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Time Series Econometrics

 Springer

Preface

Over the past decades, time series analysis has experienced a proliferous increase of applications in economics, especially in macroeconomics and finance. Today these tools have become indispensable to any empirically working economist. Whereas in the beginning the transfer of knowledge essentially flowed from the natural sciences, especially statistics and engineering, to economics, over the years theoretical and applied techniques specifically designed for the nature of economic time series and models have been developed. Thereby, the estimation and identification of structural vector autoregressive models, the analysis of integrated and cointegrated time series, and models of volatility have been extremely fruitful and far-reaching areas of research. With the award of the Nobel Prizes to Clive W. J. Granger and Robert F. Engle III in 2003 and to Thomas J. Sargent and Christopher A. Sims in 2011, the field has reached a certain degree of maturity. Thus, the idea suggests itself to assemble the vast amount of material scattered over many papers into a comprehensive textbook.

The book is self-contained and addresses economics students who have already some prerequisite knowledge in econometrics. It is thus suited for advanced bachelor, master's, or beginning PhD students but also for applied researchers. The book tries to bring them in a position to be able to follow the rapidly growing research literature and to implement these techniques on their own. Although the book is trying to be rigorous in terms of concepts, definitions, and statements of theorems, not all proofs are carried out. This is especially true for the more technically and lengthy proofs for which the reader is referred to the pertinent literature.

The book covers approximately a two-semester course in time series analysis and is divided in two parts. The first part treats univariate time series, in particular autoregressive moving-average processes. Most of the topics are standard and can form the basis for a one-semester introductory time series course. This part also contains a chapter on integrated processes and on models of volatility. The latter topics could be included in a more advanced course. The second part is devoted to multivariate time series analysis and in particular to vector autoregressive processes. It can be taught independently of the first part. The identification, modeling, and estimation of these processes form the core of the second part. A special chapter treats the estimation, testing, and interpretation of cointegrated systems. The book also contains a chapter with an introduction to state space models and the Kalman

filter. Whereas the book is almost exclusively concerned with linear systems, the last chapter gives a perspective on some more recent developments in the context of nonlinear models. I have included exercises and worked out examples to deepen the teaching and learning content. Finally, I have produced five appendices which summarize important topics such as complex numbers, linear difference equations, and stochastic convergence.

As time series analysis has become a tremendously growing field with an active research in many directions, it goes without saying that not all topics received the attention they deserved and that there are areas not covered at all. This is especially true for the recent advances made in nonlinear time series analysis and in the application of Bayesian techniques. These two topics alone would justify an extra book.

The data manipulations and computations have been performed using the software packages EVIEWS and MATLAB.¹ Of course, there are other excellent packages available. The data for the examples and additional information can be downloaded from my home page www.neusser.ch. To maximize the learning success, it is advised to replicate the examples and to perform similar exercises with alternative data. Interesting macroeconomic time series can, for example, be downloaded from the following home pages:

Germany: www.bundesbank.de

Switzerland: www.snb.ch

United Kingdom: www.statistics.gov.uk

United States: research.stlouisfed.org/fred2

The book grew out of lectures which I had the occasion to give over the years in Bern and other universities. Thus, it is a concern to thank the many students, in particular Philip Letsch, who had to work through the manuscript and who called my attention to obscurities and typos. I also want to thank my colleagues and teaching assistants Andreas Bachmann, Gregor Bäurle, Fabrice Collard, Sarah Fischer, Stephan Leist, Senada Nukic, Kurt Schmidheiny, Reto Tanner, and Martin Wagner for reading the manuscript or part of it and for making many valuable criticisms and comments. Special thanks go to my former colleague and coauthor Robert Kunst who meticulously read and commented on the manuscript. It goes without saying that all errors and shortcomings go to my expense.

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¹EVIEWS is a product of IHS Global Inc. MATLAB is a matrix-oriented software developed by MathWorks which is ideally suited for econometric and time series applications.

Contents

Part I Univariate Time Series Analysis

1	Introduction	3
1.1	Some Examples	4
1.2	Formal Definitions	7
1.3	Stationarity	11
1.4	Construction of Stochastic Processes.....	15
1.4.1	White Noise.....	15
1.4.2	Construction of Stochastic Processes: Some Examples ..	16
1.4.3	Moving-Average Process of Order One	17
1.4.4	Random Walk	19
1.4.5	Changing Mean	20
1.5	Properties of the Autocovariance Function	20
1.5.1	Autocovariance Function of MA(1) Processes	21
1.6	Exercises	22
2	ARMA Models	25
2.1	The Lag Operator.....	26
2.2	Some Important Special Cases	27
2.2.1	Moving-Average Process of Order q	27
2.2.2	First Order Autoregressive Process	29
2.3	Causality and Invertibility	32
2.4	Computation of Autocovariance Function	38
2.4.1	First Procedure.....	39
2.4.2	Second Procedure.....	41
2.4.3	Third Procedure.....	43
2.5	Exercises	44
3	Forecasting Stationary Processes	45
3.1	Linear Least-Squares Forecasts.....	45
3.1.1	Forecasting with an AR(p) Process	48
3.1.2	Forecasting with MA(q) Processes	50
3.1.3	Forecasting from the Infinite Past.....	53
3.2	The Wold Decomposition Theorem	54
3.3	Exponential Smoothing	58

3.4	Exercises	60
3.5	Partial Autocorrelation	61
3.5.1	Definition	62
3.5.2	Interpretation of ACF and PACF	64
3.6	Exercises	65
4	Estimation of Mean and ACF	67
4.1	Estimation of the Mean	67
4.2	Estimation of ACF	73
4.3	Estimation of PACF	78
4.4	Estimation of the Long-Run Variance	79
4.4.1	An Example	83
4.5	Exercises	85
5	Estimation of ARMA Models	87
5.1	The Yule-Walker Estimator	87
5.2	OLS Estimation of an AR(p) Model	91
5.3	Estimation of an ARMA(p,q) Model	94
5.4	Estimation of the Orders p and q	99
5.5	Modeling a Stochastic Process	102
5.6	Modeling Real GDP of Switzerland	103
6	Spectral Analysis and Linear Filters	109
6.1	Spectral Density	110
6.2	Spectral Decomposition of a Time Series	113
6.3	The Periodogram and the Estimation of Spectral Densities	117
6.3.1	Non-Parametric Estimation	117
6.3.2	Parametric Estimation	121
6.4	Linear Time-Invariant Filters	122
6.5	Some Important Filters	127
6.5.1	Construction of Low- and High-Pass Filters	127
6.5.2	The Hodrick-Prescott Filter	128
6.5.3	Seasonal Filters	130
6.5.4	Using Filtered Data	131
6.6	Exercises	132
7	Integrated Processes	133
7.1	Definition, Properties and Interpretation	133
7.1.1	Long-Run Forecast	135
7.1.2	Variance of Forecast Error	136
7.1.3	Impulse Response Function	137
7.1.4	The Beveridge-Nelson Decomposition	138
7.2	Properties of the OLS Estimator in the Case of Integrated Variables	141
7.3	Unit-Root Tests	145
7.3.1	Dickey-Fuller Test	147
7.3.2	Phillips-Perron Test	149

7.3.3	Unit-Root Test: Testing Strategy	150
7.3.4	Examples of Unit-Root Tests	152
7.4	Generalizations of Unit-Root Tests	153
7.4.1	Structural Breaks in the Trend Function	153
7.4.2	Testing for Stationarity	157
7.5	Regression with Integrated Variables	158
7.5.1	The Spurious Regression Problem	158
7.5.2	Bivariate Cointegration	159
7.5.3	Rules to Deal with Integrated Times Series	162
8	Models of Volatility	167
8.1	Specification and Interpretation	168
8.1.1	Forecasting Properties of AR(1)-Models	168
8.1.2	The ARCH(1) Model	169
8.1.3	General Models of Volatility	173
8.1.4	The GARCH(1,1) Model	177
8.2	Tests for Heteroskedasticity	183
8.2.1	Autocorrelation of Quadratic Residuals	183
8.2.2	Engle's Lagrange-Multiplier Test	184
8.3	Estimation of GARCH(p,q) Models	184
8.3.1	Maximum-Likelihood Estimation	184
8.3.2	Method of Moment Estimation	187
8.4	Example: Swiss Market Index (SMI)	188
 Part II Multivariate Time Series Analysis		
9	Introduction	197
10	Definitions and Stationarity	201
11	Estimation of Covariance Function	207
11.1	Estimators and Asymptotic Distributions	207
11.2	Testing Cross-Correlations of Time Series	209
11.3	Some Examples for Independence Tests	211
12	VARMA Processes	215
12.1	The VAR(1) Process	216
12.2	Representation in Companion Form	218
12.3	Causal Representation	218
12.4	Computation of Covariance Function	221
13	Estimation of VAR Models	225
13.1	Introduction	225
13.2	The Least-Squares Estimator	226
13.3	Proofs of Asymptotic Normality	231
13.4	The Yule-Walker Estimator	238

14	Forecasting with VAR Models	241
14.1	Forecasting with Known Parameters	241
14.1.1	Wold Decomposition Theorem	245
14.2	Forecasting with Estimated Parameters	245
14.3	Modeling of VAR Models	247
14.4	Example: VAR Model.....	248
15	Interpretation of VAR Models	255
15.1	Wiener-Granger Causality	255
15.1.1	VAR Approach.....	256
15.1.2	Wiener-Granger Causality and Causal Representation ...	258
15.1.3	Cross-Correlation Approach	259
15.2	Structural and Reduced Form.....	260
15.2.1	A Prototypical Example	260
15.2.2	Identification: The General Case.....	263
15.2.3	Identification: The Case $n = 2$	266
15.3	Identification via Short-Run Restrictions	268
15.4	Interpretation of VAR Models	270
15.4.1	Impulse Response Functions.....	270
15.4.2	Variance Decomposition	270
15.4.3	Confidence Intervals.....	272
15.4.4	Example 1: Advertisement and Sales.....	274
15.4.5	Example 2: IS-LM Model with Phillips Curve.....	277
15.5	Identification via Long-Run Restrictions.....	282
15.5.1	A Prototypical Example	282
15.5.2	The General Approach	285
15.6	Sign Restrictions	289
16	Cointegration	295
16.1	A Theoretical Example.....	296
16.2	Definition and Representation	302
16.2.1	Definition	302
16.2.2	VAR and VEC Models	305
16.2.3	Beveridge-Nelson Decomposition	308
16.2.4	Common Trend Representation	310
16.3	Johansen's Cointegration Test	311
16.3.1	Specification of the Deterministic Components.....	317
16.3.2	Testing Cointegration Hypotheses	318
16.4	Estimation and Testing of Cointegrating Relationships	319
16.5	An Example	321
17	Kalman Filter	325
17.1	The State Space Model.....	326
17.1.1	Examples.....	328
17.2	Filtering and Smoothing	336

17.2.1	The Kalman Filter	339
17.2.2	The Kalman Smoother	340
17.3	Estimation of State Space Models	343
17.3.1	The Likelihood Function	344
17.3.2	Identification	346
17.4	Examples	346
17.4.1	Estimation of Quarterly GDP	346
17.4.2	Structural Time Series Analysis	349
17.5	Exercises	350
18	Generalizations of Linear Models	353
18.1	Structural Breaks	353
18.1.1	Methodology	354
18.1.2	An Example	356
18.2	Time-Varying Parameters	357
18.3	Regime Switching Models	364
A	Complex Numbers	369
B	Linear Difference Equations	373
C	Stochastic Convergence	377
D	BN-Decomposition	383
E	The Delta Method	387
	Bibliography	391
	Index	403

List of Figures

Fig. 1.1	Real gross domestic product (GDP).....	5
Fig. 1.2	Growth rate of real gross domestic product (GDP).....	5
Fig. 1.3	Swiss real gross domestic product.....	6
Fig. 1.4	Short- and long-term Swiss interest rates	7
Fig. 1.5	Swiss Market Index (SMI). (a) Index. (b) Daily return	8
Fig. 1.6	Unemployment rate in Switzerland	9
Fig. 1.7	Realization of a random walk.....	12
Fig. 1.8	Realization of a branching process	12
Fig. 1.9	Processes constructed from a given white noise process. (a) White noise. (b) Moving-average with $\theta = 0.9$. (c) Autoregressive with $\phi = 0.9$. (d) Random walk	17
Fig. 1.10	Relation between the autocorrelation coefficient of order one, $\rho(1)$, and the parameter θ of a MA(1) process.....	23
Fig. 2.1	Realization and estimated ACF of MA(1) process	28
Fig. 2.2	Realization and estimated ACF of an AR(1) process.....	31
Fig. 2.3	Autocorrelation function of an ARMA(2,1) process	42
Fig. 3.1	Autocorrelation and partial autocorrelation functions. (a) Process 1. (b) Process 2. (c) Process 3. (d) Process 4	66
Fig. 4.1	Estimated autocorrelation function of a WN(0,1) process	75
Fig. 4.2	Estimated autocorrelation function of MA(1) process	76
Fig. 4.3	Estimated autocorrelation function of an AR(1) process.....	77
Fig. 4.4	Estimated PACF of an AR(1) process	78
Fig. 4.5	Estimated PACF for a MA(1) process	79
Fig. 4.6	Common kernel functions	81
Fig. 4.7	Estimated autocorrelation function for the growth rate of GDP	84
Fig. 5.1	Parameter space of causal and invertible ARMA(1,1) process	100
Fig. 5.2	Real GDP growth rates of Switzerland.....	104
Fig. 5.3	ACF and PACF of GDP growth rate	105
Fig. 5.4	Inverted roots of the ARMA(1,3) model	106
Fig. 5.5	ACF of the residuals from AR(2) and ARMA(1,3) models	107
Fig. 5.6	Impulse responses of the AR(2) and the ARMA(1,3) model	107

Fig. 5.7	Forecasts of real GDP growth rates	108
Fig. 6.1	Examples of spectral densities with $Z_t \sim \text{WN}(0, 1)$. (a) MA(1) process. (b) AR(1) process	114
Fig. 6.2	Raw periodogram of a white noise time series ($X_t \sim \text{WN}(0, 1)$, $T = 200$)	120
Fig. 6.3	Raw periodogram of an AR(2) process ($X_t = 0.9X_{t-1} - 0.7X_{t-2} + Z_t$ with $Z_t \sim \text{WN}(0, 1)$, $T = 200$)	121
Fig. 6.4	Non-parametric direct estimates of a spectral density	121
Fig. 6.5	Nonparametric and parametric estimates of spectral density	123
Fig. 6.6	Transfer function of the Kuznets filters	127
Fig. 6.7	Transfer function of HP-filter	129
Fig. 6.8	HP-filtered US GDP	130
Fig. 6.9	Transfer function of growth rate of investment in the construction sector with and without seasonal adjustment	131
Fig. 7.1	Distribution of the OLS estimator	142
Fig. 7.2	Distribution of t-statistic and standard normal distribution	144
Fig. 7.3	ACF of a random walk with 100 observations	145
Fig. 7.4	Three types of structural breaks at T_B . (a) Level shift. (b) Change in slope. (c) Level shift and change in slope	154
Fig. 7.5	Distribution of OLS-estimate $\hat{\beta}$ and t-statistic $t_{\hat{\beta}}$ for two independent random walks and two independent AR(1) processes. (a) Distribution of $\hat{\beta}$. (b) Distribution of $t_{\hat{\beta}}$. (c) Distribution of $\hat{\beta}$ and t-statistic $t_{\hat{\beta}}$	160
Fig. 7.6	Cointegration of inflation and three-month LIBOR. (a) Inflation and three-month LIBOR. (b) Residuals from cointegrating regression	163
Fig. 8.1	Simulation of two ARCH(1) processes	174
Fig. 8.2	Parameter region for which a strictly stationary solution to the GARCH(1,1) process exists assuming $\nu_t \sim \text{IIDN}(0, 1)$	180
Fig. 8.3	Daily return of the SMI (Swiss Market Index)	188
Fig. 8.4	Normal-Quantile Plot of SMI returns	189
Fig. 8.5	Histogram of SMI returns	190
Fig. 8.6	ACF of the returns and the squared returns of the SMI	190
Fig. 11.1	Cross-correlations between two independent AR(1) processes	212
Fig. 11.2	Cross-correlations between consumption and advertisement	213
Fig. 11.3	Cross-correlations between GDP and consumer sentiment	214
Fig. 14.1	Forecast comparison of alternative models. (a) $\log Y_t$. (b) $\log P_t$. (c) $\log M_t$. (d) R_t	251
Fig. 14.2	Forecast of VAR(8) model and 80 % confidence intervals	253
Fig. 15.1	Identification in a two-dimensional structural VAR	267

Fig. 15.2	Impulse response functions for advertisement and sales	276
Fig. 15.3	Impulse response functions of IS-LM model	280
Fig. 15.4	Impulse response functions of the Blanchard-Quah model	289
Fig. 16.1	Impulse responses of present discounted value model	302
Fig. 16.2	Stochastic simulation of present discounted value model	303
Fig. 17.1	State space model	326
Fig. 17.2	Spectral density of cyclical component	334
Fig. 17.3	Estimates of quarterly GDP growth rates	349
Fig. 17.4	Components of the basic structural model (BSM) for real GDP of Switzerland. (a) Logged Swiss GDP (demeaned). (b) Local linear trend (LLT). (c) Business cycle component. (d) Seasonal component	350
Fig. 18.1	Break date UK.....	357
Fig. A.1	Representation of a complex number	370

List of Tables

Table 1.1	Construction of stochastic processes	17
Table 3.1	Forecast function for a MA(1) process with $\theta = -0.9$ and $\sigma^2 = 1$	52
Table 3.2	Properties of the ACF and the PACF	65
Table 4.1	Common kernel functions	81
Table 5.1	AIC for alternative ARMA(p,q) models	105
Table 5.2	BIC for alternative ARMA(p,q) models	106
Table 7.1	The four most important cases for the unit-root test	147
Table 7.2	Examples of unit root tests	153
Table 7.3	Dickey-Fuller regression allowing for structural breaks	155
Table 7.4	Critical values of the KPSS test	158
Table 7.5	Rules of thumb in regressions with integrated processes	165
Table 8.1	AIC criterion for variance equation in GARCH(p,q) model	191
Table 8.2	BIC criterion for variance equation in GARCH(p,q) model	191
Table 8.3	One percent VaR for the next day of the return on SMI	193
Table 8.4	One percent VaR for the next 10 days of the return on SMI	193
Table 14.1	Information criteria for the VAR models of different orders	249
Table 14.2	Forecast evaluation of alternative VAR models	252
Table 15.1	Forecast error variance decomposition (FEVD) in terms of demand, supply, price, wage, and money shocks (percentages)	281
Table 16.1	Trend specifications in vector error correction models	318
Table 16.2	Evaluation of the results of Johansen's cointegration test	322

List of Definitions

1.3	Model	10
1.4	Autocovariance Function	13
1.5	Stationarity	13
1.6	Strict Stationarity	14
1.7	Strict Stationarity	14
1.8	Gaussian Process	15
1.9	White Noise	15
2.1	ARMA Models	25
2.2	Causality	32
2.3	Invertibility	37
3.1	Deterministic Process	54
3.2	Partial Autocorrelation Function I	62
3.3	Partial Autocorrelation Function II	62
6.1	Spectral Density	110
6.2	Periodogram	118
7.2	Cointegration, Bivariate	159
8.1	ARCH(1) Model	169
10.2	Stationarity	202
10.3	Strict Stationarity	203
12.1	VARMA process	215
15.2	Sign Restrictions	291
16.3	Cointegration	305
C.1	Almost Sure Convergence	378
C.2	Convergence in Probability	378
C.3	Convergence in r-th Mean	378

C.4	Convergence in Distribution	379
C.5	Characteristic Function	380
C.6	Asymptotic Normality	381
C.7	m-Dependence	381

List of Theorems

3.1	Wold Decomposition	55
4.1	Convergence of Arithmetic Average	68
4.2	Asymptotic Distribution of Sample Mean	69
4.4	Asymptotic Distribution of Autocorrelations	74
5.1	Asymptotic Normality of Yule-Walker Estimator	89
5.2	Asymptotic Normality of the Least-Squares Estimator	92
5.3	Asymptotic Distribution of ML Estimator	98
6.1	Properties of a Spectral Density	111
6.2	Spectral Representation	115
6.3	Spectral Density of ARMA Processes	121
6.4	Autocovariance Function of Filtered Process	123
7.1	Beveridge-Nelson Decomposition	139
13.1	Asymptotic Distribution of OLS Estimator	229
16.1	Beveridge-Nelson Decomposition	304
18.1	Solution TVC-VAR(1)	358
C.1	Cauchy-Bunyakovskii-Schwarz Inequality	377
C.2	Minkowski's Inequality	377
C.3	Chebyshev's Inequality	377
C.4	Borel-Cantelli Lemma	378
C.5	Kolmogorov's Strong Law of Large Numbers (SLLN)	378
C.6	Riesz-Fisher	379
C.9	Continuous Mapping Theorem	380
C.10	Slutzky's Lemma	380
C.11	Convergence of Characteristic Functions, Lévy	381
C.12	Central Limit Theorem	381
C.13	CLT for m-Dependent Processes	381
C.14	Basis Approximation Theorem	382

Notation and Symbols

r	number of linearly independent cointegration vectors
α	$n \times r$ loading matrix
β	$n \times r$ matrix of linearly independent cointegration vectors
\xrightarrow{d}	convergence in distribution
$\xrightarrow{\text{m.s.}}$	convergence in mean square
\xrightarrow{p}	convergence in probability
$\text{corr}(X, Y)$	correlation coefficient between random variables X and Y
γ_X, γ	covariance function of process $\{X_t\}$, covariance function
ρ_X, ρ	correlation function of process $\{X_t\}$, correlation function
ACF	autocorrelation function
J	long-run variance
α_X, α	partial autocorrelation function of process $\{X_t\}$
PACF	partial autocorrelation function
n	dimension of stochastic process, respectively dimension of state space
\sim	is distributed as
sgn	sign function
tr	trace of a matrix
det	determinant of a matrix
$\ \ $	norm of a matrix
\otimes	Kronecker product
\odot	Hadamard product
$\text{vec}(A)$	stacks the columns of A into a vector
$\text{vech}(A)$	stacks the lower triangular part of a symmetric matrix A into a vector
$GL(n)$	general linear group of $n \times n$ matrices
$\mathcal{O}(n)$	group of orthogonal $n \times n$ matrices
L	lag operator

$\Phi(L)$	autoregressive polynomial
$\Theta(L)$	moving-average polynomial
$\Psi(L)$	causal representation, $MA(\infty)$ polynomial
Δ	difference operator, $\Delta = 1 - L$
p	order of autoregressive polynomial
q	order of moving-average polynomial
$ARMA(p,q)$	autoregressive moving-average process of order (p, q)
$ARIMA(p,d,q)$	autoregressive integrated moving-average process of order (p, d, q)
d	order of integration
$I(d)$	integrated process of order d
$VAR(p)$	vector autoregressive process of order p
\mathbb{Z}	integer numbers
\mathbb{R}	real numbers
\mathbb{C}	complex numbers
\mathbb{R}^n	set of n -dimensional vectors
i	imaginary unit
$cov(X, Y)$	covariance between random variables X and Y
\mathbb{E}	expectation operator
\mathbb{V}	variance operator
$\Psi(1)$	persistence
$\mathbb{P}_T X_{T+h}$	linear least-squares predictor of X_{T+h} given information from period 1 up to period T
$\widetilde{\mathbb{P}}_T X_{T+h}$	linear least-squares predictor of X_{T+h} using the infinite remote past up to period T
P	Probability
$\{X_t\}$	stochastic process
$WN(0, \sigma^2)$	white noise process with mean zero and variance σ^2
$WN(0, \Sigma)$	multivariate white noise process with mean zero and covariance matrix Σ^2
$IID(0, \sigma^2)$	identically and independently distributed random variables with mean zero and variance σ^2
$IID N(0, \sigma^2)$	identically and independently normally distributed random variables with mean zero and variance σ^2
X_t	time indexed random variable
x_t	realization of random variable X_t
$f(\lambda)$	spectral density
$F(\lambda)$	spectral distribution function
I_T	periodogram
$\Psi(e^{-i\lambda})$	transfer function of filter Ψ
VaR	value at risk