Congress Control Number: 2016948105

© Springer International Publishing Switzerland 2008, 2016, corrected publication 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer International Publishing AG Switzerland

*In God we trust. All others
must have data.*

W. Edwards Deming

*In memory of Peter H. Rossi,
a mentor, colleague, and friend*

Preface to the Second Edition

Over the past 8 years, the topics associated with statistical learning have been expanded and consolidated. They have been expanded because new problems have been tackled, new tools have been developed, and older tools have been refined. They have been consolidated because many unifying concepts and themes have been identified. It has also become more clear from practice which statistical learning tools will be widely applied and which are likely to see limited service. In short, it seems this is the time to revisit the material and make it more current.

There are currently several excellent textbook treatments of statistical learning and its very close cousin, machine learning. The second edition of *Elements of Statistical Learning* by Hastie, Tibshirani, and Friedman (2009) is in my view still the gold standard, but there are other treatments that in their own way can be excellent. Examples include *Machine Learning: A Probabilistic Perspective* by Kevin Murphy (2012), *Principles and Theory for Data Mining and Machine Learning* by Clarke, Fokoué, and Zhang (2009), and *Applied Predictive Modeling* by Kuhn and Johnson (2013).

Yet, it is sometimes difficult to appreciate from these treatments that a proper application of statistical learning is comprised of (1) data collection, (2) data management, (3) data analysis, and (4) interpretation of results. The first entails finding and acquiring the data to be analyzed. The second requires putting the data into an accessible form. The third depends on extracting instructive patterns from the data. The fourth calls for making sense of those patterns. For example, a statistical learning data analysis might begin by collecting information from “rap sheets” and other kinds of official records about prison inmates who have been released on parole. The information obtained might be organized so that arrests were nested within individuals. At that point, support vector machines could be used to classify offenders into those who re-offend after release on parole and those who do not. Finally, the classes obtained might be employed to forecast subsequent re-offending when the actual outcome is not known. Although there is a chronological sequence to these activities, one must anticipate later steps as earlier steps are undertaken. Will the offender classes, for instance, include or exclude juvenile offenses or vehicular offenses? How this is decided will affect the choice of

statistical learning tools, how they are implemented, and how they are interpreted. Moreover, the preferred statistical learning procedures anticipated place constraints on how the offenses are coded, while the ways in which the results are likely to be used affect how the procedures are tuned. In short, no single activity should be considered in isolation from the other three.

Nevertheless, textbook treatments of statistical learning (and statistics textbooks more generally) focus on the third step: the statistical procedures. This can make good sense if the treatments are to be of manageable length and within the authors' expertise, but risks the misleading impression that once the key statistical theory is understood, one is ready to proceed with data. The result can be a fancy statistical analysis as a bridge to nowhere. To reprise an aphorism attributed to Albert Einstein: "In theory, theory and practice are the same. In practice they are not."

The commitment to practice as well as theory will sometimes engender considerable frustration. There are times when the theory is not readily translated into practice. And there are times when practice, even practice that seems intuitively sound, will have no formal justification. There are also important open questions leaving large holes in procedures one would like to apply. A particular problem is statistical inference, especially for procedures that proceed in an inductive manner. In effect, they capitalize on "data snooping," which can invalidate estimation, confidence intervals, and statistical tests.

In the first edition, statistical tools characterized as supervised learning were the main focus. But a serious effort was made to establish links to data collection, data management, and proper interpretation of results. That effort is redoubled in this edition. At the same time, there is a price. No claims are made for anything like an encyclopedic coverage of supervised learning, let alone of the underlying statistical theory. There are books available that take the encyclopedic approach, which can have the feel of a trip through Europe spending 24 hours in each of the major cities.

Here, the coverage is highly selective. Over the past decade, the wide range of real applications has begun to sort the enormous variety of statistical learning tools into those primarily of theoretical interest or in early stages of development, the niche players, and procedures that have been successfully and widely applied (Jordan and Mitchell, 2015). Here, the third group is emphasized.

Even among the third group, choices need to be made. The statistical learning material addressed reflects the subject-matter fields with which I am more familiar. As a result, applications in the social and policy sciences are emphasized. This is a pity because there are truly fascinating applications in the natural sciences and engineering. But in the words of Dirty Harry: "A man's got to know his limitations" (from the movie *Magnum Force*, 1973).¹ My several forays into natural science applications do not qualify as real expertise.

¹"Dirty" Harry Callahan was a police detective played by Clint Eastwood in five movies filmed during the 1970s and 1980s. Dirty Harry was known for his strong-armed methods and blunt catch-phrases, many of which are now ingrained in American popular culture.

The second edition retains its commitment to the statistical programming language R. If anything the commitment is stronger. R provides access to state-of-the-art statistics, including those needed for statistical learning. It is also now a standard training component in top departments of statistics so for many readers, applications of the statistical procedures discussed will come quite naturally. Where it could be useful, I now include the R-code needed when the usual R documentation may be insufficient. That code is written to be accessible. Often there will be more elegant, or at least more efficient, ways to proceed. When practical, I develop examples using data that can be downloaded from one of the R libraries. But, R is a moving target. Code that runs now may not run in the future. In the year it took to complete this edition, many key procedures were updated several times, and there were three updates of R itself. *Caveat emptor*. Readers will also notice that the graphical output from the many procedures used do not have common format or color scheme. In some cases, it would have been very difficult to force a common set of graphing conventions, and it is probably important to show a good approximation of the default output in any case. Aesthetics and common formats can be a casualty.

In summary, the second edition retains its emphasis on supervised learning that can be treated as a form of regression analysis. Social science and policy applications are prominent. Where practical, substantial links are made to data collection, data management, and proper interpretation of results, some of which can raise ethical concerns (Dwork et al., 2011; Zemel et al., 2013). I hope it works.

The first chapter has been rewritten almost from scratch in part from experience I have had trying to teach the material. It much better reflects new views about unifying concepts and themes. I think the chapter also gets to punch lines more quickly and coherently. But readers who are looking for simple recipes will be disappointed. The exposition is by design not “point-and-click.” There is as well some time spent on what some statisticians call “meta-issues.” A good data analyst must know what to compute and what to make of the computed results. How to compute is important, but by itself is nearly purposeless.

All of the other chapters have also been revised and updated with an eye toward far greater clarity. In many places greater clarity was sorely needed. I now appreciate much better how difficult it can be to translate statistical concepts and notation into plain English. Where I have still failed, please accept my apology.

I have also tried to take into account that often a particular chapter is downloaded and read in isolation. Because much of the material is cumulative, working through a single chapter can on occasion create special challenges. I have tried to include text to help, but for readers working cover to cover, there are necessarily some redundancies, and annoying pointers to material in other chapters. I hope such readers will be patient with me.

I continue to be favored with remarkable colleagues and graduate students. My professional life is one ongoing tutorial in statistics, thanks to Larry Brown, Andreas Buja, Linda Zhao, and Ed George. All four are as collegial as they are smart. I have learned a great deal as well from former students Adam Kapelner, Justin Bleich, Emil Pitkin, Kai Zhang, Dan McCarthy, and Kory Johnson. Arjun

Gupta checked the exercises at the end of each chapter. Finally, there are the many students who took my statistics classes and whose questions got me to think a lot harder about the material. Thanks to them as well.

But I would probably not have benefited nearly so much from all the talent around me were it not for my earlier relationship with David Freedman. He was my bridge from routine calculations within standard statistical packages to a far better appreciation of the underlying foundations of modern statistics. He also reinforced my skepticism about many statistical applications in the social and biomedical sciences. Shortly before he died, David asked his friends to “keep after the rascals.” I certainly have tried.

Philadelphia, PA, USA

Richard A. Berk

Preface to the First Edition

As I was writing my recent book on regression analysis (Berk, 2003), I was struck by how few alternatives to conventional regression there were. In the social sciences, for example, one either did causal modeling econometric style or largely gave up quantitative work. The life sciences did not seem quite so driven by causal modeling, but causal modeling was a popular tool. As I argued at length in my book, causal modeling as commonly undertaken is a loser.

There also seemed to be a more general problem. Across a range of scientific disciplines there was too often little interest in statistical tools emphasizing induction and description. With the primary goal of getting the “right” model and its associated p -values, the older and interesting tradition of exploratory data analysis had largely become an under-the-table activity; the approach was in fact commonly used, but rarely discussed in polite company. How could one be a real scientist, guided by “theory” and engaged in deductive model testing, while at the same time snooping around in the data to determine which models to test? In the battle for prestige, model testing had won.

Around the same time, I became aware of some new developments in applied mathematics, computer science, and statistics making data exploration a virtue. And with the virtue came a variety of new ideas and concepts, coupled with the very latest in statistical computing. These new approaches, variously identified as “data mining,” “statistical learning,” “machine learning,” and other names, were being tried in a number of the natural and biomedical sciences, and the initial experience looked promising.

As I started to read more deeply, however, I was struck by how difficult it was to work across writings from such disparate disciplines. Even when the material was essentially the same, it was very difficult to tell if it was. Each discipline brought it own goals, concepts, naming conventions, and (maybe worst of all) notation to the table.

In the midst of trying to impose some of my own order on the material, I came upon *The Elements of Statistical Learning* by Trevor Hastie, Robert Tibshirani, and Jerome Friedman (Springer-Verlag, 2001). I saw in the book a heroic effort to

integrate a very wide variety of data analysis tools. I learned from the book and was then able to approach more primary material within a useful framework.

This book is my attempt to integrate some of the same material and some new developments of the past six years. Its intended audience is practitioners in the social, biomedical, and ecological sciences. Applications to real data addressing real empirical questions are emphasized. Although considerable effort has gone into providing explanations of why the statistical procedures work the way they do, the required mathematical background is modest. A solid course or two in regression analysis and some familiarity with resampling procedures should suffice. A good benchmark for regression is Freedman's *Statistical Models: Theory and Practice* (2005). A good benchmark for resampling is Manly's *Randomization, Bootstrap, and Monte Carlo Methods in Biology* (1997). Matrix algebra and calculus are used only as languages of exposition, and only as needed. There are no proofs to be followed.

The procedures discussed are limited to those that can be viewed as a form of regression analysis. As explained more completely in the first chapter, this means concentrating on statistical tools for which the conditional distribution of a response variable is the defining interest and for which characterizing the relationships between predictors and the response is undertaken in a serious and accessible manner.

Regression analysis provides a unifying theme that will ease translations across disciplines. It will also increase the comfort level for many scientists and policy analysts for whom regression analysis is a key data analysis tool. At the same time, a regression framework will highlight how the approaches discussed can be seen as alternatives to conventional causal modeling.

Because the goal is to convey how these procedures can be (and are being) used in practice, the material requires relatively in-depth illustrations and rather detailed information on the context in which the data analysis is being undertaken. The book draws heavily, therefore, on datasets with which I am very familiar. The same point applies to the software used and described.

The regression framework comes at a price. A 2005 announcement for a conference on data mining sponsored by the Society for Industrial and Applied Mathematics (SIAM) listed the following topics: query/constraint-based data mining, trend and periodicity analysis, mining data streams, data reduction/preprocessing, feature extraction and selection, post-processing, collaborative filtering/personalization, cost-based decision making, visual data mining, privacy-sensitive data mining, and lots more. Many of these topics cannot be considered a form of regression analysis. For example, procedures used for edge detection (e.g., determining the boundaries of different kinds of land use from remote sensing data) are basically a filtering process to remove noise from the signal.

Another class of problems makes no distinction between predictors and responses. The relevant techniques can be closely related, at least in spirit, to procedures such as factor analysis and cluster analysis. One might explore, for

example, the interaction patterns among children at school: who plays with whom. These too are not discussed.

Other topics can be considered regression analysis only as a formality. For example, a common data mining application in marketing is to extract from the purchasing behavior of individual shoppers patterns that can be used to forecast future purchases. But there are no predictors in the usual regression sense. The conditioning is on each individual shopper. The question is not what features of shoppers predict what they will purchase, but what a given shopper is likely to purchase.

Finally, there are a large number of procedures that focus on the conditional distribution of the response, much as with any regression analysis, but with little attention to how the predictors are related to the response (Horváth and Yamamoto, 2006; Camacho et al., 2006). Such procedures neglect a key feature of regression analysis, at least as discussed in this book, and are not considered. That said, there is no principled reason in many cases why the role of each predictor could not be better represented, and perhaps in the near future that shortcoming will be remedied.

In short, although using a regression framework implies a big-tent approach to the topics included, it is not an exhaustive tent. Many interesting and powerful tools are not discussed. Where appropriate, however, references to that material are provided.

I may have gone a bit overboard with the number of citations I provide. The relevant literatures are changing and growing rapidly. Today's breakthrough can be tomorrow's bust, and work that by current thinking is uninteresting can be the spark for dramatic advances in the future. At any given moment, it can be difficult to determine which is which. In response, I have attempted to provide a rich mix of background material, even at the risk of not being sufficiently selective. (And I have probably missed some useful papers nevertheless.)

In the material that follows, I have tried to use consistent notation. This has proved to be very difficult because of important differences in the conceptual traditions represented and the complexity of statistical tools discussed. For example, it is common to see the use of the expected value operator even when the data cannot be characterized as a collection of random variables and when the sole goal is description.

I draw where I can from the notation used in *The Elements of Statistical Learning* (Hastie et al., 2001). Thus, the symbol X is used for an input variable, or predictor in statistical parlance. When X is a set of inputs to be treated as a vector, each component is indexed by a subscript (e.g., X_j). Quantitative outputs, also called response variables, are represented by Y , and categorical outputs, another kind of response variable, are represented by G with K categories. Upper case letters are used to refer to variables in a general way, with details to follow as needed. Sometimes these variables are treated as random variables, and sometimes not. I try to make that clear in context.

Observed values are shown in lower case, usually with a subscript. Thus x_i is the i th observed value for the variable X . Sometimes these observed values are nothing

more than the data on hand. Sometimes they are realizations of random variables. Again, I try to make this clear in context.

Matrices are represented in bold uppercase. For example, in matrix form the usual set of p predictors, each with N observations, is an $N \times p$ matrix \mathbf{X} . The subscript i is generally used for observations and the subscript j for variables. Bold lowercase letters are used for vectors with N elements, commonly columns of X . Other vectors are generally not represented in boldface fonts, but again, I try to make this clear in context.

If one treats Y as a random variable, its observed values y are either a random sample from a population or a realization of a stochastic process. The conditional means of the random variable Y for various configurations of X -values are commonly referred to as “expected values,” and are either the conditional means of Y for different configurations of \mathbf{X} -values in the population or for the stochastic process by which the data were generated. A common notation is $E(Y|X)$. The $E(Y|X)$ is also often called a “parameter.” The conditional means computed from the data are often called “sample statistics,” or in this case, “sample means.” In the regression context, the sample means are commonly referred to as the fitted values, often written as $\hat{y}|X$. Subscripting can follow as already described.

Unfortunately, after that it gets messier. First, I often have to decipher the intent in the notation used by others. No doubt I sometimes get it wrong. For example, it is often unclear if a computer algorithm is formally meant to be an estimator or a descriptor.

Second, there are some complications in representing nested realizations of the same variable (as in the bootstrap), or model output that is subject to several different chance processes. There is a practical limit to the number and types of bars, asterisks, hats, and tildes one can effectively use. I try to provide warnings (and apologies) when things get cluttered.

There are also some labeling issues. When I am referring to the general linear model (i.e., linear regression, analysis of variance, and analysis of covariance), I use the terms classical linear regression, or conventional linear regression. All regressions in which the functional forms are determined before the fitting process begins, I call parametric. All regressions in which the functional forms are determined as part of the fitting process, I call nonparametric. When there is some of both, I call the regressions semiparametric. Sometimes the lines among parametric, nonparametric, and semiparametric are fuzzy, but I try to make clear what I mean in context. Although these naming conventions are roughly consistent with much common practice, they are not universal.

All of the computing done for this book was undertaken in R. R is a programming language designed for statistical computing and graphics. It has become a major vehicle for developmental work in statistics and is increasingly being used by practitioners. A key reason for relying on R for this book is that most of the newest developments in statistical learning and related fields can be found in R. Another reason is that it is free.

Readers familiar with S or S-plus will immediately feel at home; R is basically a “dialect” of S. For others, there are several excellent books providing a good

introduction to data analysis using R. Dalgaard (2002), Crawley (2007), and Maindonald and Braun (2007) are all very accessible. Readers who are especially interested in graphics should consult Murrell (2006). The most useful R website can be found at <http://www.r-project.org/>.

The use of R raises the question of how much R-code to include. The R-code used to construct all of the applications in the book could be made available. However, detailed code is largely not shown. Many of the procedures used are somewhat in flux. Code that works one day may need some tweaking the next. As an alternative, the procedures discussed are identified as needed so that detailed information about how to proceed in R can be easily obtained from R help commands or supporting documentation. When the data used in this book are proprietary or otherwise not publicly available, similar data and appropriate R-code are substituted.

There are exercises at the end of each chapter. They are meant to be hands-on data analyses built around R. As such, they require some facility with R. However, the goals of each problem are reasonably clear so that other software and datasets can be used. Often the exercises can be usefully repeated with different datasets.

The book has been written so that later chapters depend substantially on earlier chapters. For example, because classification and regression trees (CART) can be an important component of boosting, it may be difficult to follow the discussion of boosting without having read the earlier chapter on CART. However, readers who already have a solid background in material covered earlier should have little trouble skipping ahead. The notation and terms used are reasonably standard or can be easily figured out. In addition, the final chapter can be read at almost any time. One reviewer suggested that much of the material could be usefully brought forward to Chap. 1.

Finally, there is the matter of tone. The past several decades have seen the development of a dizzying array of new statistical procedures, sometimes introduced with the hype of a big-budget movie. Advertising from major statistical software providers has typically made things worse. Although there have been genuine and useful advances, none of the techniques have ever lived up to their most optimistic billing. Widespread misuse has further increased the gap between promised performance and actual performance. In this book, therefore, the tone will be cautious, some might even say dark. I hope this will not discourage readers from engaging seriously with the material. The intent is to provide a balanced discussion of the limitations as well as the strengths of the statistical learning procedures.

While working on this book, I was able to rely on support from several sources. Much of the work was funded by a grant from the National Science Foundation: SES-0437169, "Ensemble Methods for Data Analysis in the Behavioral, Social and Economic Sciences." The first draft was completed while I was on sabbatical at the Department of Earth, Atmosphere, and Oceans, at the Ecole Normale Supérieure in Paris. The second draft was completed after I moved from UCLA to the University of Pennsylvania. All three locations provided congenial working environments. Most important, I benefited enormously from discussions about statistical learning with colleagues at UCLA, Penn and elsewhere: Larry Brown, Andreas Buja, Jan de

Leeuw, David Freedman, Mark Hansen, Andy Liaw, Greg Ridgeway, Bob Stine, Mikhail Traskin and Adi Wyner. Each is knowledgeable, smart and constructive. I also learned a great deal from several very helpful, anonymous reviews. Dick Koch was enormously helpful and patient when I had problems making TeXShop perform properly. Finally, I have benefited over the past several years from interacting with talented graduate students: Yan He, Weihua Huang, Brian Kriegler, and Jie Shen. Brian Kriegler deserves a special thanks for working through the exercises at the end of each chapter.

Certain datasets and analyses were funded as part of research projects undertaken for the California Policy Research Center, The Inter-America Tropical Tuna Commission, the National Institute of Justice, the County of Los Angeles, the California Department of Correction and Rehabilitation, the Los Angeles Sheriff's Department, and the Philadelphia Department of Adult Probation and Parole. Support from all of these sources is gratefully acknowledged.

Philadelphia, PA
2006

Richard A. Berk

*The original version of the book was revised:
Belated corrections have been incorporated.
The erratum to the book is available at [https://
doi.org/10.1007/978-3-319-44048-4_10](https://doi.org/10.1007/978-3-319-44048-4_10)*

Contents

1	Statistical Learning as a Regression Problem	1
1.1	Getting Started	2
1.2	Setting the Regression Context	2
1.3	Revisiting the Ubiquitous Linear Regression Model	8
1.3.1	Problems in Practice	9
1.4	Working with Statistical Models that Are Wrong	11
1.4.1	An Alternative Approach to Regression	15
1.5	The Transition to Statistical Learning	23
1.5.1	Models Versus Algorithms	24
1.6	Some Initial Concepts	28
1.6.1	Overall Goals of Statistical Learning	29
1.6.2	Data Requirements: Training Data, Evaluation Data and Test Data	31
1.6.3	Loss Functions and Related Concepts	35
1.6.4	The Bias-Variance Tradeoff	38
1.6.5	Linear Estimators	39
1.6.6	Degrees of Freedom	40
1.6.7	Basis Functions	42
1.6.8	The Curse of Dimensionality	46
1.7	Statistical Learning in Context	48
2	Splines, Smoothers, and Kernels	55
2.1	Introduction	55
2.2	Regression Splines	55
2.2.1	Applying a Piecewise Linear Basis	56
2.2.2	Polynomial Regression Splines	61
2.2.3	Natural Cubic Splines	63
2.2.4	<i>B</i> -Splines	66
2.3	Penalized Smoothing	69
2.3.1	Shrinkage and Regularization	70

- 2.4 Smoothing Splines 81
 - 2.4.1 A Smoothing Splines Illustration 84
- 2.5 Locally Weighted Regression as a Smoother 86
 - 2.5.1 Nearest Neighbor Methods 87
 - 2.5.2 Locally Weighted Regression 88
- 2.6 Smoothers for Multiple Predictors 92
 - 2.6.1 Smoothing in Two Dimensions 93
 - 2.6.2 The Generalized Additive Model 96
- 2.7 Smoothers with Categorical Variables 103
 - 2.7.1 An Illustration Using the Generalized Additive Model
with a Binary Outcome 103
- 2.8 An Illustration of Statistical Inference After Model Selection 106
- 2.9 Kernelized Regression 114
 - 2.9.1 Radial Basis Kernel 118
 - 2.9.2 ANOVA Radial Basis Kernel 120
 - 2.9.3 A Kernel Regression Application 120
- 2.10 Summary and Conclusions 124
- 3 Classification and Regression Trees (CART) 129**
 - 3.1 Introduction 129
 - 3.2 The Basic Ideas 131
 - 3.2.1 Tree Diagrams for Understanding Conditional
Relationships 132
 - 3.2.2 Classification and Forecasting with CART 136
 - 3.2.3 Confusion Tables 137
 - 3.2.4 CART as an Adaptive Nearest Neighbor Method 139
 - 3.3 Splitting a Node 140
 - 3.4 Fitted Values 144
 - 3.4.1 Fitted Values in Classification 144
 - 3.4.2 An Illustrative Prison Inmate Risk Assessment
Using CART 145
 - 3.5 Classification Errors and Costs 148
 - 3.5.1 Default Costs in CART 149
 - 3.5.2 Prior Probabilities and Relative Misclassification
Costs 151
 - 3.6 Pruning 157
 - 3.6.1 Impurity Versus $R_z(T)$ 159
 - 3.7 Missing Data 159
 - 3.7.1 Missing Data with CART 161
 - 3.8 Statistical Inference with CART 163
 - 3.9 From Classification to Forecasting 165
 - 3.10 Varying the Prior and the Complexity Parameter 166
 - 3.11 An Example with Three Response Categories 170
 - 3.12 Some Further Cautions in Interpreting CART Results 173
 - 3.12.1 Model Bias 173

- 3.12.2 Model Variance. 173
- 3.13 Regression Trees. 175
 - 3.13.1 A CART Application for the Correlates
of a Student’s GPA in High School 177
- 3.14 Multivariate Adaptive Regression Splines (MARS) 179
- 3.15 Summary and Conclusions 181
- 4 Bagging 187**
 - 4.1 Introduction 187
 - 4.2 The Bagging Algorithm 188
 - 4.3 Some Bagging Details 189
 - 4.3.1 Revisiting the CART Instability Problem 189
 - 4.3.2 Some Background on Resampling. 190
 - 4.3.3 Votes and Probabilities 193
 - 4.3.4 Imputation and Forecasting 193
 - 4.3.5 Margins 193
 - 4.3.6 Using Out-Of-Bag Observations as Test Data 195
 - 4.3.7 Bagging and Bias 195
 - 4.3.8 Level I and Level II Analyses with Bagging 196
 - 4.4 Some Limitations of Bagging 197
 - 4.4.1 Sometimes Bagging Cannot Help 197
 - 4.4.2 Sometimes Bagging Can Make the Bias Worse 197
 - 4.4.3 Sometimes Bagging Can Make the Variance Worse 198
 - 4.5 A Bagging Illustration 199
 - 4.6 Bagging a Quantitative Response Variable 200
 - 4.7 Summary and Conclusions 201
- 5 Random Forests. 205**
 - 5.1 Introduction and Overview 205
 - 5.1.1 Unpacking How Random Forests Works. 206
 - 5.2 An Initial Random Forests Illustration 208
 - 5.3 A Few Technical Formalities 210
 - 5.3.1 What Is a Random Forest? 211
 - 5.3.2 Margins and Generalization Error for Classifiers
in General 211
 - 5.3.3 Generalization Error for Random Forests 212
 - 5.3.4 The Strength of a Random Forest 214
 - 5.3.5 Dependence. 214
 - 5.3.6 Implications. 214
 - 5.3.7 Putting It All Together 215
 - 5.4 Random Forests and Adaptive Nearest Neighbor Methods. 217
 - 5.5 Introducing Misclassification Costs. 221
 - 5.5.1 A Brief Illustration Using Asymmetric Costs 222
 - 5.6 Determining the Importance of the Predictors. 224
 - 5.6.1 Contributions to the Fit 224

- 5.6.2 Contributions to Prediction 225
- 5.7 Input Response Functions. 230
 - 5.7.1 Partial Dependence Plot Examples 234
- 5.8 Classification and the Proximity Matrix 237
 - 5.8.1 Clustering by Proximity Values. 238
- 5.9 Empirical Margins 242
- 5.10 Quantitative Response Variables. 243
- 5.11 A Random Forest Illustration Using a Quantitative
Response Variable 245
- 5.12 Statistical Inference with Random Forests 250
- 5.13 Software and Tuning Parameters 252
- 5.14 Summary and Conclusions 255
 - 5.14.1 Problem Set 2 256
 - 5.14.2 Problem Set 3 257
- 6 Boosting** 259
 - 6.1 Introduction 259
 - 6.2 Adaboost 260
 - 6.2.1 A Toy Numerical Example of Adaboost.M1 261
 - 6.2.2 Why Does Boosting Work so Well
for Classification? 263
 - 6.3 Stochastic Gradient Boosting 266
 - 6.3.1 Tuning Parameters. 271
 - 6.3.2 Output 273
 - 6.4 Asymmetric Costs. 274
 - 6.5 Boosting, Estimation, and Consistency 276
 - 6.6 A Binomial Example 276
 - 6.7 A Quantile Regression Example 281
 - 6.8 Summary and Conclusions 286
- 7 Support Vector Machines** 291
 - 7.1 Support Vector Machines in Pictures 292
 - 7.1.1 The Support Vector Classifier 292
 - 7.1.2 Support Vector Machines 295
 - 7.2 Support Vector Machines More Formally. 295
 - 7.2.1 The Support Vector Classifier Again:
The Separable Case 296
 - 7.2.2 The Nonseparable Case 297
 - 7.2.3 Support Vector Machines 299
 - 7.2.4 SVM for Regression 301
 - 7.2.5 Statistical Inference for Support Vector Machines. 301
 - 7.3 A Classification Example 302
 - 7.4 Summary and Conclusions 308

- 8 Some Other Procedures Briefly** 311
 - 8.1 Neural Networks 311
 - 8.2 Bayesian Additive Regression Trees (BART) 316
 - 8.3 Reinforcement Learning and Genetic Algorithms 320
 - 8.3.1 Genetic Algorithms 320
- 9 Broader Implications and a Bit of Craft Lore** 325
 - 9.1 Some Integrating Themes 325
 - 9.2 Some Practical Suggestions 326
 - 9.2.1 Choose the Right Procedure 326
 - 9.2.2 Get to Know Your Software 328
 - 9.2.3 Do Not Forget the Basics 329
 - 9.2.4 Getting Good Data 330
 - 9.2.5 Match Your Goals to What You Can Credibly Do 331
 - 9.3 Some Concluding Observations 331
- Erratum to: Statistical Learning from a Regression Perspective** E1
- References** 333
- Index** 343