Complex Numbers

A

The simple quadratic equation $x^2 + 1 = 0$ has no solution in the field of real numbers, $\mathbb{R}$. Thus, it is necessary to envisage the larger field of complex numbers $\mathbb{C}$. A complex number z is an ordered pair (a, b) of real numbers where ordered means that we regard (a, b) and (b, a) as distinct if $a \neq b$. Let $x = (a, b)$ and $y = (c, d)$ be two complex numbers. Then we endow the set of complex numbers with an addition and a multiplication in the following way:

$$\text{addition:} \qquad x + y = (a, b) + (c, d) = (a + c, b + d)$$
$$\text{multiplication:} \qquad xy = (a, b)(c, d) = (ac - bd, ad + bc).$$

These two operations will turn $\mathbb{C}$ into a field where $(0, 0)$ and $(1, 0)$ play the role of 0 and 1.[1] The real numbers $\mathbb{R}$ are embedded into $\mathbb{C}$ because we identify any $a \in \mathbb{R}$ with $(a, 0) \in \mathbb{C}$.

The number $\imath = (0, 1)$ is of special interest. It solves the equation $x^2 + 1 = 0$, i.e. $\imath^2 = -1$. The other solution being $-\imath = (0, -1)$. Thus any complex number (a, b) may be written as $(a, b) = a + \imath b$ where a, b are arbitrary real numbers.[2]

[1] Substraction and division can be defined accordingly:

$$\text{subtraction:} \qquad (a, b) - (c, d) = (a - c, b - d)$$
$$\text{division:} \qquad (a, b)/(c, d) = \frac{(ac + bd, bc - ad)}{(c^2 + d^2)}, \quad c^2 + d^2 \neq 0.$$

[2] A more detailed introduction of complex numbers can be found in Rudin (1976) or any other mathematics textbook.

K. Neusser, *Time Series Econometrics*, Springer Texts in Business and Economics,
DOI 10.1007/978-3-319-32862-1

An element z in this field can be represented in two ways:

$$z = a + \imath b \qquad \text{Cartesian coordinates}$$
$$= re^{\imath\theta} = r(\cos\theta + \imath\sin\theta) \qquad \text{polar coordinates.}$$

In the representation in Cartesian coordinates $a = \mathrm{Re}(z) = \Re(z)$ is called the *real part* whereas $b = \mathrm{Im}(z) = \Im(z)$ is called the *imaginary part* of z.

A complex number z can be viewed as a point in the two-dimensional Cartesian coordinate system with coordinates (a, b). This geometric interpretation is represented in Fig. A.1.

The *absolute value* or *modulus* of z, denoted by $|z|$, is given by $r = \sqrt{a^2 + b^2}$. Thus, the absolute value is nothing but the distance of z viewed as a point in the complex plane (the two-dimensional Cartesian coordinate system) to the origin (see Fig. A.1). θ denotes the angle to the positive real axis (x-axis) measured in radians. It is denoted by $\theta = \arg z$. It holds that $\tan\theta = \frac{b}{a}$. Finally, the conjugate of z, denoted by $\bar{z}$, is defined by $\bar{z} = a - \imath b$.

Setting $r = 1$ and $\theta = \pi$, gives the following famous formula:

$$e^{\imath\pi} + 1 = (\cos\pi + \imath\sin\pi) + 1 = -1 + 1 = 0.$$

This formula relates the most famous numbers in mathematics.

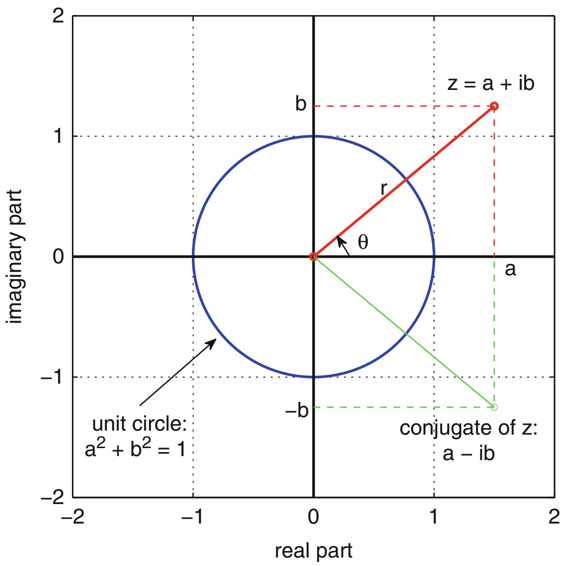

Fig. A.1 Representation of a complex number

From the definition of complex numbers in polar coordinates, we immediately derive the following implications:

$$\cos\theta = \frac{e^{\imath\theta} + e^{-\imath\theta}}{2} = \frac{a}{r},$$
$$\sin\theta = \frac{e^{\imath\theta} - e^{-\imath\theta}}{2\imath} = \frac{b}{r}.$$

Further implications are de Moivre's formula and Pythagoras' theorem (see Fig. A.1):

$$\text{de Moivre's formula} \qquad \left(re^{\imath\theta}\right)^n = r^n e^{\imath n\theta} = r^n(\cos n\theta + \imath\sin n\theta)$$
$$\text{Pythagoras' theorem} \qquad 1 = e^{\imath\theta}e^{-\imath\theta} = (\cos\theta + \imath\sin\theta)(\cos\theta - \imath\sin\theta)$$
$$= \cos^2\theta + \sin^2\theta$$

From Pythagoras' theorem it follows that $r^2 = a^2 + b^2$. The representation in polar coordinates allows to derive many trigonometric formulas.

Consider the polynomial $\Phi(z) = \phi_0 - \phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p$ of order $p \geq 1$ with $\phi_0 = 1$.[3] The *fundamental theorem of algebra* then states that every polynomial of order $p \geq 1$ has exactly p roots in the field of complex numbers. Thus, the field of complex numbers is algebraically complete. Denote these roots by $\lambda_1, \ldots, \lambda_p$, allowing that some roots may appear several times. The polynomial can then be factorized as

$$\Phi(z) = \left(1 - \lambda_1^{-1}z\right)\left(1 - \lambda_2^{-1}z\right)\ldots\left(1 - \lambda_p^{-1}z\right).$$

This expression is well-defined because the assumption of a nonzero constant ($\phi_0 = 1 \neq 0$) excludes the possibility of roots equal to zero. If we assume that the coefficients ϕ_j, $j = 0, \ldots, p$, are real numbers, the complex roots appear in conjugate pairs. Thus if $z = a + \imath b$, $b \neq 0$, is a root then $\bar{z} = a - \imath b$ is also a root.

[3]The notation with "$-\phi_j z^j$" instead of "$\phi_j z^j$" was chosen to conform to the notation of AR-models.

Linear Difference Equations

B

Linear difference equations play an important role in time series analysis. We therefore summarize the most important results.[1] Consider the following linear difference equation of order p with constant coefficients. This equation is defined by the recursion:

$$X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p}, \qquad \phi_p \neq 0, t \in \mathbb{Z}.$$

Thereby $\{X_t\}$ represents a sequence of real numbers and $\phi_1, \ldots, \phi_p$ are p constant coefficients. The above difference equation is called *homogeneous* because it involves no other variable than X_t. A solution to this equation is a function $F : \mathbb{Z} \to \mathbb{R}$ such that its values $F(t)$ or F_t reduce the difference equation to an identity.

It is easy to see that if $\{X_t^{(1)}\}$ and $\{X_t^{(2)}\}$ are two solutions than $\{c_1 X_t^{(1)} + c_2 X_t^{(2)}\}$, for any $c_1, c_2 \in \mathbb{R}$, is also a solution. The set of solutions is therefore a linear space (vector space).

Definition B.1. *A set of solutions* $\{\{X_t^{(1)}\}, \ldots, \{X_t^{(m)}\}\}$, $m \leq p$, *is called* linearly independent *if*

$$c_1 X_t^{(1)} + \ldots + c_m X_t^{(m)} = 0, \qquad \textit{for } t = 0, 1, \ldots, p-1$$

implies that $c_1 = \ldots = c_m = 0$. *Otherwise we call the set* linearly dependent*.*

Given arbitrary starting values $x_0, \ldots, x_{p-1}$ for $X_0, \ldots, X_{p-1}$, the difference equation determines all further through the recursion:

$$X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} \qquad t = p, p+1, \ldots.$$

[1]For more detailed presentations see Agarwal (2000), Elaydi (2005) or Neusser (2009).

K. Neusser, *Time Series Econometrics*, Springer Texts in Business and Economics,
DOI 10.1007/978-3-319-32862-1

Similarly for X_t mit $t = -1, -2, \ldots$. Suppose we have p linearly independent solutions $\{\{X_t^{(1)}\}, \ldots, \{X_t^{(p)}\}\}$ then there exists exactly p numbers $c_1, \ldots, c_p$ such that the solution

$$X_t = c_1 X_t^{(1)} + c_2 X_t^{(2)} + \ldots + c_p X_t^{(p)}$$

is compatible with arbitrary starting values $x_0, \ldots, x_{p-1}$. These starting values then determine uniquely all values of the sequence $\{X_t\}$. Thus $\{X_t\}$ is the only solution compatible with starting values. The goal therefore consists in finding p linearly independent solutions.

We guess that the solutions are of the form $X_t = z^{-t}$ where z may be a complex number. If this guess is right then we must have for $t = 0$:

$$1 - \phi_1 z - \ldots - \phi_p z^p = 0.$$

This equation is called the *characteristic equation*.[2] Thus z must be a root of the polynomial $\Phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p$. From the fundamental theorem of algebra we know that there are exactly p roots in the field of complex numbers. Denote these roots by $z_1, \ldots, z_p$.

Suppose that these roots are different from each other. In this case $\{\{z_1^{-t}\}, \ldots, \{z_p^{-t}\}\}$ constitutes a set of p linearly independent solutions. To show this it is sufficient to verify that the determinant of the matrix

$$W = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ z_1^{-1} & z_2^{-1} & \ldots & z_p^{-1} \\ z_1^{-2} & z_2^{-2} & \ldots & z_p^{-2} \\ \vdots & \vdots & & \vdots \\ z_1^{-p+1} & z_2^{-p+1} & \ldots & z_p^{-p+1} \end{pmatrix}$$

is different from zero. This determinant is known as Vandermonde's determinant and is equal to $\det W = \prod_{1 \le i < j \le p} (z_i - z_j)$. This determinant is clearly different from zero because the roots are different from each other. The general solution to the difference equation therefore is

$$X_t = c_1 z_1^{-t} + \ldots + c_p z_p^{-t} \tag{B.1}$$

where the constants $c_1, \ldots, c_p$ are determined from the starting values (initial conditions).

In the case where some roots of the characteristic polynomial are equal, the general solution becomes more involved. Let $z_1, \ldots, z_r$, $r < p$, be the roots which

[2] Sometimes one can find $z^p - \phi_1 z^{p-1} - \ldots - \phi_p = 0$ as the characteristic equation. The roots are of the two characteristic equations are then reciprocal to each other.

are different from each other and denote their corresponding multiplicities by $m_1, \ldots, m_r$. It holds that $\sum_{j=1}^{r} = p$. The general solution is then given by

$$X_t = \sum_{j=1}^{r} \left(c_{j0} + c_{j1} t + \ldots + c_{jm_j-1} t^{m_j-1}\right) z_j^{-t} \tag{B.2}$$

where the constants c_{ji} are again determined from the starting values (initial conditions).

C Stochastic Convergence

This appendix presents the relevant concepts and theorems from probability theory. The reader interested in more details should consult corresponding textbooks, for example Billingsley (1986), Brockwell and Davis (1991), Hogg and Craig (1995), or Kallenberg (2002) among many others.

In the following, all real random variables or random vectors X are defined with respect to some probability space $(\Omega, \mathfrak{A}, \mathbf{P})$. Thereby, Ω denotes an arbitrary space with σ-field $\mathfrak{A}$ and probability measure $\mathbf{P}$. A random variable, respectively random vector, X is then defined as a measurable function from Ω to $\mathbb{R}$, respectively $\mathbb{R}^n$. The probability space Ω plays no role as it is introduced just for the sake of mathematical rigor. The interest rather focuses on the distributions induced by $\mathbf{P} \circ X^{-1}$.

We will make use of the following important inequalities.

Theorem C.1 (Cauchy-Bunyakovskii-Schwarz Inequality)**.** *For any two random variables X and Y,*

$$|\mathbb{E}(XY)| \leq \sqrt{\mathbb{E}X^2}\sqrt{\mathbb{E}Y^2}.$$

The equality holds if and only if $X = \frac{\mathbb{E}(XY)}{\mathbb{E}(Y^2)}Y$.

Theorem C.2 (Minkowski's Inequality)**.** *Let X and Y be two random variables with $\mathbb{E}|X|^2 < \infty$ and $\mathbb{E}|Y|^2 < \infty$, then*

$$\left(\mathbb{E}|X+Y|^2\right)^{1/2} \leq \left(\mathbb{E}|X|^2\right)^{1/2} + \left(\mathbb{E}|Y|^2\right)^{1/2}.$$

Theorem C.3 (Chebyschev's Inequality)**.** *If $\mathbb{E}|X|^r < \infty$ for $r \geq 0$ then for every $r \geq 0$ and any $\varepsilon > 0$*

$$\mathbf{P}[|X| \geq \varepsilon] \leq \varepsilon^{-r}\mathbb{E}|X|^r.$$

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DOI 10.1007/978-3-319-32862-1

Theorem C.4 (Borel-Cantelli Lemma). *Let $A_1, A_2, \ldots \in \mathfrak{A}$ be an infinite sequence of events in some probability space $(\Omega, \mathfrak{A}, \mathbf{P})$ such that $\sum_{k=1}^{\infty} \mathbf{P}(A_k) < \infty$. Then, $\mathbf{P}\{A_k \text{ i.o.}\} = 0$. The event $\{A_k \text{ i.o.}\}$ is defined by $\{A_k \text{ i.o.}\} = \limsup_k \{A_k\} = \bigcap_{k=1}^{\infty} \bigcup_{j=k}^{\infty} A_j$ where i.o. stands for infinitely often.*

On several occasions it is necessary to evaluate the limit of a sequence of random variables. In probability theory several concepts of convergence are discussed: *almost sure convergence*, *convergence in probability*, *convergence in r-th mean* (*convergence in quadratic mean*), *convergence in distribution*. We only give definitions and the most important theorems leaving an in-depth discussion to the relevant literature. Although not explicitly mentioned, many of the theorem below also hold in an analogous way in a multidimensional context.

Definition C.1 (Almost Sure Convergence). *For random variables X and $\{X_t\}$ defined on the same probability space $(\Omega, \mathfrak{A}, \mathbf{P})$, we say that $\{X_t\}$* converges almost surely *or* with probability one *to X if*

$$\mathbf{P}\left\{\omega \in \Omega : \lim_{t\to\infty} X_t(\omega) = X(\omega)\right\} = 1.$$

This fact is denoted by $X_t \xrightarrow{a.s.} X$ or $\lim X_t = X$a.s.

Theorem C.5 (Kolmogorov's Strong Law of Large Numbers (SLLN)). *Let $X, X_1, X_2, \ldots$ be identically and independently distributed random variables. Then, the arithmetic average $\overline{X}_T = \frac{1}{T}\sum_{t=1}^{T} X_t$ converges almost surely to $\mathbb{E}X$ if and only if $\mathbb{E}|X| < \infty$.*

Definition C.2 (Convergence in Probability). *For random variables X and $\{X_t\}$ defined on the same probability space, we say that $\{X_t\}$ converges in* probability *to X if*

$$\lim_{t\to\infty} \mathbf{P}[|X_t - X| > \varepsilon] = 0 \qquad \text{for all } \varepsilon > 0.$$

This fact is denoted by $X_t \xrightarrow{p} X$ or $\operatorname{plim} X_t = X$.

Remark C.1. If X and $\{X_t\}$ are real valued random vectors, we replace the absolute value in the definition above by the Euclidean norm $\|.\|$. This is, however, equivalent to saying that every component X_{it} converges in probability to X_i, the i-th component of X.

Definition C.3 (Convergence in r-th Mean). *A sequence $\{X_t\}$ of random variables converges in* r-th mean *to a random variable X if*

$$\lim_{t\to\infty} \mathbb{E}(|X_t - X|^r) = 0 \qquad \text{for } r > 0.$$

We denote this fact by $X_t \xrightarrow{r} X$. *If* $r = 1$ *we say that the sequence converges absolutely; and if* $r = 2$ *we say that the sequence converges in mean square which is denoted by* $X_t \xrightarrow{m.s.} X$.

Remark C.2. In the case $r = 2$, the corresponding definition for random vectors is

$$\lim_{t\to\infty} \mathbb{E}(\|X_t - X\|^2) = \lim_{t\to\infty} \mathbb{E}(X_t - X)'(X_t - X) = 0.$$

Theorem C.6 (Riesz-Fisher)**.** *Let* $\{X_t\}$ *be a sequence of random variables such* $\sup_t \mathbb{E}|X_t|^2 < \infty$. *Then there exists a random variable* X *with* $\mathbb{E}|X|^2 < \infty$ *such that*

$$X_t \xrightarrow{m.s.} X \quad \textit{if and only if} \quad \mathbb{E}|X_t - X_s|^2 \to 0 \quad \textit{for } t, s \to \infty.$$

This version of the Riesz-Fisher theorem provides a condition, known as the Cauchy criterion, which is often easier to verify when the limit is unknown.

Definition C.4 (Convergence in Distribution)**.** *A sequence* $\{X_t\}$ *of random vectors with corresponding distribution functions* $\{F_{X_t}\}$ *converges in* distribution*, if there exists an random vector* X *with distribution function* F_X *such that*

$$\lim_{t\to\infty} F_{X_t}(x) = F_X(x) \qquad \textit{for all } x \in \mathcal{C}$$

where $\mathcal{C}$ *denotes the set of points for which* $F_X(x)$ *is continuous. We denote this fact by* $X_t \xrightarrow{d} X$.

Note that, in contrast to the previously mentioned modes of convergence, convergence in distribution does not require that all random vectors are defined on the same probability space. The convergence in distribution states that, for large enough t, the distribution of X_t can be approximated by the distribution of X.

The following Theorem relates the four convergence concepts.

Theorem C.7. (i) *If* $X_t \xrightarrow{a.s.} X$ *then* $X_t \xrightarrow{p} X$.

(ii) *If* $X_t \xrightarrow{p} X$ *then there exists a subsequence* $\{X_{t_n}\}$ *such that* $X_{t_n} \xrightarrow{a.s.} X$.

(iii) *If* $X_t \xrightarrow{r} X$ *then* $X_t \xrightarrow{p} X$ *by Chebyschev's inequality (Theorem C.3).*

(iv) *If* $X_t \xrightarrow{p} X$ *then* $X_t \xrightarrow{d} X$.

(v) *If* X *is a fixed constant, then* $X_t \xrightarrow{d} X$ *implies* $X_t \xrightarrow{p} X$. *Thus, the two concepts are equivalent under this assumption.*

These facts can be summarized graphically:

$$
\begin{array}{ccccc}
 & & X_{t_n} \xrightarrow{a.s.} X & & \\
 & & \Uparrow & & \\
X_t \xrightarrow{a.s.} X & \Longrightarrow & X_t \xrightarrow{p} X & \Longrightarrow & X_t \xrightarrow{d} X \\
 & & \Uparrow & & \\
 & & X_t \xrightarrow{r} X & &
\end{array}
$$

A further useful theorem is:

Theorem C.8. *If* $\mathbb{E}X_t \longrightarrow \mu$ *and* $\mathbb{V}X_t \longrightarrow 0$ *then* $X_t \xrightarrow{m.s.} \mu$ *and consequently* $X_t \xrightarrow{p} \mu$.

Theorem C.9 (Continuous Mapping Theorem)**.** *For any continuous function* $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ *and random vectors* $\{X_t\}$ *and* X *defined on some probability space, the following implications hold:*

(i) $X_t \xrightarrow{a.s.} X$ *implies* $f(X_t) \xrightarrow{a.s.} f(X)$.
(ii) $X_t \xrightarrow{p} X$ *implies* $f(X_t) \xrightarrow{p} f(X)$.
(iii) $X_t \xrightarrow{d} X$ *implies* $f(X_t) \xrightarrow{d} f(X)$.

An important application of the Continuous Mapping Theorem is the so-called Delta method which can be used to approximate the distribution of $f(X_t)$ (see Appendix E).

A further useful result is given by:

Theorem C.10 (Slutzky's Lemma)**.** *Let* $\{X_t\}$ *and* $\{Y_t\}$ *be two sequences of random vectors such that* $X_t \xrightarrow{d} X$ *and* $Y_t \xrightarrow{d} c$, *c constant, then*

(i) $X_t + Y_t \xrightarrow{d} X + c$,
(ii) $Y_t' X_t \xrightarrow{d} c'X$.
(iii) $X_t / Y_t \xrightarrow{d} X/c$ *if c is a nonzero scalar.*

Like the (cumulative) distribution function, the characteristic function provides an alternative way to describe a random variable.

Definition C.5 (Characteristic Function)**.** *The* characteristic function *of a real random vector X, denoted by* φ_X, *is defined by*

$$\varphi_X(s) = \mathbb{E}e^{\imath\lambda' X}, \qquad \lambda \in \mathbb{R}^n,$$

where $\imath$ *is the imaginary unit.*

If, for example, $X \sim \mathrm{N}(\mu, \sigma^2)$, then $\varphi_X(s) = \exp(\imath s\mu - \frac{1}{2}\sigma^2 s^2)$. The characteristic function uniquely determines the distribution of X. Thus, if two random variables have the same characteristic function, they have the same distribution. Moreover, convergence in distribution is equivalent to convergence of the corresponding characteristic functions.

Theorem C.11 (Convergence of Characteristic Functions, Lévy)**.** *Let $\{X_t\}$ be a sequence of real random variables with corresponding characteristic functions φ_{X_t} then*

$$X_t \xrightarrow{d} X \quad \text{if and only if} \quad \lim_{t\to\infty} \varphi_{X_t}(\lambda) = \varphi_X(\lambda), \quad \text{for all } \lambda \in \mathbb{R}^n.$$

In many cases the limiting distribution is a normal distribution. In which case one refers to the asymptotic normality.

Definition C.6 (Asymptotic Normality)**.** *A sequence of random variables $\{X_t\}$ with "means" μ_t and "variances" $\sigma_t^2 > 0$ is said to be* asymptotically normally *distributed if*

$$\sigma_t^{-1}(X_t - \mu_t) \xrightarrow{d} X \sim \mathrm{N}(0,1).$$

Note that the definition does not require that $\mu_t = \mathbb{E}X_t$ nor that $\sigma_t^2 = \mathbb{V}(X_t)$. Asymptotic normality is obtained if the X_t's are identically and independently distributed with constant mean and variance. In this case the Central Limit Theorem (CLT) holds.

Theorem C.12 (Central Limit Theorem)**.** *Let $\{X_t\}$ be a sequence of identically and independently distributed random variables with constant mean μ and constant variance σ^2 then*

$$\sqrt{T}\,\frac{\overline{X}_T - \mu}{\sigma} \xrightarrow{d} \mathrm{N}(0,1),$$

where $\overline{X}_T = T^{-1}\sum_{t=1}^{T} X_t$ is the arithmetic average.

It is possible to relax the assumption of identically distributed variables in various ways so that there exists a variety of CLT's in the literature. For our purpose it is especially important to relax the independence assumption. A natural way to do this is by the notion of m-*dependence*.

Definition C.7 (m-Dependence)**.** *A strictly stationary random process $\{X_t\}$ is called* m-dependent *for some nonnegative integer m if and only if the two sets of random variables $\{X_\tau, \tau \le t\}$ and $\{X_\tau, \tau \ge t+m+1\}$ are independent.*

Note that for such processes $\Gamma(j) = 0$ for $j > m$. This type of dependence allows to proof the following generalized Central Limit Theorem (see Brockwell and Davis 1991).

Theorem C.13 (CLT for m-Dependent Processes)**.** *Let $\{X_t\}$ be a strictly stationary mean zero m-dependent process with autocovariance function $\Gamma(h)$ such that $\mathbf{V}_m = \sum_{h=-m}^{m} \Gamma(h) \neq 0$ then*

(i) $\lim_{T\to\infty} T\mathbb{V}(\overline{X}_T) = \mathbf{V}_m$ *and*
(ii) $\sqrt{T}\overline{X}_T$ *is asymptotically normal* $\mathrm{N}(0, \mathbf{V}_m)$.

Often it is difficult to derive the asymptotic distribution of $\{X_t\}$ directly. This situation can be handled by approximating the original process $\{X_t\}$ by a process $\{X_t^{(m)}\}$ which is easier to handle in terms of its asymptotic distribution and where the precision of the approximation can be "tuned" by the parameter m.

Theorem C.14 (Basis Approximation Theorem)**.** *Let* $\{X_t\}$ *and* $\{X_t^{(m)}\}$ *be two random vectors process such that*

(i) $X_t^{(m)} \xrightarrow{d} X^{(m)}$ *as* $t \to \infty$ *for each* $m = 1, 2, \ldots$,
(ii) $X^{(m)} \xrightarrow{d} X$ *as* $m \to \infty$, *and*
(iii) $\lim_{m\to\infty} \limsup_{t\to\infty} \mathbf{P}[|X_t - X_t^{(m)}| > \epsilon] = 0$ *for every* $\epsilon > 0$.

Then

$$X_t \xrightarrow{d} X \qquad \text{as } t \to \infty.$$

D Beveridge-Nelson Decomposition

The Beveridge-Nelson decomposition proves to be an indispensable tool. Based on the seminal paper by Phillips and Solo (1992), we proof the following Theorem for matrix polynomials where $\|.\|$ denotes the matrix norm (see Definition 10.6 in Chapter 10). The univariate version is then a special case with the absolute value replacing the norm.

Theorem D.1. *Any a lag polynomial* $\Psi(\mathrm{L}) = \sum_{j=0}^{\infty} \Psi_j \mathrm{L}^j$ *where* Ψ_j *are* $n \times n$ *matrices with* $\Psi_0 = I_n$ *can be represented by*

$$\Psi(\mathrm{L}) = \Psi(1) - (I_n - \mathrm{L})\widetilde{\Psi}(\mathrm{L}) \tag{D.1}$$

where $\widetilde{\Psi}(\mathrm{L}) = \sum_{j=0}^{\infty} \widetilde{\Psi}_j \mathrm{L}$ *with* $\widetilde{\Psi}_j = \sum_{i=j+1}^{\infty} \Psi_i$. *Moreover,*

$$\sum_{j=1}^{\infty} j^2 \|\Psi_j\|^2 < \infty \quad \textit{implies} \quad \sum_{j=0}^{\infty} \|\widetilde{\Psi}_j\|^2 < \infty \textit{ and } \|\Psi(1)\| < \infty.$$

Proof. The first part of the Theorem is obtained by the algebraic manipulations below:

$$\begin{aligned}
\Psi(\mathrm{L}) - \Psi(1) &= I_n + \Psi_1 \mathrm{L} + \Psi_2 \mathrm{L}^2 + \ldots \\
&\quad - I_n - \Psi_1 - \Psi_2 - \ldots \\
&= \Psi_1(\mathrm{L} - I_n) + \Psi_2(\mathrm{L}^2 - I_n) + \Psi_3(\mathrm{L}^3 - I_n) + \ldots \\
&= (\mathrm{L} - I_n)\Psi_1 + (\mathrm{L} - I_n)\Psi_2(\mathrm{L} + I_n) \\
&\quad + (\mathrm{L} - I_n)\Psi_3(\mathrm{L}^2 + \mathrm{L} + I_n) + \ldots
\end{aligned}$$

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DOI 10.1007/978-3-319-32862-1

$$= -(I_n - \mathrm{L})(\underbrace{(\Psi_1 + \Psi_2 + \Psi_3 + \ldots)}_{\widetilde{\Psi}_0} +$$

$$\underbrace{(\Psi_2 + \Psi_3 + \ldots)}_{\widetilde{\Psi}_1} \mathrm{L} + \underbrace{(\Psi_3 + \ldots)}_{\widetilde{\Psi}_2} \mathrm{L}^2 + \ldots)$$

Taking any $\delta \in (1/2, 1)$, the second part of the Theorem follows from

$$\begin{aligned}
\sum_{j=0}^{\infty} \|\widetilde{\Psi}_j\|^2 = \sum_{j=0}^{\infty} \left\| \sum_{i=j+1}^{\infty} \Psi_j \right\|^2 &\leq \sum_{j=0}^{\infty} \left(\sum_{i=j+1}^{\infty} \|\Psi_j\| \right)^2 \\
&= \sum_{j=0}^{\infty} \left(\sum_{i=j+1} i^{\delta} \|\Psi_j\| i^{-\delta} \right)^2 \\
&\leq \sum_{j=0}^{\infty} \left(\sum_{i=j+1}^{\infty} i^{2\delta} \|\Psi_j\|^2 \right) \left(\sum_{i=j+1}^{\infty} i^{-2\delta} \right) \\
&\leq (2\delta - 1)^{-1} \sum_{j=0}^{\infty} \left(\sum_{i=j+1}^{\infty} i^{2\delta} \|\Psi_i\|^2 \right) j^{1-2\delta} \\
&= (2\delta - 1)^{-1} \sum_{i=0}^{\infty} \left(\sum_{j=0}^{i-1} j^{1-2\delta} \right) i^{2\delta} \|\Psi_i\|^2 \\
&\leq [(2\delta - 1)(2 - 2\delta)]^{-1} \sum_{j=0}^{\infty} j^{2\delta} \|\Psi_i\|^2 j^{2-2\delta} \\
&= [(2\delta - 1)(2 - 2\delta)]^{-1} \sum_{j=0}^{\infty} j^2 \|\Psi_i\|^2 < \infty.
\end{aligned}$$

The first inequality follows from the triangular inequality for the norm. The second inequality is Hölder's inequality (see, for example, Naylor and Sell 1982, p. 548) with $p = q = 2$. The third and the fourth inequality follow from the Lemma below. The last inequality, finally, follows from the assumption.

The last assertion follows from

$$\|\Psi(1)\| \leq \sum_{j=0}^{\infty} \|\Psi_j\| = \|I_n\| + \sum_{j=1}^{\infty} j\|\Psi_j\|j^{-1}$$

$$\leq \|I_n\| + \left(\sum_{j=1}^{\infty} j^2\|\Psi_j\|^2\right)^2 \left(\sum_{j=1}^{\infty} j^{-2}\right)^2 < \infty.$$

The last inequality is again a consequence of Hölder's inequality. The summability assumption then guarantees the convergence of the first term in the product. Cauchy's condensation test finally establishes the convergence of the last term. □

Lemma D.1. *The following results are useful:*

(i) For any $b > 0$, $\sum_{i=j+1}^{\infty} i^{-1-b} \leq b^{-1}j^{-b}$.
(ii) For any $c \in (0,1)$, $\sum_{j=1}^{i} j^{c-1} \leq c^{-1}i^c$.

Proof. Let k be a number greater than j, then $k^{-1-b} \leq j^{-1-b}$ and

$$k^{-1-b} \leq \int_{k-1}^{k} j^{-1-b}\mathrm{d}j = b^{-1}(k-1)^{-b} - b^{-1}k^{-b}.$$

This implies that $\sum_{k=j+1}^{\infty} k^{-1-b} \leq b^{-1}j^{-b}$. This proves part (i) by changing the summation index back from k to j. Similarly, $k^{c-1} \leq j^{c-1}$ and

$$k^{c-1} \leq \int_{k-1}^{k} j^{c-1}\mathrm{d}j = c^{-1}k^c - c^{-1}(k-1)^c.$$

Therefore $\sum_{k=1}^{i} k^{c-1} \leq c^{-1}i^c$ which proves part (ii) by changing the summation index back from k to j. □

Remark D.1. An alternative common assumption is $\sum_{j=1}^{\infty} j\|\Psi_j\| < \infty$. It is, however, easy to see that this assumption is more restrictive as it implies the one assumed in the Theorem, but not vice versa. See Phillips and Solo (1992) for more details.

The Delta Method

E

It is often the case that it is possible to obtain an estimate $\hat{\beta}_T$ of some parameter β, but that one is really interested in a function f of β. The Continuous Mapping Theorem then suggests to estimate $f(\beta)$ by $f(\hat{\beta}_T)$. But then the question arises how the distribution of $\hat{\beta}_T$ is related to the distribution of $f(\hat{\beta}_T)$.

Expanding the function into a first order Taylor approximation allows to derive the following theorem.

Theorem E.1. *Let* $\{\hat{\beta}_T\}$ *be a* K*-dimensional sequence of random variables with the property* $\sqrt{T}(\hat{\beta}_T - \beta) \xrightarrow{\mathrm{d}} N(0, \Sigma)$ *then*

$$\sqrt{T}\left(f(\hat{\beta}_T) - f(\beta)\right) \xrightarrow{\mathrm{d}} \mathrm{N}\left(0, \nabla f(\beta)\ \Sigma\ \nabla f(\beta)'\right),$$

where $f : \mathbb{R}^K \longrightarrow \mathbb{R}^J$ *is a continuously differentiable function with Jacobian matrix (matrix of first order partial derivatives)* $\nabla f(\beta) = \partial f(\beta)/\partial \beta'$.

Proof. See Serfling (Serfling 1980, 122–124). □

Remark E.1. In the one-dimensional case where $\sqrt{T}(\hat{\beta}_T - \beta) \xrightarrow{\mathrm{d}} \mathrm{N}(0, \sigma^2)$ and $f : \mathbb{R} \longrightarrow \mathbb{R}$ the above theorem becomes:

$$\sqrt{T}\left(f(\hat{\beta}_T) - f(\beta)\right) \xrightarrow{\mathrm{d}} \mathrm{N}\left(0, [f'(\beta)]^2\sigma^2\right)$$

where $f'(\beta)$ is the first derivative evaluated at β.

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DOI 10.1007/978-3-319-32862-1

Remark E.2. The $J \times K$ Jacobian matrix of first order partial derivatives is defined as

$$\nabla f(\beta) = \partial f(\beta)/\partial \beta' = \begin{pmatrix} \frac{\partial f_1(\beta)}{\partial \beta_1} & \cdots & \frac{\partial f_1(\beta)}{\partial \beta_K} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(\beta)}{\partial \beta_1} & \cdots & \frac{\partial f_J(\beta)}{\partial \beta_K} \end{pmatrix}.$$

Remark E.3. In most applications β is not known so that one evaluates the Jacobian matrix at $\hat{\beta}_T$.

Example: Univariate

Suppose we have obtained an estimate of β equal to $\hat{\beta} = 0.6$ together with an estimate for its variance $\hat{\sigma}^2_{\hat{\beta}} = 0.2$. We can then approximate the variance of $f(\hat{\beta}) = 1/\hat{\beta} = 1.667$ by

$$\hat{\mathrm{V}}(f(\hat{\beta})) = \left[\frac{-1}{\hat{\beta}^2}\right]^2 \hat{\sigma}^2_{\hat{\beta}} = 1.543.$$

Example: Multivariate

In the process of computing the impulse response function of a VAR(1) model with $\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$ one has to calculate $\Psi_2 = \Phi^2$. If we stack all coefficients of Φ into a vector $\beta = \mathrm{vec}(\Phi) = (\phi_{11}, \phi_{21}, \phi_{12}, \phi_{22})'$ then we get:

$$f(\beta) = \mathrm{vec}\, \Psi_2 = \mathrm{vec}\, \Phi^2 = \begin{pmatrix} \psi^{(2)}_{11} \\ \psi^{(2)}_{21} \\ \psi^{(2)}_{12} \\ \psi^{(2)}_{22} \end{pmatrix} = \begin{pmatrix} \phi_{11}^2 + \phi_{12}\phi_{21} \\ \phi_{11}\phi_{21} + \phi_{21}\phi_{22} \\ \phi_{11}\phi_{12} + \phi_{12}\phi_{22} \\ \phi_{12}\phi_{21} + \phi_{22}^2 \end{pmatrix},$$

where $\Psi_2 = \left[\psi^{(2)}_{ij}\right]$. The Jacobian matrix then becomes:

$$\nabla f(\beta) = \begin{pmatrix} \frac{\partial \psi^{(2)}_{11}}{\partial \phi_{11}} & \frac{\partial \psi^{(2)}_{11}}{\partial \phi_{21}} & \frac{\partial \psi^{(2)}_{11}}{\partial \phi_{12}} & \frac{\partial \psi^{(2)}_{11}}{\partial \phi_{22}} \\ \frac{\partial \psi^{(2)}_{21}}{\partial \phi_{11}} & \frac{\partial \psi^{(2)}_{21}}{\partial \phi_{21}} & \frac{\partial \psi^{(2)}_{21}}{\partial \phi_{12}} & \frac{\partial \psi^{(2)}_{21}}{\partial \phi_{22}} \\ \frac{\partial \psi^{(2)}_{12}}{\partial \phi_{11}} & \frac{\partial \psi^{(2)}_{12}}{\partial \phi_{21}} & \frac{\partial \psi^{(2)}_{12}}{\partial \phi_{12}} & \frac{\partial \psi^{(2)}_{12}}{\partial \phi_{22}} \\ \frac{\partial \psi^{(2)}_{22}}{\partial \phi_{11}} & \frac{\partial \psi^{(2)}_{22}}{\partial \phi_{21}} & \frac{\partial \psi^{(2)}_{22}}{\partial \phi_{12}} & \frac{\partial \psi^{(2)}_{22}}{\partial \phi_{22}} \end{pmatrix} = \begin{pmatrix} 2\phi_{11} & \phi_{12} & \phi_{21} & 0 \\ \phi_{21} & \phi_{11} + \phi_{22} & 0 & \phi_{21} \\ \phi_{12} & 0 & \phi_{11} + \phi_{22} & \phi_{12} \\ 0 & \phi_{12} & \phi_{21} & 2\phi_{22} \end{pmatrix}.$$

In Section 15.4.4 we obtained the following estimate for a VAR(1) model for $\{X_t\} = \{(\ln(\mathrm{A}_t), \ln(\mathrm{S}_t))'\}$:

$$X_t = \hat{c} + \hat{\Phi} X_{t-1} + \hat{Z}_t = \begin{pmatrix} -0.141 \\ 0.499 \end{pmatrix} + \begin{pmatrix} 0.316 & 0.640 \\ -0.202 & 1.117 \end{pmatrix} X_{t-1} + \hat{Z}_t.$$

The estimated covariance matrix of $\operatorname{vec}\hat{\Phi}$, $\hat{\mathrm{V}}(\operatorname{vec}\hat{\Phi})$, was:

$$\hat{\mathrm{V}}(\hat{\beta}) = \hat{\mathrm{V}}(\operatorname{vec}\hat{\Phi}) = \begin{pmatrix} 0.0206 & 0.0069 & -0.0201 & -0.0067 \\ 0.0069 & 0.0068 & -0.0067 & -0.0066 \\ -0.0201 & -0.0067 & 0.0257 & 0.0086 \\ -0.0067 & -0.0066 & 0.0086 & 0.0085 \end{pmatrix}.$$

We can then approximate the variance of $f(\hat{\beta}) = \operatorname{vec}(\Phi^2)$ by

$$\hat{\mathrm{V}}(f(\operatorname{vec}\hat{\Phi})) = \hat{\mathrm{V}}(\operatorname{vec}\hat{\Phi}^2) = \nabla f(\operatorname{vec}\Phi)|_{\Phi=\hat{\Phi}}\, \hat{\mathrm{V}}(\operatorname{vec}\hat{\Phi})\, \nabla f(\operatorname{vec}\Phi)|'_{\Phi=\hat{\Phi}}.$$

This leads :

$$\hat{\mathrm{V}}(f(\operatorname{vec}\hat{\Phi})) = \begin{pmatrix} 0.0245 & 0.0121 & -0.0245 & -0.0119 \\ 0.0121 & 0.0145 & -0.0122 & -0.0144 \\ -0.0245 & -0.0122 & 0.0382 & 0.0181 \\ -0.0119 & -0.0144 & 0.0181 & 0.0213 \end{pmatrix}.$$

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K. Neusser, *Time Series Econometrics*, Springer Texts in Business and Economics,
DOI 10.1007/978-3-319-32862-1

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