

# Solutions to Selected Problems

## Problems of Chap. 1

### 1.1 Physical constants

- (a)  $c = 2.99792458 \times 10^8 \text{ m s}^{-1}$  (exact).
- (b)  $h = 6.6261 \times 10^{-34} \text{ J s}$ .
- (c)  $\hbar = 1.0545 \times 10^{-34} \text{ J s}$ .
- (d)  $e = 1.6022 \times 10^{-19} \text{ C}$ .
- (e)  $m_0 = 0.9109 \times 10^{-30} \text{ kg}$ .
- (f)  $\mu_0 = 4\pi \times 10^{-7} \text{ V s A}^{-1} \text{ m}^{-1}$ .
- (g)  $\epsilon_0 = 1/(\mu_0 c^2) = 0.8854 \times 10^{-11} \text{ A s V}^{-1} \text{ m}^{-1}$ .
- (h)  $k = 1.3807 \times 10^{-23} \text{ J K}^{-1}$ .
- (i)  $N_A = 6.022 \times 10^{26}$  molecules per Mole.
- (j)  $R = kN_A = 8.315 \times 10^3 \text{ J K}^{-1}$  per Mole.
- (k)  $L_0 = 2.687 \times 10^{25}$  molecules per  $\text{m}^3$  at  $0^\circ\text{C}$  and normal pressure.

### 1.2 Frequency, wavelength, wavenumber and energy scale

- (a)  $1 \mu\text{m}$ ;  $300 \text{ THz}$ ;  $10^4 \text{ cm}^{-1}$ ;  $1.9878 \times 10^{-20} \text{ J}$ ;  $1.2407 \text{ eV}$ .
- (b)  $300 \mu\text{m}$ ;  $1 \text{ THz}$ ;  $3300 \text{ m}^{-1} = 33.33 \text{ cm}^{-1}$ ;  $6.626 \times 10^{-23} \text{ J}$ ;  $4.136 \text{ meV}$ .
- (c)  $1 \text{ nm}$ ;  $300 \text{ PHz}$ ;  $2.0 \times 10^{-17} \text{ J}$ ;  $1.240 \text{ keV}$ .
- (d)  $1 \text{ m}^{-1}$ ;  $1 \text{ m}$ ;  $300 \text{ MHz}$ ;  $1.24 \mu\text{eV}$ .
- (e)  $1.2407 \mu\text{m}$ ;  $241.8 \text{ THz}$ ;  $1.6022 \times 10^{-19} \text{ J}$ ;  $1 \text{ eV}$ .

### 1.3

- (a)  $T = 300 \text{ K} \equiv kT = 4.142 \times 10^{-21} \text{ J} \equiv kT/e = 25.85 \text{ meV} \equiv \nu = kT/h = 6.625 \text{ THz} \equiv kT/(hc) = 208 \text{ cm}^{-1} \equiv hc/(kT) = 48 \mu\text{m}$ .
- (b)  $1 \text{ meV} \equiv 8.06 \text{ cm}^{-1} \equiv 0.2418 \text{ THz}$ .
- (c)  $1 \text{ cm}^{-1} \equiv 30 \text{ GHz}$ .
- (c)  $10 \text{ cm}^{-1} \equiv 1.2408 \text{ meV}$ .

### 1.4 Power of the sun light and laser power

- (a)  $140 \text{ mW}$ . (b)  $1.8 \text{ kW/cm}^2$ . (c)  $1.3 \text{ kW/cm}^2$ .

## Problems of Chap. 2

**2.1**  $5.6 \times 10^{22} \text{ m}^{-3}$ ;  $5.6 \times 10^{25} \text{ m}^{-3}$ ;  $5.6 \times 10^{28} \text{ m}^{-3}$ .

### 2.2 Field amplitude

(a)  $\varepsilon_0 A^2/2 = u$ ;  $A = \sqrt{2u/\varepsilon_0}$ ;  $\varepsilon_0 = 0.89 \times 10^{-11} \text{ As V}^{-1} \text{ m}^{-1}$ ;  $A = 4.7 \times 10^5 \text{ V m}^{-1}$ .

(b)  $Z = 10^6 \text{ m}^{-3}$ ;  $A = \sqrt{2h\nu Z/\varepsilon_0} = 6.3 \times 10^{-2} \text{ V m}^{-1}$ ;  $u = 2 \times 10^{-14} \text{ J m}^{-3}$ .

(c)  $Z = 2 \times 10^{13} \text{ m}^{-3}$ ;  $A = 180 \text{ V m}^{-1}$ ;  $u = 1 \mu \text{ J m}^{-3}$ .

### 2.3 Occupation number

(b)  $kT = 4.14 \times 10^{-21} \text{ J} = 25.8 \text{ meV}$ ;  $f_1^{\text{Boltz}} - f_2^{\text{Boltz}} \sim 1.8 \times 10^{-35}$ .

(c)  $f_1^{\text{Boltz}} - f_2^{\text{Boltz}} \sim 0.54 - 0.46 = 0.08$ .

### 2.4 Oscillation condition

(a) In one case, the condition is  $G_1 G_1 V u/2 = u/2$  and in an other case,  $V G_1 G_1 u/2 = u/2$ . Show that both cases lead to  $GV = 1$ .

(b) For both directions, we obtain the product  $GV$  and the same sum of the phases.

### 2.5 Brewster angle

(a)  $54.4^\circ$ . (b)  $56.3^\circ$ . (c)  $61.2^\circ$ . (d)  $60.4^\circ$ .

## Problems of Chap. 3

**3.1**  $\delta\nu/(c/2L) = 2 \times 10^5$ .

**3.2**  $V = R_1 R_2$ ;  $s_{\text{eff}} = 10$ ;  $\tau_p = 6.7 \text{ ns}$ ;  $l_p = 2 \text{ m}$ ;  $Q = 63$ .

### 3.3 Resonator with air

(a)  $\nu_1 = c/(2nL)$ .

(b)  $\delta\nu = c/(2L) - (c/n)/(2L) = c/(2L)(1 - 1/n) = 160 \text{ kHz}$ ;  $\delta\nu/\nu \sim 3 \times 10^{-9}$ .

**3.4** Energy =  $\varepsilon_0 a_1 T^{-1} \int_0^L \int_0^T E^2(z, t) dz dt = \varepsilon_0 a_1 a_2 A^2 T^{-1} \int_0^L \int_0^T \sin^2 kz \sin^2 \omega t dz dt = \varepsilon_0 a_1 a_2 L A^2/4$ ;  $u = \varepsilon_0 A^2/4$ .

**3.5**  $V = R_1 R_2$ ;  $\tau_p = 1/(1 - V)$ .

### 3.6 Photon density

(a)  $Z_{\text{FP}}/\tau_p = Z/T$ ;  $Z_{\text{FP}} = Q/(2\pi)Z$ ;  $Q = 2\pi l/(1 - R)$ .

(b) We obtain the same result, but  $Q = \pi l/(1 - R)$ .

**3.7**  $R_{\text{FP}} = 1 - T_{\text{FP}} = 4R(1 - R)^{-2} \sin^2 \delta/2[1 + 4R(1 - R)^{-2} \sin^2 \delta/2]^{-1}$ .

**3.8**  $T_{\text{FP}} = 1/[1 + 4R(1 - R)^{-2} \sin^2 \delta/2]$ , where  $R = \sqrt{R_1 R_2}$ .

**3.9 Fabry–Perot interferometer with absorbing mirrors**

- (a)  $T_{\text{FP}} = (1 + (A_m^2/T_m^2))^{-1} (1 + 4R(1 - R)^{-2} \sin^2 \delta/2)^{-1}$ ;  $T_{\text{FP,max}} = (1 + A_m^2/T_m^2)^{-1}$ .  
 (b)  $1/(1 + A_m^2/T_m^2) < 0.98$ ;  $A_m/T_m < 0.1$ .

**3.10 Fabry–Perot interferometer for obliquely incident radiation**

- (a)  $\delta = k \times 2L \cos \theta + 2\varphi$ .  
 (b)  $\varphi = 0$ ;  $\delta = k \times 2L \cos \theta = zl \times 2\pi$ ;  $2L \cos \theta = zl \times \lambda$ .

**Problems of Chap. 4****4.2 Absolute number of two-level systems**

- (a)  $N_{\text{tot}} = 10^{15}$ . (b)  $N_{\text{tot}} = 10^{10}$ . (c)  $N_{\text{tot}} = 10^4$ .

**Problems of Chap. 5**

**5.1**  $L + (n - 1)L' = 57.6 \text{ cm}$ .

**5.2 Photon density**

- (a) The laser beam has only a slightly larger diameter at 10 m distance from the laser and the laser power is of the order of 1 W.  
 (b) Assuming that the luminescence radiation is emitted isotropically, the power reaching an area of 1 cm diameter is  $P_{\text{fluor}} = P_0 \times 2\pi \sin^2(\alpha/2)$ , where  $\alpha$  is the angle corresponding to the area. It follows that  $\alpha \sim 5 \times 10^{-4}$ ;  $P_{\text{fluor}} \sim P_0 \times 2\pi \times \alpha^2/2 \sim 0.4 \mu\text{W}$ .

**5.3**  $g(\lambda)d\lambda = g(\nu)d\nu$ ;  $g(\nu) = g(\lambda)/|d\nu/d\lambda|$ ;  $\nu = c/\lambda$ ;  $d\nu/d\lambda = -c/\lambda^2$ ;  $g(\nu) = \lambda^2 g(\lambda)/c$ .

**5.4 Population of the upper laser level**

- (a)  $r = N_2/\tau_{\text{rel}}^* = 3.3 \times 10^{29} \text{ m}^{-3} \text{ s}^{-1}$ ; volume =  $\pi r^2 L' = 7.9 \times 10^{-10} \text{ m}^3$ ;  
 $P_{\text{pump}} = 1.5 \times 3.3 \times 10^{29} \times 7.9 \times 10^{-10} \times 2.4 \times 10^{-19} \text{ W} = 9.4 \text{ W}$ ; the factor 1.5 takes account of the quantum efficiency.  
 (b)  $N_{\text{tot}} = 10^{24} \times 7.9 \times 10^{-10} = 7.9 \times 10^{14}$ .  
 (c) Energy =  $10^{24} \times 7.9 \times 10^{-10} \times 1.5 \times 1.6 \times 10^{-19} \text{ J} = 190 \mu\text{J}$ ; energy density = energy/volume =  $(N_{\text{tot}}/\text{volume}) \times h\nu = 240 \text{ kJ/m}^3 = 240 \text{ J per liter}$ .  
 (d)  $P_{\text{pump}} = 94 \text{ W}$ . [The reason is the stimulated emission (Sect. 8.8).]

**Problems of Chap. 6****6.1 Photon density**

(a)  $Z = D(\nu)d\nu\bar{n} = (8\pi\nu^2/c^3)kT/h\nu \sim 6 \times 10^7 \text{ m}^{-3}$ .

(b)  $Z = 6 \times 10^{10} \text{ m}^{-3}$ .

(c)  $Z = (8\pi\nu^2/c^3)d\nu \exp(-h\nu/kT) = 4 \times 10^{-34} \text{ m}^{-3}$ .

### 6.2 Number of thermal photons in a mode of a laser resonator

(a)  $\bar{n} = \exp(-h\nu/kT) \sim 2 \times 10^{-29}$ .

(b)  $\bar{n} = 1/[\exp(h\nu/kT) - 1] \sim 0.25$ .

(c)  $\bar{n} = kT/h\nu \sim 6$ .

## Problems of Chap. 7

### 7.1 Amplification of radiation in titanium–sapphire

(a)  $\alpha = 8 \text{ m}^{-1}$ . (b)  $G_1 - 1 = 0.5$ .

(c)  $\alpha(1 \mu\text{m})/\alpha(\lambda_0) \sim 0.5$ ;  $\alpha(1 \mu\text{m}) \sim 4 \text{ m}^{-1}$ ;  $G_1 - 1 = 0.25$ .

**7.2**  $\sigma_{\text{nat}} = (\lambda/n)^2/2\pi = 3.2 \times 10^{-14} \text{ m}^2$ ;  $\tau_{\text{sp}} = 3.8 \mu\text{s}$ ;  $\Delta\nu_{\text{nat}} = 1/2\pi\tau_{\text{sp}} = 4.2 \times 10^4 \text{ Hz}$ ;  $\Delta\nu_0 = 1.1 \times 10^{14} \text{ Hz}$ ;  $\sigma_{21}/\sigma_{\text{nat}} = 1.5\Delta\nu_{\text{nat}}/\Delta\nu_0 = 5.7 \times 10^{-10}$ ;  $\sigma_{21} = 1.8 \times 10^{-23} \text{ m}^2$ .

### 7.3 Two-dimensional gain medium

(a)  $\alpha = 2,000 \text{ m}^{-1} = 20 \text{ cm}^{-1}$ . (b)  $G_1 - 1 = 1.5 \times 10^{-3}$ .

## Problems of Chap. 8

**8.1**  $\tau_p = (2nL/c)(1 - R)^{-1} = 6 \times 10^{-8} \text{ s}$ ;  $l_p = (c/n)\tau_p = 10 \text{ m}$ ;  $(N_2 - N_1)_{\text{th}} = 1/l_p\sigma_{21} = 3 \times 10^{21} \text{ m}^{-3}$ .

**8.2**  $r_{\text{th}} = (N_2 - N_1)_{\text{th}}\tau_{\text{rel}}^* = 8 \times 10^{26} \text{ m}^{-3} \text{ s}^{-1}$ ;

$Z_\infty = (10r_{\text{th}} - r_{\text{th}})\tau_p = 9r_{\text{th}}\tau_p = 4.3 \times 10^{19} \text{ m}^{-3}$ ;  $P_{\text{out}} = Z_\infty a_1 a_2 L h\nu/\tau_p = 9 \text{ W}$ ;  
 $r_{\text{out}}/r = Z_\infty/(\tau_p r) = 9r_{\text{th}}/10r_{\text{th}} = 0.9$ .

**8.3**  $(N_2 - N_1)_0 = 10 \times (N_2 - N_1)_{\text{th}} = 3 \times 10^{22} \text{ m}^{-3}$ ;  $\gamma_0 = b_{21}(N_2 - N_1)_0 = 1.3 \times 10^8 \text{ s}^{-1}$ ;  $\kappa = 1/\tau_p = 1.6 \times 10^7 \text{ s}^{-1}$ ;  $Z_0 = 1/a_1 a_2 L = 2 \times 10^7 \text{ m}^{-3}$ ;  $t_{\text{on}} = 18 \text{ ns}$ .

**8.4** If the active medium has a smaller length than the resonator, the threshold condition is  $(N_2 - N_1)_\infty = -\ln V/(2nL'\sigma_{21})$ , where  $L'$  is the length of the active medium. It follows for that case that the gain coefficient  $\alpha = (N_2 - N_1)\sigma_{21}$  has to be larger than the reciprocal of the effective photon path length in the crystal,  $l'_p = l_p L'/L$ ,  $\alpha \geq 1/l'_p = -\ln V/(2nL')$  or  $2\alpha L' \geq -\ln V$ . We find, with  $G = \exp(2\alpha L')$ , that the condition of gain,  $GV \geq 1$ , is fulfilled.

## Problems of Chap. 10

**10.1**  $\nu_{110} = \nu_{101} = \nu_{011} = c/(\sqrt{2}a) = 21.2 \text{ GHz}$ ;  $\nu_{111} = \sqrt{3}c/(2a) = 26 \text{ GHz}$ .

**10.2 Degeneracy of modes of a rectangular cavity resonator**

(a) 3. (b) 2. (c) No degeneracy. (d) 2.

**10.3 Density of modes of a cavity resonator**

- (a)  $D(\nu) = 8\pi n^3 \nu^2 / c^3 = 1.7 \times 10^5 \text{ m}^{-3} \text{ Hz}^{-1}$  for  $\nu = 4.3 \times 10^{14} \text{ Hz}$ ;  $n = 1$ .  
 (b)  $1.0 \times 10^6 \text{ m}^{-3} \text{ Hz}^{-1}$ .  
 (c)  $8.3 \times 10^6 \text{ m}^{-3} \text{ Hz}^{-1}$ .

**10.5 Mode density on different scales**

- (a)  $D(\nu)d\nu = D(h\nu)d(h\nu)$ ;  $D(h\nu) = D(\nu)d\nu/d(h\nu) = D(\nu)/h$ .  
 (b)  $D(\nu)d\nu = D(\omega)d\omega$ ;  $D(\omega) = D(\nu)d\nu/d\omega = D(\nu)/(2\pi)$ .  
 (c)  $D(\nu)d\nu = D(\lambda)d\lambda$ ;  $D(\lambda) = D(\nu) \times d\nu/d\lambda = cD(\nu)/\lambda^2$ .

**10.6**  $\nu = (c/2)\sqrt{a_1^{-2} + L^{-2}} = c/(2a_1)\sqrt{1 + a_1^2/L^2} \sim c/(2a_1)[1 + a_1/(2L^2)]$ ;  
 $d\nu/dL \sim -(ca_1/(2L^3))$ ;  $d\nu/\nu \sim (a_1^2/L^2)dL/L$ .

**10.7 Density of modes in free space**

We consider a propagating wave  $E = A \exp[i(\omega t - \mathbf{k}r)]$ . We apply periodic boundary conditions:  $E(x + L, y + L, z + L) = E(x, y, z)$  for each value of  $t$ ;  $L$  is the length of the periodicity interval assumed to be equal in all spatial directions. This leads to the conditions:  $\exp(ik_x L) = 1$ ;  $\exp(ik_y L) = 1$ ;  $\exp(ik_z L) = 1$ . It follows that:  $k_x = l \times 2\pi/L$ ;  $k_y = m \times 2\pi/L$ ;  $k_z = n \times 2\pi/L$ ;  $k^2 = (2\pi/L)^2(l^2 + m^2 + n^2)$ , with  $l, m, n = 0, \pm 1, \pm 2, \dots$ . We find, with  $\omega = ck$ , that  $\omega^2 = (2\pi c/L)^2(l^2 + m^2 + n^2)$ . The mode density in  $k$  space is  $D(k) = (L^3/\pi^2)k^2$  and in  $\omega$  space  $D^*(\omega) = \omega^2 L^3/(\pi^2 c^3)$ . With  $D^*(\omega)d\omega = D^*(\nu)d\nu$ , we obtain  $D^*(\nu) = (8\pi \nu^2/c^3)L^3$ .

**Problems of Chap. 11****11.1 Gaussian beam**

- (b) The ratio of the intensity of the radiation within the beam radius  $r_0$  to the total intensity is

$$\int_0^{r_0} 2\pi r \exp(-r^2/r_0^2)dr / \int_0^\infty 2\pi r \exp(-r^2/r_0^2)dr = 1 - 1/e = 0.63.$$

We used  $\int 2xe^{-x^2} dx = -\exp(-x^2)$ .

- (c)  $r_p = I_p/I_{\text{tot}} = 1 - \exp(-r_p^2/r_0^2)$ ;  $r_p/r_0 = \sqrt{-\ln(1-p)}$ .  
 (d)  $r_p = 1.52 r_0$ .  
 (e)  $r_p = 1.73 r_0$ .  
 (f) A Taylor expansion of  $p(r_p)$  with respect to  $r_p$  yields  $p \sim 1 - r_p^2/r_0^2$ .

**11.2**  $\theta_{0,u} = \sqrt{2}\lambda/(\pi r_0) = 1.3 \times 10^{-3} (= 4.5 \text{ arc minutes})$

**11.3** The angle of divergence is  $\Theta = 0.1(\sqrt{2}/\pi)\lambda/r_0 = 2 \times 10^{-6}$ .

**11.4 Density of photons in a Gaussian beam.** The number of photons emitted per second by the laser is  $P_{\text{out}}/(h\nu) = 6 \times 10^{14} \text{ s}^{-1}$ . A detector of diameter  $D$  monitors radiation within the angle  $\vartheta = D/d$ , where  $d$  is the distance from the laser. The portion of radiation within the angle  $\vartheta$  is  $\sin^2 \vartheta / \sin^2 \theta^2 \sim \vartheta^2 / \theta^2$ . It follows for the number of photons per second:

- (a)  $\vartheta = 10^{-7}$ ;  $\vartheta^2 / \theta^2 = 2 \times 10^{-3}$ ;  $3 \times 10^{11} \text{ s}^{-1}$ .  
 (b)  $\vartheta = 7 \times 10^{-11}$ ;  $\vartheta^2 / \theta^2 \sim 10^{-9}$ ;  $\sim 10^6 \text{ s}^{-1}$ .

## Problems of Chap. 13

### 13.1 Ultrashort pulses

(a)  $t_p \sim 1/\Delta\nu_0 \sim 0.3 \text{ ps}$ . (b)  $t_p \sim 10 \text{ ps}$ . (c)  $t_p = 30 \text{ ps}$ .

**13.2** Excited  $\text{Ti}^{3+}$  ions are collected during the round trip time  $T = 10^{-8} \text{ s}$ . The density of excited  $\text{Ti}^{3+}$  is  $rT = 3 \times 10^{20} \text{ m}^{-3}$ . Accordingly, the energy in a pulse is  $rTa_1a_2L' \times h\nu = 19 \text{ nJ}$  and the pulse power =  $1.9 \text{ MW}$ . The average power is  $1.9 \text{ W}$ .

**13.3**  $2 \times 10^7 \text{ W}$ .

## Problems of Chap. 14

### 14.1 Helium–neon laser: line broadening and gain cross section

- (a)  $\Delta\nu_D = 1.5 \times 10^9 \text{ Hz}$ . (b)  $\Delta\nu_c \sim 10^6 \text{ Hz}$ . (c)  $\Delta\nu_{\text{nat}} = 1.6 \times 10^6 \text{ Hz}$ .  
 (d)  $\Delta\nu_0 = 1.6 \times 10^7 \text{ Hz}$ . (e)  $\sigma_{21}(\nu_0) = (1/c)h\nu B_{21}g_G(\nu_0) = 1.0 \times 10^{-16} \text{ m}^2$ .

### 14.2 Helium–neon laser: threshold condition, output power and oscillation onset time

- (a)  $\tau_p = T/(1 - R_1R_2) = 1.5 \times 10^{-7} \text{ s}$ ;  $l_p = c\tau_p = 45 \text{ m}$ .  $(N_2 - N_1)_{\text{th}} = 1/(\sigma_{21}l_p) = 2 \times 10^{14} \text{ m}^{-3}$ .  
 (b)  $(N_2 - N_1)_{\text{th}} \times a_1a_2L = 2 \times 10^8$ ; number of excited neon atoms in the laser.  
 (c)  $r_{\text{th}} = (N_2 - N_1)_{\text{th}} \times a_1a_2L/\tau_{\text{rel}}^* = 2 \times 10^{15} \text{ s}^{-1}$ ;  $r_{\text{out}} \times a_1a_2L = 9r_{\text{th}}a_1a_2L = 2 \times 10^{16} \text{ s}^{-1}$ ;  $P_{\text{out}} = r_{\text{out}} \times a_1a_2Lh\nu = 13 \text{ mW}$ .  
 (d)  $Z_0 = (a_1a_2L)^{-1} = 5 \times 10^5 \text{ m}^{-3}$ ;  $Z_{\infty} = r_{\text{out}}\tau_p = 2.7 \times 10^9 \text{ m}^{-3}$ ;  $\alpha_{\text{th}} = (N_2 - N_1)_{\text{th}} \times \sigma_{21} = 0.02 \text{ m}^{-1}$ ;  $\gamma_{\text{th}} = c\alpha_{\text{th}} = 6 \times 10^6 \text{ s}^{-1}$ ;  $\kappa = 6 \times 10^6 \text{ s}^{-1}$  (because  $\kappa = \gamma_{\text{th}}$  at threshold);  $\gamma_0 = 10 \gamma_{\text{th}} = 6 \times 10^7 \text{ s}^{-1}$ ;  $t_{\text{on}} = \ln(Z_{\infty}/Z_0)/(\gamma_0 - \kappa) = 160 \text{ ns}$ .

### 14.3 Doppler effect

- (a)  $\nu = \nu_0(1 \pm v/c)$ ;  $\delta\nu = (2v/c)\nu_0 = 3 \times 10^9 \text{ Hz}$ .  
 (b) The homogeneous width of the line due to  $2 \rightarrow 1$  spontaneous transitions is  $\Delta\nu = 1/(2\pi\tau_{\text{rel}}) = 1.6 \times 10^8 \text{ Hz}$ , where  $\tau_{\text{rel}}$  is the lifetime of the lower laser level. This corresponds to a velocity range  $-v, v$  or to  $|v| = c\delta\nu/\nu_0 = 100 \text{ m s}^{-1}$ .

- (c) The gain curve has a minimum of a halfwidth of 160 kHz. In the line center, the gain is smaller than outside because outside (80 kHz away from the center) ions of the velocity  $+v$  contribute to gain in half a round trip and ions of the velocity  $-v$  contribute during the other half round trip. The Lamb dip can be used for frequency stabilization of a helium–neon laser.

#### 14.4 CO<sub>2</sub> laser

- (a)  $\Delta\nu_D = 2\nu_0\sqrt{(2kT/mc^2)\ln 2} = 5.6 \times 10^7 \text{ Hz}$ ;  $m = m_C + 2m_O = 44 m_p = 7.3 \times 10^{-26} \text{ kg}$ ;  $m_p = \text{proton mass}$ .  $\sigma_{21}(\nu_0) = 0.94c^2 A_{21}/(8\pi\nu^2 \Delta\nu_D) = 1 \times 10^{-21} \text{ m}^2$ .  $G_{\text{th}}V = 1$ ;  $V = 0.7$ ;  $G_{\text{th}} = 1.43$ ;  $G_{\text{th}} = \exp[\alpha_{\text{th}} \times 2L]$ ;  $\alpha_{\text{th}} = \ln(G_{\text{th}})/2L = 0.18 \text{ m}^{-1}$ .  $(N_2 - N_1)_{\text{th}} = \alpha_{\text{th}}/\sigma_{21} = 1.8 \times 10^{20} \text{ m}^{-3}$ .  $r_{\text{th}} = (N_2 - N_1)_{\text{th}}/\tau_{\text{rel}}^* = 4.5 \times 10^{19} \text{ m}^{-3} \text{ s}^{-1}$ ;  $P = ra_1 a_2 L h\nu$ ;  $r = P/a_1 a_2 L h\nu = 3 \times 10^{25} \text{ m}^{-3} \text{ s}^{-1}$ ;  $r \sim 10^6 r_{\text{th}}$ ; the pump rate is about  $10^6$  times larger than the threshold pump rate.
- (b) Because of the extremely long lifetime of the upper laser level with respect to spontaneous emission, the oscillation builds up as soon as the population difference exceeds  $(N_2 - N_1)_{\text{th}}$ . Stronger pumping then leads to generation of laser radiation. By collisions of the CO<sub>2</sub> molecules with each other, a quasithermal distribution of the populations of the different rotational levels of the excited state is maintained and the pump energy is converted to laser radiation (and energy of relaxation).
- (c) We treat, for simplicity, the CO<sub>2</sub> gas as an ideal gas. At 273 K and normal pressure, an ideal gas (mole volume 22.4 l) contains  $6 \times 10^{23}$  molecules. This corresponds to about  $3 \times 10^{25} \text{ m}^{-3}$ . We use this number for CO<sub>2</sub> at room temperature and normal pressure (1 bar). At a pressure of 10 mbar, the density of available CO<sub>2</sub> molecules is  $3 \times 10^{22} \text{ m}^{-3}$ . At room temperature, excited CO<sub>2</sub> molecules are in different rotational states. About 1% of the molecules are in a particular rotational state. Thus, about  $3 \times 10^{21}$  molecules per  $\text{m}^3$  are available for laser transitions. Assuming that half of the molecules are in an excited state we find that the density of molecules in a vibrational-rotational state is  $1.5 \times 10^{21} \text{ m}^{-3}$ . This leads to  $\alpha \sim 8 \times \alpha_{\text{th}} \sim 1.4 \text{ m}^{-1}$  and to a single path gain of  $G_1 = \exp(\alpha L) = 4$ .
- (d) For a collision-broadened line, the gain cross section is  $\sigma_{21} = c^2 A_{21}/(8\pi\nu^2) g(\nu)$ . With increasing pressure  $g(\nu)$  broadens and the cross section in the line center,  $\sigma_{21}(\nu_0) = c^2 A_{21}/(8\pi\nu^2) \times 2/(\pi \Delta\nu_c)$ , is inversely proportional to the gas pressure  $p$ . It follows that  $\alpha(\nu_0)$  is independent of pressure above a pressure of about 10 mbar. At this pressure,  $2\Delta\nu_c \sim \Delta\nu_D$ , the gain coefficient we calculated is the maximum gain coefficient for the TEA and the high-pressure CO<sub>2</sub> laser.

In a TEA laser, the pulse duration of the radiation is about 200 ns. It is much larger than the duration (20 ns) of the electrical excitation pulse. During about 20 round trip transits of the radiation through the active medium, a fast redistribution occurs for the population of the levels involved in a laser oscillation. If we assume that about 1% of the excited molecules contribute to the laser oscillation, we find the pulse energy  $E_{\text{pulse}} = 0.3 \text{ J}$  and the pulse power  $E_{\text{pulse}}/t_{\text{pulse}} \sim 1 \text{ MW}$ .

(e)  $Z_0 = (a_1 a_2 L)^{-1} = 10^4 \text{ m}^{-3}$ ;  $\tau_p = (2L/c)/(1 - R) = 2.2 \times 10^{-8} \text{ s}$ ;  $Z_\infty = P_{\text{out}} \tau_p / (a_1 a_2 L h \nu) = 1.1 \times 10^{21} \text{ m}^{-3}$ ;  $t_{\text{on}} = T \ln (Z_\infty / Z_0) / (GV) = 24 \text{ ns}$ .

## Problems of Chap. 15

**15.2**  $\sigma_{21}(\text{YAG})/\sigma_{21}(\text{TiS}) = \Delta\nu_0(\text{TiS})/\Delta\nu_0(\text{YAG}) \times \tau_{\text{sp}}(\text{TiS})/\tau_{\text{sp}}(\text{YAG}) \times \lambda^2(\text{YAG})/\lambda^2(\text{TiS}) \sim 10$ .

**15.3** An  $\text{N}_2$  molecule consists of two atoms, a molecule has a single vibrational frequency (and all molecules in an  $\text{N}_2$  gas have the same frequency) while the  $\text{Ti}^{3+}$  ions in  $\text{Al}_2\text{O}_3$  belong to a system with a large number of atoms (ions), namely of  $N \sim 10^{25} \text{ m}^{-3}$ , with  $3N$  vibrational frequencies.

### 15.4 Laser tandem pumping

- (a)  $\eta = \eta_1 \times \eta_2 \times \eta_3 \times \eta_4 \sim 25\%$ ;  $\eta_1 \sim 0.8$  (efficiency of a semiconductor laser);  $\eta_2 \sim 0.8$  ( $\text{Nd}^{3+}:\text{YVO}_4$  laser);  $\eta_3 = 0.5$  (frequency doubling);  $\eta_4 = 0.53 \mu\text{m}/0.68 \mu\text{m} = 0.78$ .
- (b) The  $\text{Nd}^{3+}:\text{YVO}_4$  laser produces a laser beam with a small angle of aperture. Therefore, a column of a small diameter can be excited in titanium–sapphire allowing generation of a narrow laser beam. Direct pumping with a semiconductor laser beam, which has a large divergence, leads to excitation of the whole titanium–sapphire crystal. This results in strong heating.

## Problems of Chap. 16

### 16.1 Dye laser

- (a)  $GV; V = 0.7; G_{\text{th}} = 1.43; G_{\text{th}} = \exp(\alpha_{\text{th}} L')$ ;  $\alpha_{\text{th}} = (1/L') \ln G_{\text{th}} = 350 \text{ m}^{-1}$ ;  $(N_2 - N_1)_{\text{th}} = \alpha_{\text{th}}/\alpha_{21} = 3.5 \times 10^{21} \text{ m}^{-3}$ .
- (b)  $r_{\text{th}} = (N_2 - N_1)_{\text{th}}/\tau_{\text{rel}}^* = 7 \times 10^{29} \text{ m}^{-3} \text{ s}^{-1}$ ;  $r_{\text{th}} a_1 a_2 L' = 3 \times 10^{19} \text{ s}^{-1}$ ;  $P_{\text{out}} = 9 r_{\text{th}} a_1 a_2 L' h \nu = 0.8 \text{ W}$ .

**16.2**  $P_{\text{pulse}} = 190 \text{ MW}$ ;  $P = 1.9 \text{ mW}$ .

## Problems of Chap. 19

### 19.1 Acceleration energies

(a)  $E = \sqrt{\lambda_w/\lambda} m_0 c^2 = 2.9 \text{ MeV}$ . (b)  $E = 500 \text{ MeV}$ .

**19.2**  $E = \sqrt{\lambda_w(1 + k^2)8m_0 c^2}/2/\sqrt{\lambda}$ ;  $dE = d\lambda = \sqrt{\dots}(-1)/(2\lambda^{3/2})$ ;  $dv/v = -d\lambda/\lambda = 2dE/E$ .

## Problems of Chap. 20

**20.1**  $\lambda_{\text{deBroglie}} = h/(mv)$ . The ratio is  $m_0 = m_e = 1/0.07 = 14$ .

### 20.2 Number of states

- (a)  $\epsilon = 26 \text{ meV} = 4.2 \times 10^{-21} \text{ J}$ ;  $d\epsilon = 1.6 \times 10^{-22} \text{ J}$ ;  $D(\epsilon)d\epsilon = 1.5 \times 10^{23} \text{ m}^{-3}$ .  
 (b)  $D^{2D}(\epsilon)d\epsilon = 1.0 \times 10^{16} \text{ m}^{-2}$ .  
 (c)  $D^{1D}(\epsilon)d\epsilon = 7.6 \times 10^7 \text{ m}^{-1}$ .

**20.3**  $\delta\nu = c/(2nL) = 41.6 \text{ GHz}$ .

## Problems of Chap. 21

### 21.1 Wave vector of nonequilibrium electrons in GaAs

$k = (1/\hbar)\sqrt{2m_e\epsilon} = 4.3 \times 10^8 \text{ m}^{-1}$ ;  $1.4 \times 10^8 \text{ m}^{-1}$ ;  $4.3 \times 10^7 \text{ m}^{-1}$ ;  $\lambda_g = hc/E_g = 870 \text{ nm}$ ; gap wavelength (vacuum wavelength);  $q_p = 2n\pi/\lambda_g = 2.6 \times 10^7 \text{ m}^{-1}$ . It follows that  $q_p$  is small compared to  $k$  for the first two values of  $k$ .

(b)  $q_p = k$  for  $\epsilon = \hbar^2 q_p^2 / (2m_e) = 0.4 \text{ meV}$ .

**21.2**  $q_p \ll k_1 = (1/\hbar)\sqrt{2m_e\epsilon_c}$ ;  $\epsilon_c \gg \hbar^2 q_p^2 / (2m_e) = 0.4 \text{ meV}$ ;  $q_p \ll k_2 = (1/\hbar)\sqrt{2m_h\epsilon_v}$ ;  $\epsilon_v \gg \hbar^2 q_p^2 / (2m_h) = 0.4/6 \text{ meV}$ .

### 21.3 Electron and holes in an undoped GaAs quantum film in thermal equilibrium

- (a)  $E_{Fe} = E_{Fh} = E_F$ .  
 (b) Since  $E_{Fe} = E_c + \epsilon_{Fe} = E_F$  and  $E_{Fv} = E_v - \epsilon_{Fh} = E_F$ , it follows that  $E_c + \epsilon_{Fe} = E_v - \epsilon_{Fh}$  and  $-\epsilon_{Fe} - \epsilon_{Fh} = E_g$ . The gap energy is positive because  $\epsilon_{Fe}$  and  $\epsilon_{Fh}$  have negative signs for small electron and hole densities.  
 (c)  $N_{\text{thermal}}^{2D} = \sqrt{D_c^{2D} D_h^{2D} kT} \exp[-E_g^{2D}/(2kT)] = 2 \times 10^4 \text{ m}^{-2}$ ; with  $kT = 26 \text{ meV}$ ;  $E_g^{2D} = 1.4 \text{ eV}$ ;  $N_{\text{thermal}}^{2D}$  is by many orders of magnitude smaller than  $N_{\text{tr}}^{2D} = 1.4 \times 10^{16} \text{ m}^{-2}$ .

## Problems of Chap. 22

**22.1**  $N^{2D} \sim 3.3 \times 10^{16} \text{ m}^{-2}$ ;  $\epsilon_{Fv} \sim 6 \text{ meV}$ ;  $\epsilon_{Fc} \sim 84 \text{ meV}$ ;  $E_{Fc} - E_{Fv} \sim 78 \text{ meV}$ .

### 22.2 Quantum well laser

Average photon path length  $l_p = 0.9L = 0.45 \text{ mm}$ ;  $N_{\text{th}}^{2D} - N_{\text{tr}}^{2D} = (1/3) \times 1.3(\sigma_{\text{eff}} l_p / a_1)^{-1} = 1.3 \times 10^{15} \text{ m}^{-2}$ ;  $j = 3N_{\text{th}}^{2D} e / \tau_{\text{sp}} = 3.1 \times 10^6 \text{ Am}^{-2}$ ;  $I = 0.3 \text{ A}$ .

### 22.3 Photons in a quantum well laser

- (a)  $h\nu = 1.42 \text{ eV} = 2.3 \times 10^{-19} \text{ J}$ ;  $\tau_p = 5 \times 10^{-12} \text{ s}$ ;  $Za_1 a_2 L h\nu / \tau_p = 2P_{\text{out}}$ ;  
 $Z = 2.0 \times 10^{18} \text{ m}^{-3}$ .  
 (b)  $Z_{\text{tot}} = Za_1 a_2 L = 10^5$ .

(c)  $N_{\text{tot}} \sim N_{\text{tr}}^{2\text{D}} a_2 L = 1.2 \times 10^9$ ;  $Z_{\text{tot}}$  is much smaller than  $N_{\text{tot}}$ .

## Problems of Chap. 23

**23.1**  $q_{\text{p}} = 2\pi n/\lambda_{\text{g}} = 2.6 \times 10^7 \text{ m}^{-1}$ ;  $\pi/(a/2) = 1.1 \times 10^{10} \text{ m}^{-1} \gg q_{\text{p}}$ .

### 23.2 Indirect gap semiconductor

(a)  $h\nu = E_{\text{g}}^{\text{ind}} + \hbar\omega_{\text{phonon}}$ ;  $0 = 2\pi/a + q_{\text{phonon}}$ .

(b)  $E_{\text{g}}^{\text{ind}} = h\nu + \hbar\omega_{\text{phonon}}$ ;  $2\pi/a = q_{\text{phonon}}$ .

## Problems of Chap. 24

### 24.1 GaN quantum well

(a)  $D^{2\text{D}}(\text{GaN}) = 3 \times D^{2\text{D}}(\text{GaAs})$ .

(b) To obtain the same occupation number difference, the nonequilibrium electron density has to be larger by a factor of three. If the Einstein coefficient  $B_{21}$  has the same value, the gain is by a factor  $\nu_2 = \nu_1$  larger for GaN ( $\nu_2 =$  frequency of a laser with a GaN-based quantum well and  $\nu_1 =$  frequency of a laser with a GaAs-based quantum well).

## Problems of Chap. 26

**26.1** We consider a two-dimensional plane wave,

$$\psi = \psi_0 e^{i[\mathbf{k}\mathbf{r} - (E/\hbar)t]},$$

where  $\mathbf{k}$  and  $\mathbf{r}$  are two-dimensional vectors within the plane. We apply periodic boundary conditions for the  $x$  and  $y$  direction,  $k_x L = m \times 2\pi$  and  $k_y L = n \times 2\pi$ , where  $m$  and  $n$  are integers and  $L$  is the periodicity length (for the directions along  $x$  and  $y$ ). In  $k$  space, the area of a ring of radius  $k$  and width  $dk$  is  $2\pi k dk$ . The area containing one  $k$  point is  $(2\pi/L)^2$ . The density of  $k$  states is  $\bar{D}^{2\text{D}}(k) = kd k / (2\pi)L^2$ . The density of states in the energy space follows from the relation  $\bar{D}^{2\text{D}}(\epsilon) d\epsilon = 2\bar{D}^{2\text{D}}(k) dk$ , where the factor 2 takes into account that there are two spin directions for an electron. Making use of the dispersion relation  $\epsilon = \hbar^2 k^2 / (2m)$  and of  $d\epsilon/dk = \hbar^2 k / m$  we find  $\bar{D}^{2\text{D}} = 2\bar{D}^{2\text{D}}(k) dk/d\epsilon = m/(\pi\hbar^2)L^2$  and  $D^{2\text{D}}(\epsilon) = m/(\pi\hbar^2)$  (=density of states per unit of energy and unit of area).

## 26.2 Subpicosecond quantum well laser

- (a) Yes. It is in principle possible to have a gain profile of a halfwidth of about 50 meV. The necessary frequency width is  $\Delta\nu_0 = \Delta E_0/h = 12 \text{ THz}$ ;  $t_{\text{pulse}} \sim 1/\Delta\nu_0 \sim 10^{-13} \text{ s} = 100 \text{ fs}$ .
- (b) The pulse separation is  $T = 2nL/c = 2 \times 10^{-11} \text{ s}$ . Most likely, the number of photons available in a pulse is not sufficient for the saturation of a semiconductor reflector.
- (c) If an external reflector is used, an active Q-switching technique should be applicable and the generation of subpicosecond pulses should be possible.

## Problems of Chap. 27

**27.1** The periodic boundary condition for a one-dimensional system yields the  $k$  values  $k = s \times 2\pi/L$ . The density of states in  $k$  space is  $\bar{D}^{1D}(k) = 2 \times L/(2\pi)$  because there are two states ( $\pm k$ ) in an interval  $2\pi/L$ .

$$\epsilon = \hbar^2 k^2/2m; d\epsilon/dk = (\hbar/m)k = \hbar\sqrt{2\epsilon/m}; \bar{D}^{1D}(\epsilon) d\epsilon = 2\bar{D}^{1D}(k) dk;$$

$$\bar{D}^{1D}(\epsilon) = (2L/\pi) dk/d\epsilon = L/(\pi\hbar)\sqrt{2m/\epsilon} \text{ and } D^{1D} = (\pi/\hbar)\sqrt{2m/\epsilon}.$$

**27.2**  $f_2 = 0$ ;  $f_1 = 1$ ;  $G - 1 = (n/a_2c)h^2 v B_{21} n_{0,\text{nat}}^{1D}/\Delta\epsilon_{\text{nat}} = 0.26$ .

**27.3**  $I_{\text{th}} = 0.8 \times Ne/\tau_{\text{sp}} = 0.75 \text{ nA}$ .

### 27.4 Bipolar laser as two-level laser

- (a)  $D_r(E)$ ;  $E = E_g + \epsilon =$  pair level energy;  $\epsilon = \epsilon_2 + \epsilon_1$ ;  $\epsilon_2$  and  $\epsilon_1$  are the energies of the electron and the hole that constitute a radiative pair.  $D_r(\epsilon)$  is the 3D, 2D, 1D or 0D density of states, depending on the dimensionality of the semiconductor.
- (b) The gain characteristic  $H_{21}$  is proportional to  $f_p - \bar{f}_p$ , where  $f_p$  is the probability that the pair level is occupied and  $\bar{f}_p$  is the probability that the pair level is empty;  $f_p - \bar{f}_p = 2f_p$ .
- (c)  $f_p - \bar{f}_p > 1/2$ , since the absorption coefficient is proportional to  $\bar{f}$  and the stimulated emission to  $f$ .

The condition must correspond to the condition  $f_2 - f_1 = 0$  or  $f_2 - f_1 = f_e - (1 - f_h) = 0$ . It follows that  $f_p - \bar{f}_p = f_2 - f_1 + 1/2 = f_e + f_h - 1/2$ .  $f_2$  and  $f_1$  are the occupation numbers for the electrons in the conduction band and the valence band and  $f_e$  and  $f_h$  are, in the electron-hole picture, the occupation numbers for the electrons and holes, respectively (see Sect. 21.10).

### 27.5 Laser operated with a gain medium with a naturally broadened line

- (a) To the knowledge of the author: no.

An electron-hole pair can be considered as an occupied single electron pair level (=occupied upper laser level). The lower laser level is the vacuum level. The lifetime of the vacuum level is infinitely large (or large compared to the spontaneous lifetime of the pair). Thus, we have no lifetime broadening of the

lower laser level, supposed that the population of the levels of the electron and the hole, which constitute a radiative electron-hole pair, occurs sufficiently fast.

(b)  $\sigma_{21} = (\lambda/n)^2/2\pi = 9 \times 10^{-15} \text{ m}^2$  for  $n = 3.6$ .

## Problems of Chap. 31

### 31.3

(a)  $A_\infty = 0.18 \text{ V}$ .

(b)  $\gamma_0 = a/C = 10 \times 10^9 \text{ s}^{-1}$ ;  $\kappa = G/C = 0.77 \times 10^9 \text{ s}^{-1}$ ;  $\gamma_0 - \kappa = 2.3 \times 10^8 \text{ s}^{-1}$   
 $\ll \omega_0 = 6 \times 10^{10} \text{ s}^{-1}$ .

### 31.4 Van der Pol equation

(a) The equation follows from (31.47) by introducing  $\epsilon = (\gamma_0 - k)/\omega_0$  and  $y = U\sqrt{(3\omega_0 b/C)(\gamma_0 - k)^{-1}}$ .

(b) The ansatz  $y = A \cos \tau$  leads to  $A = 2$ .

# References

1. A.E. Siegman, *Lasers* (University Science, 1986)
2. W.T. Silfvast, *Laser Fundamentals*, 2nd edn. (Cambridge University Press, 2003)
3. O. Svelto, *Principles of Lasers*, 5th edn. (Plenum Press, 2010)
4. S. Hooker, C. Webb, *Laser Physics* (Oxford University Press, 2010)
5. P.W. Milonni, J.H. Eberly, *Lasers*, 1st edn. (Wiley, 1998)
6. A. Yariv, *Optical Electronics*, 3rd edn. (Holt, Rinehart & Winston, 1985)
7. W. Koechner, *Solid-State Laser Engineering*, 6th edn. (Springer, 2005)
8. J.T. Verdeyen, *Laser Electronics*, 3rd edn. (Prentice Hall, 1995)
9. C.C. Davis, *Lasers and Electro-Optics* (Cambridge University Press, 1996)
10. W. Demtröder, *Laser spectroscopy*, vol. 1, *Basic Principles*, 5th edn. (Springer, 2014); vol. 2, *Experimental Techniques* (Springer, 2015)
11. F. Kneubühl, M. Sigrist, *Laser*, 6th edn. (Teubner Studienbücher der Physik, 2005)
12. P. Meystre, M. Sargent III, *Elements of Quantum Optics*, 4th edn. (Springer, 2007)
13. M. Fox, *Quantum Optics* (Oxford University Press, 2006)
14. M.R. Spiegel, S. Lipschutz, J. Liu, *Mathematical Handbook of Formulas and Tables*, 3rd edn. (McGraw Hill, 2008)
15. I. Gradstein, I. Ryshik, *Tables of Series, Products and Integrals* (Harri Deutsch, 1981)
16. M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (U.S. Govt. Printing Office, Washington, DC, 1984)
17. R.N. Bracewell, *The Fourier Transform and its Applications*, 2nd edn. (McGraw Hill, 1986)
18. J. Mathews, R.L. Walker, *Mathematical Methods of Physics*, 2nd edn. (Benjamin, 1970)
19. I.N. Bronstein, K.A. Semendjajew, *Taschenbuch der Mathematik*, 6th edn. (Harry Deutsch, 1961)
20. K. Jänich, *Mathematik 1* (Springer, 2001)
21. B.A. Lengyel, Evolution of masers and lasers 1966. *A. J. Phys.* **34**, 903 (1966)
22. C.H. Townes, *How the Laser Happened* (Oxford University Press, 1999)
23. N. Bloembergen, Physical review records the birth of the laser area. *Phys. Today* 28 (1993)
24. J. Hecht, *The Laser Guidebook* (McGraw Hill, 1986)
25. J. Hecht, *Laser Pioneers* (Academic, 1992)
26. M. Meschede, *Optics, Light and Lasers* (Wiley, 2004)
27. E. Hecht, *Optics*, 4th edn. (Addison Wesley, 2002)
28. M. Born, W. Wolf, *Principles of Optics*, 6th edn. (Pergamon Press, 1987)
29. M.V. Klein, *Optics* (Wiley, 1970)
30. H. Paul, *Photonen* (Teubner, 1995)

31. M. Fox, *Optical Properties of Solids* (Oxford University Press, 2007)
32. J.P. Gordon, H.J. Zeiger, C.H. Townes, The maser-new type of microwave amplification, frequency standard, and spectrometer. *Phys. Rev.* **99**, 1264 (1954)
33. P.F. Moulton, Spectroscopic and laser characteristics of  $Ti:Al_2O_3$ . *J. Opt. Soc. Am. B* **3**, 125 (1986)
34. H.S. Carslaw, J.C. Jaeger, *Conduction of Heat in Solids*, 2nd edn. (Clarendon, 1959)
35. C.J. Foot, *Atomic Physics* (Oxford University Press, 2008)
36. R. Loudon, *The Quantum Theory of Light*, 2nd edn. (Clarendon Press, 1983)
37. W. Brunner, W. Radloff, K. Junge, *Quantenelektronik* (VEB Deutscher Verlag der Wissenschaften, 1975)
38. A. Einstein, Zur Quanttheorie der Strahlung, *Mitt. Phys. Ges. Zürich* **16**, 47 (1916) and *Physikalische Zeitschrift* **18**, 121 (1917); English translations in B.L. van Werden (Ed.), *Sources of Quantum Mechanics* (North-Holland 1967) and in D. ter Haar (Ed.), *The Old Quantum Theory* (Pergamon, 1967)
39. A.L. Schawlow, C.H. Townes, Infrared and optical masers. *Phys. Rev.* **112**, 1940 (1958)
40. K. Shimoda, *Introduction to Laser Physics*, 2nd edn. (Springer, 1986)
41. H. Haken, *Laser Theory, Handbuch der Physik*, vol. XXV/2c (Springer, 1970)
42. M. Sargent, M.O. Scully, W.E. Lamb, Jr., *Laser Physics* (Addison-Wesley, 1974)
43. S. Strogatz, *Nonlinear Systems and Chaos* (Perseus Publishing, 1994)
44. E. Ott, *Chaos in Dynamical Systems* (Cambridge University Press, 2002)
45. J. Ohtsubo, *Semiconductor Lasers; Stability, Instability and Chaos*, 2nd edn. (Springer, 2008)
46. H.A. Kramers, La diffusion de la lumi'ere, *Atti Cong. Inter. Fisica* (Transactions of Volta Centenary Congress), Como, vol. 2, p. 545 (1927)
47. R. de L. Kronig, On the theory of the dispersion of X-rays. *J. Opt. Soc. Am.* **12**, 547 (1926)
48. C. Füchtbauer, Die Absorption in Spektrallinien im Lichte der Quantentheorie. *Physik. Zeitschr.* **21**, 322 (1920)
49. M. Czerny, Messungen im Rotationsspektrum des HCl im langwelligen Ultrarot. *Zeitschr. Physik.* **34**, 227 (1925)
50. R. Tolman, Duration of molecules in upper quantum states. *Phys. Rev.* **23**, 693 (1924)
51. H.A. Kramers, The quantum theory of dispersion. *Nature* **113**, 673 (1924)
52. R. Ladenburg, Untersuchungen über die anomale dispersion angeregter Gase. *Z. Physik.* **48**, 15 (1928)
53. H. Kopfermann, R. Ladenburg, Anomale dispersion in angeregtem Neon. *Z. Physik.* **48**, 26 (1928)
54. H. Kopfermann, R. Ladenburg, Experimental proof of negative dispersion. *Nature* **122**, 438 (1928)
55. R. Ladenburg, Negative dispersion in angeregtem Neon. *Z. Physik.* **65**, 167 (1930)
56. R. Ladenburg, Dispersion in electrically excited gases. *Rev. Mod. Phys.* **5**, 243 (1933)
57. L. Hoddeson, E. Braun, J. Teichmann, S. Weart, *Out of the Crystal Maze* (Oxford University Press, 1992)
58. L. Allen, J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover Publications, 1987)
59. J.D. Jackson, *Classical Electrodynamics* (Wiley, 1962)
60. S. Ramo, J.R. Whinnery, T. Van Duzer, *Fields and Waves in Communication Electronics*, 3rd edn. (Wiley, 1993)
61. D.M. Pozar, *Microwave Engineering* (Addison Wesley Publishing Company, 1990)
62. A.D. Olver, *Microwave and Optical Transmission* (Wiley, 1992)
63. P.F. Goldsmith, *Quasioptical Systems* (IEEE PRESS, 1998)
64. W.J. Smith, *Modern Optical Engineering*, 2nd edn. (McGraw-Hill, 1990)
65. A. Vanderlugt, *Optical Signal Processing* (Wiley, 1992)
66. W.S. Chang, *Principles of Lasers and Optics* (Cambridge University Press, 2005)
67. A.G. Fox, T. Li, Resonant modes in a laser interferometer. *Bell Syst. Tech. J.* **40**, 453 (1961)
68. G.D. Boyd, J.P. Gordon, Confocal multimode resonator for millimeter through optical wavelengths. *Bell Syst. Tech. J.* **40**, 489 (1961)
69. H. Kogelnik, T. Li, Laser beams and resonators. *Proc. IEEE* **5**, 1550 (1966)

70. L.G. Gouy, Sur une propri ete nouvelle des ondes lumineuses. C. R. Acad. Sci. Paris **110**, 1251 (1890)
71. L.G. Gouy, Sur la propagation anormale des ondes. C. R. Acad. Sci. Paris **111**, 33 (1890)
72. L.G. Gouy, Sur la propagation anormale des ondes, Annales de Chimie et de Physique. Ser. **6**, 145 (1891)
73. F. Reiche,  ber die anomale Fortpflanzung von Kugelwellen beim Durchgang durch Brennpunkte. Ann. der Physik. **29**, 65 (1909)
74. C.R. Carpenter, Am. J. Phys. **27**, 98 (1958)
75. F. Lindner et al., Gouy phase shift for a few-cycle laser pulses. Phys. Rev. Lett. **92**, 113001 (2004)
76. A.B. Ruffin et al., Direct observation of the Gouy phase shift with single-cycle terahertz pulses. Phys. Rev. Lett. **83**, 3410 (1999)
77. H.H. Telle, A.G. Ure a, R. J. Donovan, *Laser Chemistry* (Wiley, 2007)
78. D. B uerle, *Laser Processing and Chemistry*, 3rd edn. (Springer, 2000)
79. M.W. Berns, K.O. Greulich (Eds.), *Laser Manipulation of Cells and Tissues* (Elsevier, 2007)
80. A. Katzir, *Lasers and Optical Fibers in Medicine* (Academic Press, 1993)
81. A.N. Chester, S. Martucelli, A.M. Scheggi, *Laser Systems for Photobiology and Photomedicine* (Plenum Press, 1991)
82. H. Niemz, *Laser-Tissue Interactions* (Springer, 1996)
83. J. Eichler, G. Ackermann, *Holographie* (Springer, 1993)
84. M. Fran on, *Holographie* (Springer, 1972)
85. H.A. Bachor, *A Guide to Experiments in Quantum Optics* (Wiley, 1988)
86. J.C. Diels, W. Rudolph, *Ultrashort Laser Pulse Phenomena* (Elsevier, 2006)
87. M.E. Fermann, A. Galvanaushas, G. Sucha (Eds.), *Ultrafast Lasers* (Marcel Dekker, 2003)
88. A.M. Weiner, *Ultrafast Optics* (Wiley, 2009)
89. J. Ye, S.T. Cundiff, *Femtosecond Optical Frequency Comb Technology* (Springer, 2005)
90. C. Rulliere (Ed.), *Femtosecond Laser Pulses* (Springer, 1988)
91. R. Ell et al., Generation of 5-fs pulses and octave-spanning spectra directly from a Ti:sapphire laser. Opt. Lett. **26**, 373 (2001)
92. D. Mittleman (Ed.), *Sensing with Terahertz Radiation* (Springer, 2003)
93. R.E. Miles, P. Harrison, D. Lippens (Eds.), *Terahertz Sources and Systems* (Kluwer Academic Publishers, 2001)
94. K. Sakai (Ed.), *Terahertz Optoelectronics* (Springer, 2005)
95. C. K ubler, R. Huber, A. Leitenstorfer, Ultrabroad terahertz pulses: generation and field-resolved detection. Semicond. Sci. Technol. **20**, 128 (2005)
96. A. Sell, A. Leitenstorfer, R. Huber, Phase-locked generation and field-resolved detection of widely tunable terahertz pulses with amplitudes exceeding 100 MV/cm. Opt. Lett. **33**, 2767 (2008)
97. D.H. Auston, Subpicosecond electro-optic shock waves. Appl. Phys. Lett. **43**, 713 (1983)
98. D.H. Auston, K.P. Cheung, P.R. Smith, Picosecond photoconducting Hertzian dipoles. Appl. Phys. Lett. **45**, 284 (1984)
99. D.H. Auston, K.P. Cheung, Coherent time-domain far-infrared spectroscopy. J. Opt. Soc. Am. B **2**, 606 (1985)
100. B.B. Hu, M.C. Nuss, Imaging with terahertz waves. Opt. Lett. **20**, 1716 (1985)
101. T. Brabec, F. Krausz, Intense few-cycle laser fields: frontiers of nonlinear optics. Rev. Mod. Phys. **72**, 545 (2000)
102. A. Baltuska et al., Attosecond control of electronic processes by intense light fields. Nature **421**, 611 (2003)
103. H.C. Kapteyn, M.M. Murnane, I.P. Christov, Extreme nonlinear optics: coherent X-rays from lasers. Phys. Today **39** (2005)
104. A.V. Oppenheim, A.S. Wilsky, *Signals and Systems* (Prentice Hall, 1983)
105. R.A. Alfano (Ed.), *The Supercontinuum Laser Source*, 2nd edn. (Springer, 2006)
106. T.P. Softley, *Atomic Spectra* (Oxford University Press, 1994)
107. B.H. Bransden, C.J. Joachain, *Physics of Atoms and Molecules*, 2nd edn. (Prentice Hall, 2003)

108. L.I. Schiff, *Quantum Mechanics* (McGraw-Hill, 1955)
109. C. Cohen-Tannoudji, B. Diu, F. Laloe, *Quantum Mechanics*, 1st edn. (Wiley, 1977)
110. A. Corney, *Atomic and Laser Spectroscopy* (Clarendon Press, 1977)
111. H.G. Kuhn, *Atomic Spectra*, 2nd edn. (Longmans, 1969)
112. G. Herzberg, *Atomic Spectra and Atomic Structure* (Dover, 1944)
113. G. Herzberg, *Molecular Spectra and Molecular Structure* (D. van Nostrand, 1967)
114. H. Haken, H. Wolf, *The Physics of Atoms and Quanta*, 6th edn. (Springer, 2000)
115. M. Karplus, R.N. Porter, *Atoms and Molecules* (Benjamin, 1970)
116. Javan et al., Population inversion and continuous optical maser oscillations in a gas discharge containing a He-Ne mixture. *Phys. Rev. Lett.* **6**, 106 (1961)
117. B.A. Lengyel, *Introduction to Laser Physics* (Wiley, 1966)
118. W.L. Faust, R.A. McFarlane, Line strengths for noble-gas maser transitions; calculations of gain/inversion at various wavelengths. *J. Appl. Phys.* **35**, 2010 (1964)
119. S.D. Ganichev, W. Prettl, *Intense Terahertz Excitation of Semiconductors* (Oxford University Press, 2006)
120. T.H. Maiman, Optical maser action in Ruby. *Nature* **187**, 493 (1960)
121. S. Hüfner, *Optical Spectra of Transparent Rare Earth Compounds* (Academic Press, 1978)
122. G.H. Dieke, H.M. Crosswhite, The spectra of the doubly and triply ionized rare earths. *Appl. Opt.* **2**, 675 (1963)
123. E.H. Carlson, G.H. Dieke, The state of the  $\text{Nd}^{3+}$  ion as derived from the absorption and fluorescence spectra of  $\text{NdCl}_3$  and their zeeman effects. *J. Chem. Phys.* **34**, 1602 (1961)
124. T. Kushida, H.M. Marcos, J.E. Geusic, Laser transition cross section and fluorescence branching ratio for  $\text{Nd}^{3+}$  in yttrium aluminum garnet. *Phys. Rev.* **716**, 289 (1968)
125. W.M. Yen, P.M. Selzer (Eds.), *Laser Spectroscopy of Solids* (Springer, 1981)
126. W.M. Yen, *Laser Spectroscopy of Solids II* (Springer, 1989)
127. A. Klein, W. Bäumlner, M. Landthaler, P. Babilas, Laser and IPL treatment of port-wine stains: therapy, options, limitations, and practical aspects. *Lasers Med. Sci.* **26**, 845 (2011)
128. W. Bäumlner, H. Ulrich, A. Hartl, M. Landthaler, G. Sharifstein, Optical parameters for the treatment of leg veins using Nd:YAG lasers at 1.064 nm. *Dermatol. Surg.* **155**, 363 (2006)
129. T. Maisch et al., The role of singlet oxygen and oxygen concentration in photodynamic inactivation of bacteria. *PNAS* **104** (2007)
130. F.P. Schäfer (Ed.), *Dye Lasers*, 3rd edn. (Springer, 1990)
131. B.R. Benware et al., Demonstration of a high average power tabletop soft X-ray laser. *Phys. Rev. Lett.* **81**, 5804 (1998)
132. J.J. Rocca et al., Energy extraction and achievement of the saturation limit in a discharge-pumped table-top soft X-ray amplifier. *Phys. Rev. Lett.* **77**, 1476 (1996)
133. J. Dunn et al., Gain saturation regime for laser-driven tabletop, transient Ni-like ion X-ray lasers. *Phys. Rev. Lett.* **84**, 4834 (2000)
134. M.A. Noginov, *Solid-State Random Lasers* (Springer, 2005)
135. L.V. Keldysh, Kinetic theory of impact ionization in semiconductors. *Sov. Phys. JETP* **21**, 509 (1960)
136. L.V. Keldysh, Concerning the theory of impact ionization in semiconductors. *Sov. Phys. JETP* **37**, 1135 (1965)
137. B. Henderson, G.F. Imbusch, *Optical Spectroscopy of Inorganic Solids* (Clarendon Press, 1989)
138. B.F. Gächter, J.A. Koningstein, *J. Chem. Phys.* **60**, 2003 (1974)
139. P. Albers, E. Stark, G. Huber, Continuous-wave laser operation and quantum efficiency of titanium-doped sapphire. *J. Opt. Soc. Am. B* **134**, 134 (1986)
140. M. J. F. Digonnet, *Rare-Earth-Doped Fiber Lasers and Amplifiers* (Marcel Dekker, 2001)
141. E. Desurvire, *Erbium-Doped Fiber Amplifiers* (Wiley, 2002)
142. A. Mendez, T.F. Morse, *Speciality Optical Fibers Handbook* (Elsevier, 2007)
143. J.P. Dakin, R.G.W. Brown (Eds.), *Handbook of Optoelectronics*, vols. I and II (Taylor and Francis, 2006)

144. G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, 1989)
145. K.F. Renk, Role of excited-impurity quasiparticles for amplification of radiation in an erbium-doped glass fiber amplifier. *Appl. Phys. Lett.* **96**, 131104 (2010)
146. G.H. Dieke, S. Singh, Absorption and fluorescence spectra with magnetic properties of  $\text{ErCl}_3$ . *J. Chem. Phys.* **35**, 555 (1961)
147. F. Varsanyi, G.H. Dieke, Energy levels of hexagonal  $\text{ErCl}_3$ . *J. Chem. Phys.* **36**, 2951 (1962)
148. T. Holstein, S.K. Lyo, R. Orbach, Spectral-spatial diffusion in inhomogeneously broadened systems. *Phys. Rev. Lett.* **36**, 891 (1976)
149. T. Holstein, S.K. Lyo, R. Orbach, Phonon-assisted energy transport in inhomogeneously broadened systems. *Phys. Rev. B* **15**, 4693 (1977)
150. R.M. Macfarlane, R.M. Shelby, Homogeneous line broadening of optical transitions of ions and molecules in glasses. *J. Lumin.* **36**, 179 (1987)
151. W.M. Yen, R.T. Brundage, Fluorescence line narrowing in inorganic glasses: linewidth measurements. *J. Lumin.* **36**, 209 (1987)
152. T. Förster, Zwischenmolekulare Energiewanderung und Fluoreszenz. *Ann. Phys.* **2**, 55 (1948)
153. E. Desurvire, J.L. Zyskind, J.R. Simpson, Spectral gain hole-burning at 1.53  $\mu\text{m}$  in erbium-doped fiber amplifiers. *IEEE Photon. Technol. Lett.* **2**, 246 (1990)
154. J.N. Sandoe, P.H. Sarkies, S. Parke, Variation of the  $\text{Er}^{3+}$  cross section for stimulated emission with glass composition. *J. Phys. D: Appl. Phys.* **5**, 1788 (1972)
155. E. Desurvire, J.R. Simpson, P.C. Becker, High-gain erbium-doped traveling-wave fiber amplifier. *Opt. Lett.* **12**, 888 (1987)
156. Y. Sun, J.L. Zyskind, A. Srivastava, Average inversion level, modeling, and physics of erbium-doped fiber amplifiers. *IEEE J. Sel. Top. Quan. Electron.* **3**, 991 (1997)
157. C.R. Giles, E. Desurvire, Modeling erbium-doped fiber amplifiers. *IEEE J. Lightwave Technol.* **9**, 271 (1991)
158. G.C. Valley, Modeling cladding-pumped Er/Yb fiber amplifiers. *Opt. Fiber Technol.* **7**, 21 (2001)
159. J. Limpert et al., High-power femtosecond Yb-doped fiber amplifier. *Opt. Expr.* **10**, 628 (2002)
160. P.F. Wysocki, N. Park, D. DiGiovanni, Erbium-ytterbium codoped fiber amplifier. *Opt. Lett.* **21**, 1744 (1996)
161. S.D. Jackson, A. Sabella, D.G. Lancaster, Application and development of high-power and highly efficient silica-based fiber lasers operating at 2  $\mu\text{m}$ . *IEEE J. Sel. Top. in Quan. Elect.* **34**, 991 (2009)
162. G.D. Goodno et al., Low-phase-noise, single-frequency, single-mode 608 W thulium fiber amplifier. *Opt. Lett.* **34**, 1204 (2009)
163. S.D. Jackson, E. Bugge, G. Ebert, Directly diode-pumped holmium fiber lasers. *Opt. Lett.* **32**, 2496 (2007)
164. S.D. Jackson, A. Sabella, D.G. Lancaster, Diode-pumped 1.7-W erbium 3- $\mu$  fiber laser. *Opt. Lett.* **24**, 1133 (1999)
165. X. Zhu, R. Jain, Watt-level Er-doped and Er-Pr-codoped ZBLAN fiber amplifiers at the 2.7-2.8  $\mu\text{m}$  wavelength range. *Opt. Lett.* **33**, 208 (2008)
166. C.A. Brau, *Free-Electron Lasers* (Academic Press, 1990)
167. C.E. Webb, J.D.C. Jones (Eds.), *Handbook of Laser Technology and Applications* (Jones Publications, 2004) (Chapter B 5.1, free electron lasers and synchrotron light sources)
168. L.R. Elias, W.M. Fairbank, J.M. Madey, H.A. Schwettman, T.I. Smith, Observation of stimulated emission of radiation by relativistic electrons in a spatially periodic transverse magnetic field. *Phys. Rev. Lett.* **36**, 717 (1976)
169. D.A. Deacon, L.R. Elias, J.M. Madey, G.J. Ramian, H.A. Schwettman, T.I. Smith, First operation of a free-electron laser. *Phys. Rev. Lett.* **38**, 892 (1977)
170. S.V. Milton, Exponential gain and saturation of a self-amplified spontaneous emission freeelectron laser. *Science* **292**, 2037 (2001)
171. V. Ayvazyan et al., Generation of GW radiation pulses from a VUV free-electron laser operating in the femtosecond regime. *Phys. Rev. Lett.* **88**, 104802 (2002)

172. V. Ayvazyan et al., First operation of a free-electron laser generating GW power radiation at 32 nm wavelength. *Eur. Phys. J. D* **37**, 297 (2006)
173. Z. Huang, K. Kim, Review of X-ray free-electron laser theory. *Phys. Rev. Spec. Top. Accel. Beams* **10**, 034801 (2007)
174. B.W. McNeil, N.R. Thompson, X-ray free-electron lasers. *Nature Photon.* **4**, 814 (2010)
175. J.M. Madey, Stimulated emission of Bremsstrahlung in a periodic magnetic field. *J. Appl. Phys.* **42**, 1906 (1971)
176. F.A. Hopf, P. Meystre, M.O. Scully, Classical theory of a free-electron laser. *Opt. Comm.* **18**, 413 (1976)
177. C. Kittel, *Introduction to Solid State Physics*, 7th edn. (Wiley, 1996)
178. J. Singleton, *Band Theory and Electronic Properties of Solids* (Oxford University Press, 2001)
179. H. Ibach, H. Lüth, *Solid State Physics*, 3rd edn. (Springer, 2003)
180. J.M. Ziman, *Principle of the Theory of Solids*, 2nd edn. (Cambridge University Press)
181. L.D. Landau, E.M. Lifshitz, L.P. Pitaevski, *Statistical Physics: Theory of the Condensed State*, vol. 2; *Course of theoretical physics*, vol. 9 (Pergamon Press, 1980)
182. U. Rössler, *Solid State Theory: An Introduction*, 2nd edn. (Springer, 2009)
183. H. Haug, S.W. Koch, *Quantum Theory of Optical and Electronic Properties of Semiconductors*, 4th edn. (World Scientific, 2004)
184. C.F. Klingshirn, *Semiconductor Optics* (Springer, 1997)
185. K.F. Brennan, *The Physics of Semiconductors* (Cambridge University Press, 1999)
186. K. Seeger, *Semiconductor Physics: An Introduction*, 4th edn. (Springer, 1999)
187. H.C. Casey Jr., M.B. Panish, *Heterostructure Lasers (Fundamental Principles)* (Academic Press, Part A, 1987)
188. S. Zory, *Quantum Well Lasers* (Academic Press, 1993)
189. L.A. Coldren, S.W. Corzine, *Diode Lasers and Photonic Integrated Circuits* (Wiley, 1995)
190. J. Agrawal, *Semiconductor Lasers* (AIP Press, 1995)
191. T. Numai, *Fundamental of Semiconductor Lasers* (Springer, 2004)
192. W.W. Chow, S.W. Koch, *Semiconductor-Laser Fundamentals* (Springer, 1999)
193. C.Y. Chang, F. Kai, *GaAs High-Speed Devices* (Wiley, 1994)
194. J. Cheng, N.K. Dutta, *Vertical-Cavity Surface-Emitting Lasers: Technology and Applications* (Gordon and Breach Science Publisher, 2000)
195. S.F. Yu, *Analysis and Design of Vertical Cavity Surface Emitting Lasers* (Wiley, 2003)
196. S. Nakamura, G. Fasol, *The Blue Laser Diode* (Springer, 1997)
197. H.K. Choi, *Long-Wavelength Infrared Semiconductor Lasers* (Wiley, 2004)
198. S.L. Chuang, *Physics of Optoelectronic Devices* (Wiley, 1995)
199. N.K. Dutta, M. Wang, *Semiconductor Optical Amplifiers* (World Scientific, 2006)
200. W.B. Leigh, *Devices for Optoelectronics* (Marcel Dekker, 1996)
201. M.G. Bernard, G. Duraffourg, Laser conditions in semiconductors. *Phys. Stat. Sol.* **1**, 699 (1961)
202. H. Welker, Über neue halbleitende Verbindungen. *Z. Naturforschung* **7a**, 744 (1952)
203. S. Nakamura et al., Room temperature continuous-wave operation of InGaN multi-quantum-well- structure laser diodes with a long lifetime. *Appl. Phys. Lett.* **70**, 868 (1997)
204. N.M. Johnson, A.V. Nurmikko, S.P. DenBaars, Blue diode lasers. *Phys. Today* **31** (2000)
205. M.A. Haase, J. Qiu, J.M. DePuydt, H. Cheng, *Appl. Phys. Lett.* **59**, 1272 (1991)
206. G.F. Neumark, R.M. Park, M. DePuydt, Blue-green diode lasers. *Phys. Today* **26** (1994)
207. P. Yeh, *Optical Waves in Layered Media* (Wiley, 1988)
208. A. Yariv, P. Yeh, *Photonics*, 6th edn. (Oxford University Press, 2007)
209. E. Rosencher, B. Vinter, *Optoelectronics* (Cambridge University Press, 2002)
210. B.E.A. Saleh, M.C. Teich, *Photonics*, 2nd edn. (Wiley, 2007)
211. C. Yeh, F. Shimabukuro, *The Essence of dielectric Waveguides* (Springer, 2008)
212. J.D. Joannopoulos, R.D. Meade, J.N. Winn, *Photonic Crystals* (Princeton University Press, 1995)
213. H. Benesty, V. Berger, J.-M. G' erard, D. Maystre, A. Tchelnokov, *Photonic Crystals* (Springer, 2005)

214. K. Vahala, *Optical Microcavities* (World Scientific, 2004)
215. J. Carrol, J. Whiteaway, D. Plump, *Distributed Feedback Semiconductor Lasers* (SPIE Optical Engineering Press, 1998)
216. L. Novotny, B. Hecht, *Nano-Optics* (Cambridge University Press, 2006)
217. J.W.S. Rayleigh, On the remarkable phenomenon of crystalline reflection described by Prof. Stokes. *Phil. Mag.* **26**, 256 (1888)
218. E. Yablonovitch, Inhibited spontaneous emission in solid state physics and electronics. *Phys. Rev. Lett.* **58**, 2059 (1987)
219. W. Culshaw, High resolution millimeter wave Fabry–Perot interferometer. *IRE Trans. Microw. Theory Tech.* **MTT-8**, 182 (1960)
220. K.F. Renk, L. Genzel, Interference filters and Fabry-Perot interferometers for the far infrared. *Appl. Opt.* **1**, 642 (1962)
221. V.M. Ustinov, A.W. Zhukov, A. Yu, Egorov, N.A. Maleev, *Quantum Dot Lasers* (Oxford University Press, 2003)
222. P. Harrison, *Quantum Wells, Wires and Dots*, 3rd edn. (Wiley, 2009)
223. J. Faist, F. Capasso, D.L. Sivco, C. Sirtori, A.L. Hutchinson, A.Y. Cho, Quantum cascade laser *Science* **264**, 553 (1998)
224. R. Köhler, A. Tredicucci, F. Beltram et al., Terahertz heterostructure laser. *Nature* **417**, 156 (2002)
225. B.S. Williams, Terahertz quantum-cascade lasers. *Nature Photon.* **1**, 517 (2007)
226. R. Shankar, *Principles of Quantum Mechanics*, 2nd edn. (Springer, 2008)
227. E. Merzbacher, *Quantum Mechanics*, 2nd edn. (Wiley, 1970)
228. L. Liboff, *Introductory Quantum Mechanics* (Holden-Day, 1980)
229. F. S. Levin, *Quantum Theory* (Cambridge University Press, 2002)
230. K. Kopitzki, P. Herzog, *Einführung in die Festkörperphysik*, 4th edn. (Teubner, 2002)
231. R. de L. Kronig, W.G. Penney, Quantum mechanics of electrons in crystal lattices. *Proc. Roy. Soc. (London)* **A 130**, 499 (1931)
232. G. Bastard, *Wave Mechanics Applied to Semiconductor Heterostructures* (Les Editions de Physique, 1988)
233. G. Bastard, Superlattice band structure in the envelope-function approximation. *Phys. Rev. B* **24**, 5693 (1981)
234. S. Yngvesson, *Microwave Semiconductor Devices* (Kluwer Academic Publishers, 1991)
235. D.M. Pozar, *Microwave Engineering* (Addison Wesley Publishing Company, 1990)
236. S.M. Sze, *Physics of Semiconductor Devices*, 2nd edn. (Wiley, 1981)
237. S.M. Sze (Ed.), *Modern Semiconductor Device Physics*, 2nd edn. (Wiley, 1998)
238. M. Shur, *GaAs Devices and Circuits* (Plenum, 1987)
239. H. Kroemer, Gunn effect—bulk instabilities, in *Topics in Solid State and Quantum Electronics*, ed. by W.D. Hershberger (Wiley, 1972)
240. I. Bahl, P. Bhartia, *Microwave Solid State Circuit Design* (Wiley, 1988)
241. J.C. Slater, *Microwave Electronics* (Dover Publications, 1950)
242. M.J. Howes, D.V. Morgan (Eds.), *Microwave Devices* (Wiley, 1976)
243. B. van der Pol, *Proc. IRE* **22**, 1051 (1934)
244. K. Kurokawa, Some basic characteristics of broadband negative resistance oscillator circuits. *Bell Syst. Tech. J.* **48**, 1937 (1969)
245. K. Kurokawa, *An Introduction to the Theory of Microwave Circuits* (Academic, 1969)
246. K.F. Renk, B.I. Stahl, Operation of a semiconductor superlattice oscillator. *Phys. Lett. A* **375**, 2644 (2011)
247. T.C. Sollner, E.R. Brown, W.D. Goodhue, H.Q. Le, Microwave and millimeter-wave resonant-tunneling devices, in *Physics of Quantum Electron Devices*, ed. by F. Capasso (Springer, 1990)
248. E.R. Brown, J.R. Söderström, C.D. Parker, L.L. Mahoney, K.M. Molvar, McGill, Oscillations up to 712 GHz in InAs/AlSb resonant-tunneling diodes. *Appl. Phys. Lett.* **58**, 2291 (1991)
249. E.R. Brown, T.C. Sollner, C.D. Parker, W.D. Goodhue, C.L. Chen, Oscillations up to 420 GHz in GaAs/AlAs resonant-tunneling diodes. *Appl. Phys. Lett.* **55**, 1777 (1989)
250. V.I. Sugakov, *Lectures in Synergetics* (World Scientific, 1998)

251. F. Bloch, Über die Quantenmechanik der Elektronen in Kristallgittern. *Zeitschrift für Physik.* **52**, 555 (1928)
252. C. Zener, A theory of the electrical breakdown of solid dielectrics. *Proc. Roy. Soc. London Ser. A* **145**, 523 (1934)
253. L.V. Keldish, Effect of ultrasonics on the electron spectrum of crystals. *Sov. Phys. Solid State* **4**, 1658 (1962)
254. L. Esaki, R. Tsu, Superlattice and negative differential conductivity in semiconductors. *IBM J. Res. Dev.* **14**, 61 (1970)
255. S.A. Ktitorov, G.S. Simin, V. Ya, Sindalovskii, Bragg reflections and the high-frequency conductivity of an electronic solid-state plasma. *Sov. Phys. Sol. State* **13**, 1872 (1972)
256. A.A. Ignatov, K.F. Renk, E.P. Dodin, Esaki-Tsu superlattice oscillator: Josephson-like dynamics of carriers. *Phys. Rev. Lett.* **70**, 1996 (1993)
257. A.A. Ignatov, E. Schomburg, J. Grenzer, K.F. Renk, E.P. Dodin, THz-field induced nonlinear transport and dc voltage generation in a semiconductor superlattice due to Bloch oscillations. *Z. Phys. B* **98**, 187 (1995)
258. H. Kroemer, On the nature of the negative-conductivity resonance in a superlattice Bloch oscillator, cond-mat/0007428 (2000)
259. E. Schomburg, N.V. Demarina, K.F. Renk, Amplification of a terahertz field in a semiconductor superlattice via phase-locked k-space bunches of Bloch oscillating electrons. *Phys. Rev. B* **67**, 155302 (2003)
260. N.V. Demarina, K.F. Renk, Bloch gain for terahertz radiation in semiconductor superlattices of different miniband widths mediated by acoustic and optical phonons. *Phys. Rev. B* **67**, 155302 (2003)
261. A. Sibille, J.F. Palmier, H. Wang, F. Mollot, Observation of Esaki-Tsu negative differential velocity in GaAs/AlAs superlattices. *Phys. Rev. Lett.* **64**, 52 (1990)
262. A.A. Ignatov, E. Schomburg, K.F. Renk, W. Schatz, J.F. Palmier, F. Mollot, Response of a Bloch oscillator to a THz-field. *Ann. Physik.* **3**, 137 (1994)
263. K. Unterrainer, S.J. Allen et al., Inverse Bloch oscillator: strong terahertz-photocurrent resonances at the Bloch frequency. *Phys. Rev. Lett.* **76**, 2973 (1996)
264. P.G. Savvidis, B. Kolasa, G. Lee, S.J. Allen, Resonant crossover of terahertz loss to gain in a Bloch oscillating InAs/AlSb super-superlattice. *Phys. Rev. Lett.* **92**, 196802 (2004)
265. A. Lisauskas, E. Mohler, H. Roskos, N. Demarina, Towards superlattice terahertz amplifiers and lasers, in *Terahertz Frequency Detection and Identification of Materials and Objects*, ed. by R.E. Miles et al. (Springer, 2007), p. 31
266. H. Kroemer, Large-amplitude oscillation dynamics and domain suppression in a superlattice Bloch oscillator, cond-mat/0009311 (2000)
267. J. Feldmann, K. Leo et al., Optical investigation of Bloch oscillations in a semiconductor superlattice. *Phys. Rev. B* **46**, 7252 (1992)
268. W. Waschke, H.G. Roskos, R. Schwegler, K. Leo, H. Kurz, K. Köhler, Coherent submillimeter wave emission from Bloch oscillations in a semiconductor superlattice. *Phys. Rev. Lett.* **70**, 3319 (1993)
269. K. Leo, *High-Field Transport in Semiconductor Superlattices* (Springer, 2003)
270. A. Wacker, Semiconductor superlattices; a model system for nonlinear transport. *Phys. Rep.* **357**, 1 (2002)
271. E.E. Mendez, G. Bastard, Wannier-stark ladders and Bloch oscillations in superlattices. *Phys. Today* **34** (1993)
272. H. Willenberg, G.H. Döhler, J. Faist, Intersubband gain in a Bloch oscillator and quantum cascade laser. *Phys. Rev. B* **67**, 08531 (2003)
273. J. Stark, Beobachtungen über den Effekt des elektrischen Feldes auf Spektrallinien. *Annalen der Physik.* **43**, 965 (1914)
274. G.H. Wannier, Wave functions and effective Hamiltonian for Bloch electrons in an electric field. *Phys. Rev.* **117**, 432 (1960)
275. G.H. Wannier, Dynamics of band electrons in electric and magnetic fields. *Rev. Mod. Phys.* **34**, 645 (1962)

276. R.G. Hunsperger, *Integrated Optics: Theory and Technology*, 6th edn. (Springer, 2009)
277. R. Menzel, *Photonics*, 2nd edn. (Springer, 2007)
278. F.G. Smith, T.A. King, *Optics and Photonics: An Introduction* (Wiley, 2000)
279. X. Li, *Optoelectronic Devices* (Cambridge University Press, 2009)
280. R.G. Hunsperger (Ed.), *Photonic Devices and Systems* (Marcel Dekker, 1994)
281. T. Schneider, *Nonlinear Optics in Telecommunications* (Springer, 2004)
282. J. Eberspächer, H.-J. Vögel, C. Bettstetter, C. Hartmann, *GSM—Global System for Communications* (Wiley, 2008)
283. E.F. Schubert, *Light-Emitting Diodes* (Cambridge University Press, 2005)
284. K. Müller, U. Scherf, *Organic Light-Emitting Devices* (Wiley, 2006)
285. M. Reufer et al., Low-threshold polymeric distributed feedback lasers with metallic contacts. *Appl. Phys. Lett.* **84**, 3262 (2004)
286. D. Schneider et al., Deep blue widely tunable organic solid-state laser based on a spirobifluorene derivative. *Appl. Phys. Lett.* **84**, 4693 (2004)
287. E. Holzer, A. Penzkofer et al., Corrugated neat thin-film conjugated polymer distributed feedback lasers. *Appl. Phys. B* **74**, 333 (2002)
288. P. Andrew, W.L. Barnes, Förster energy transfer in an optical microcavity. *Science* **290**, 785 (2000)
289. S.V. Frolov et al., Lasing and stimulated emission in  $\pi$ -conjugated polymers. *IEEE J. Quant. Electr.* **36**, 2 (2000)
290. V. Bulovic et al., Transform-limited. Narrow-linewidth lasing action in organic semiconductor microcavities. *Science* **279**, 553 (1998)
291. A. Haugeneder et al., Mechanism of gain narrowing in conjugated polymer thin films. *Appl. Phys. B* **66**, 389 (1998)
292. V.G. Kozlov et al., Study of lasing action based on Förster energy transfer in optically pumped organic semiconductor thin films. *J. Appl. Phys.* **84**, 4096 (1998)
293. V.G. Kozlov et al., Laser action in organic semiconductor waveguide and doubleheterostructure devices. *Nature* **389**, 362 (1997)
294. M. Berggren et al., Stimulated emission and lasing in dye-doped organic thin films with Förster transfer. *Appl. Phys. Lett.* **71**, 2230 (1997)
295. N. Tessler et al., Lasing from conjugated-polymer microcavities. *Nature* **382**, 695 (1996)
296. F. Hide et al., Semiconducting polymers: a new class of solid-state laser materials. *Science* **273**, 1833 (1996)
297. R.W. Boyd, *Nonlinear Optics*, 2nd edn. (Academic Press, 2002)
298. N. Bloembergen, *Nonlinear Optics* (Benjamin, 1965)
299. F. Zernike, J.E. Midwinter, *Applied Nonlinear Optics* (Wiley, 1973)
300. E. Hamamura, Y. Kawabe, A. Yamanaka, *Quantum Nonlinear Optics* (Academic Press, 1989)
301. G.P. Agrawal, *Nonlinear Fiber Optics* (Springer, 2007)
302. D.L. Mills, *Nonlinear Optics* (Springer, 1991)
303. T. Schneider, *Nonlinear Optics in Telecommunications* (Springer, 2004)
304. B.E.A. Saleh, M.C. Teich, *Photonics*, 2nd edn. (Wiley, 2007)
305. P.P. Banerjee, *Nonlinear Optics* (Marcel Dekker, 2004)
306. Y. Guo, C.K. Kao, E.H. Li, K.S. Chiang, *Nonlinear Photonics* (Springer, 2002)
307. E. Hanamura, Y. Kawave, A. Yomanaka, *Quantum Nonlinear Optics* (Springer, 2007)
308. M. Eichhorn, *Laser Physics* (Springer, 2014)
309. W. Nagourney, *Quantum Electronics for Atomic Physics and Telecommunication*, 2nd edn. (Oxford University Press, 2014)
310. G. Brooker, *Modern Classical Optics* (Oxford University Press, 2008)
311. B.P. Abbott et al., Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.* **116**, 061102 (2016)
312. T.C. Marshall, *Free-Electron Lasers* (Macmillan Publishing Company, 1985)
313. C. Pellegrini, A. Marinelli, S. Reiche, The physics of free-electron lasers. *Rev. Mod. Phys.* **88**, 015006 (2016)

314. C. Bostedt, S. Boutet, D.M. Fritz et al., Linac coherent light source: the first five years. *Rev. Mod. Phys.* **88**, 015007 (2016)
315. L. Young, E.P. Kanter, B. Krässig et al., *Nature* **476**, 09177 (2010)
316. G. Doumy, C. Roedig, S.-K. Son et al., Nonlinear atomic response to intense ultrashort X-rays. *Phys. Rev. Lett.* **106**, 083002 (2011)
317. E. Allaria et al., Highly coherent and stable pulses from the FERMI seeded free-electron laser in the extreme ultraviolet. *Nature Photon.* **6**, 699 (2012)
318. T. Sato, A. Iwasaki, S. Owada et al., Full-coherent free-electron laser seeded by 13th and 15th order harmonics of near-infrared femtosecond laser pulses. *J. Phys. B: Mol. Opt. Phys.* **46**, 164006 (2013)
319. S. Ackermann, A. Azima, S. Bajt et al., Generation of 19- and 38-nm radiation at a free-electron laser directly seeded at 38 nm. *Phys. Rev. Lett.* **106**, 114801 (2013)
320. T. Maltzopoulos, M. Mittenzwey, A. Azima et al., A high-harmonic generation source for seeding a free-electron laser at 38 nm. *Appl. Phys. B* **115**, 45 (2014)
321. D.G. Gauthier, P.R. Ribic et al., Generation of phase-locked pulses from a seeded free-electron laser. *Phys. Rev. Lett.* **116**, 024801 (2016)
322. E.A. Schneidmiller, M.V. Yurkov, *Physical review special topics—accelerators and beams.* **15**, 080702 (2012)
323. G. Penn, Simple method to suppress the fundamental in a harmonic free-electron laser. *Phys. Rev. Spec. Top. Accel. Beams* **18**, 060703 (2015)
324. G.R.M. Robb, R. Bonifazio, Coherent and spontaneous emission in the quantum free-electron laser. *Phys. Plasmas* **19**, 073101 (2012)
325. P. Kling, E. Giese, R. Endrich et al., What defines the quantum regime of the free-electron laser? *New J. Phys.* **17**, 123019 (2015)
326. H.P. Freund, M. Shinn, S.V. Benson, Simulation of high-average power free-electron laser oscillator. *Phys. Rev. Spec. Top. Accel. Beams* **10**, 030702 (2007)
327. W.B. Colson, Classical free-electron laser theory, in *Laser Handbook*, vol. 6, ed. by W.B. Colson, C. Pellegrini, A. Renieri (Elsevier Science Publishers, 1990), p. 115
328. J.B. Murphy, C. Pellegrini, Introduction to the physics of the free-electron laser, in *Laser Handbook*, vol. 6, ed. by W.B. Colson, C. Pellegrini, A. Renieri (Elsevier Science Publishers, 1990), p. 9
329. W.B. Colson, S.K. Ride, The free-electron laser: Maxwell's equations driven by single-particle currents. *Phys. Quan. Electron.* **7**, 377 (1980)
330. N.M. Kroll, P.L. Morton, M.N. Rosenbluth, Free-electron lasers with variable parameter wigglers, *IEEE J. Quan. Electron.* **QE-17**, 1436 (1981)
331. K. Tiedke, A. Azima, N. von Barga et al., The soft X-ray free-electron laser FLASH at DESY: beamlines, diagnostics, and end-stations. *New J. Phys.* **11**, 023029 (2009)
332. W. Steiner, *Transoral Laser Microsurgery for Cancer in the Upper Aerodigestive Tract* (Straub Druck+Medien, Schramberg, Germany, 2014)
333. M. Holthaus, Collapse of minibands in far infrared irradiated superlattices. *Phys. Rev. Lett.* **69**, 351 (1992)

# Index

## A

ABCD matrix, 226–230  
Absorption, 23, 88  
  band, 80  
  coefficient, 21, 22, 102, 110, 142, 146, 482, 535  
  cross section, 103  
Acceleration theorem, 597, 618  
Accelerator, 351, 352  
Acousto-optic modulator, 251  
Active medium, 22–26, 58–64  
Airy formula, 49  
AIs, 482  
Alexandrite laser, 295, 296  
AlN, 487  
Amplified spontaneous emission, 312, 352  
Angle of divergence, 206, 230  
Anharmonicity, 110, 320, 321  
Anisotropy of a quantum well, 421, 473, 518  
Annihilation, 63  
Antenna, 191, 570  
Antireflecting coating, 37, 493, 507, 630  
Applications, 8, 9, 261, 262, 278, 280, 281, 296, 299, 489, 623–627, 637–639, 641  
Argon ion laser, 278, 279  
Attosecond pulse, 265  
Auston switch, 264  
Autocorrelator, 259

## B

Backward wave oscillator, 543  
Beam divergence, 206  
Beam waist, 203, 208  
Beat frequency, 638  
Bernard-Duraffourg relation, 434

Biology, 9, 235, 242, 243, 261, 300, 353  
Bipolar  
  laser medium, 450–452  
  semiconductor laser, 5, 18, 37, 107, 110, 420–493  
Birefringent filter, 237  
Blackbody radiation, 86  
Bloch  
  frequency, 592, 597  
  gain, 601–605  
  laser, 11, 424, 591–619  
  oscillation, 397, 596–598, 618  
  theorem, 502, 507  
  wave, 502  
Blue laser diode, 487, 488  
Bohr radius, 636  
Bohr's energy-frequency relation, 22  
Boltzmann transport equations, 618  
Boltzmann's statistics, 25, 86  
Bose–Einstein factor, 92  
Boson, 282  
Boundary conditions for  
  electromagnetic fields, 27, 182, 190, 209, 498, 499  
  electron waves, 563  
Bragg  
  frequency, 505  
  reflection of electromagnetic waves, 505  
  reflection of electrons, 598, 611  
  reflector, 493, 494, 497, 505  
Brewster  
  angle, 37, 38, 174, 237, 252, 276  
  window, 37, 198, 275  
Brightness, 195  
Brilliance, 230  
Brillouin zone, 481, 505, 548, 598, 611  
Broadband laser, 295

Buildup of laser oscillation, *see* Onset of oscillation

## C

Carrier envelope phase, 255  
 Carrier frequency, 248, 250, 323  
 Cavity dumping, 244  
 Cavity resonator, 181–191, 244, 570  
 Centrifugal distortion, 283  
 Chemical laser, 311  
 Chemical vapor deposition (CVD), 479, 485, 630  
 Chemistry, 9, 235, 239, 242, 261, 263, 300  
 CH<sub>3</sub>F laser, 284, 285  
 Circular polarization, 42  
 Cladding, 624  
 Clamping of  
   gain factor, 34  
   luminescence, 442  
   population difference, 122, 126  
   quasi-Fermi energy, 442  
 Classical  
   absorption coefficient, 146  
   oscillator, 71–75, 572–576, 583–588  
   oscillator model of an atom, 69–71, 142, 322  
 Cleaving surface, 491  
 CO<sub>2</sub> laser, 5, 36, 37, 69, 105, 109, 281–284  
 Coherence length, 136  
 Coherent wave, 18, 222  
 Collision broadening, 273, 274  
 Color center laser, 304  
 Comparison of lasers, 108, 425, 538–540  
 Complex  
   beam parameter, 203, 218, 228  
   quantities, 20  
 Compton scattering, 356  
 Concentric resonator, 212, 213  
 Condition of gain, 333–339, 429–434, 438, 445  
 Conduction band, 417  
 Conductivity, 138  
 Configuration coordinate, 302  
 Confinement factor, 463  
 Confocal  
   parameter, 206  
   resonator, 203–213, 221  
 Contact, 533, 534  
 Continuous wave laser, 3  
 Copper vapor laser, 277, 278  
 Coumarin, 309  
 Coupling strength, 394, 602

Cr:LiCaF laser, 295  
 Cr:LiSAF laser, 295  
 Critical  
   field, 599  
   modulation index, 391, 606  
 Cross relaxation, 301, 302, 329  
 Crystal surface, 44  
 Current, 145  
 Cutoff frequency of a resonator, 185

## D

Damping, 154, 586, 625  
 Data of lasers, 5, 6, 36, 37, 60, 69, 91, 107, 108, 275, 295, 306, 359, 470, 472, 473, 594, 595  
 De Broglie wavelength, 454, 597, 598  
 Decibel, 315  
 Defect centers in solids, 304–306  
 Degenerate  
   energy levels, 24, 101  
   mode, 184  
 Density of states of  
   electrons, 418, 419, 534  
   photons, *see* mode density  
 Dephasing, 74, 159, 262, 391, 602, 610  
 Depletion layer, 533  
 Depth of focus, 206  
 Dichroitic mirror, 79  
 Dielectric  
   constant, 138  
   multilayer mirror, 44, 494  
   polarization, *see* polarization  
   susceptibility, 138–145  
 Difference frequency generation, 264, 638  
 Differential gain coefficient, 106, 370, 606, 613  
 Diffraction, 189  
   loss, 223  
 Diffusion, 287  
 Dipole  
   matrix element, 96  
   moment, 143  
   oscillator, 69–72, 164  
 Direct gap semiconductor, 421, 481  
 Disk  
   laser, 8, 299, 304  
   of light, 19, 101, 112, 114  
 Dispersion, 148, 254, 255, 257, 259, 625, 626  
   relation for electromagnetic radiation, 19, 27, 625  
   relation for electrons, 417, 481, 515, 553, 556, 562, 563

- Displacement current, 139
- Distributed
  - Bragg reflector, 493, 634
  - distortion energy, 609
  - feedback laser, 8, 493
- Divergence, 206
- Domains, 570
- Doppler broadening, 271–273
- Double heterostructure laser, 423, 534, 536, 539
- Drude theory, 175
- Dye, 310
  - laser, 309–311
- Dynamical conductivity, 364
  
- E**
- Echelette grating, 237, 284, 493
- EDFA, 626
- Edge-emitting laser, 8, 44, 473, 491
- Effective
  - gain cross section, 107, 340, 438, 446, 447, 606
  - growth rate constant, 438, 450, 469
  - mass, 417, 477
  - refractive index, 504
  - wave vector, 219
  - wavelength, 219
- Eigenfrequency, 28, 70
- Eigenvalue problem, 27, 208, 228
- Eigenvalues, 555
- Einstein
  - coefficients, 87–95, 334, 391, 394, 421, 429–431, 440, 463, 613, 616
  - relations, 89–91, 95, 429–431
- Electric
  - current, 142
  - current dipole, 70
  - dipole moment, 70, 143
  - susceptibility, 146, 147, 264
  - wave, 190
- Electro-luminescence, 537
- Electromagnetic wave, 18
- Electron
  - bunching, 390
  - collisions, 275
  - gas, 75, 418, 428, 429
  - hole pair, 451, 452, 523, 627
  - hole recombination, 451, 538, 630, 632, 634
  - phonon scattering, 62, 429, 438–441, 525
  - subband, 458, 511–514
- Energy
  - band, 416, 481, 515, 556
  - band engineering, 425
  - degeneracy, 515
  - density of an electromagnetic field, 19, 30
  - flux density, 19, 507
  - gap, 416
  - ladder based laser, 58
  - ladder system, 58, 74, 391–397, 612–617
  - level broadening, 334, 433
  - relaxation, 618
  - transfer, 301, 302, 330, 633
- Erbium-doped
  - fiber amplifier, 325–343, 626
  - fiber laser, 63, 301, 327, 343
- Er:YAG laser, 297
- Esaki–Tsu characteristic, 577, 599
- Etalon, 236
- Excimer laser, 279, 280
- Extended Kronig–Penney model, 563
  
- F**
- Fabry–Perot
  - interferometer, 48, 49, 52
  - resonator, 26–36, 38–40, 43, 45–53, 154, 213, 422
  - resonator containing a gain medium, 52, 53
- Far-field range, 205
- Far infrared laser, 285, 538
- Far infrared quantum cascade laser, 550
- Feedback, 32
- FEL, *see* Free-electron laser
- Femtochemistry, 261
- Femtosecond laser, 245–266
- Fermi–Dirac distribution, 331, 342, 430, 459
- Fermi energy, 331, 430
- Fiber
  - amplifier, 302, 314, 325–343, 626
  - laser, 5, 8, 44, 105, 109, 300, 304, 342, 343
- Filling factor, 332, 339
- Finesse, 50
- Flash lamp, 292
- Floquet states, 615
- Fluctuations, 581
- Fluorescence, 60, 80, 341, 342
- Focused Gaussian beam, 228, 230
- Forbidden mode, 187
- Förster mechanism, 330, 633
- Four-level laser, 59

- Four wave mixing, 641–643
- Four-level laser, 7, 8
- Fourier transform, 250
- Franck-Condon principle, 279, 281, 293, 309, 319
- Fraunhofer range, 205
- Free electron, 415, 424, 514, 533
  - in a crystal, 418
  - laser, 10, 210, 348–412
  - laser medium, 349
  - oscillation, 349, 424
  - wave, 417
- Free spectral range, 49, 51
- Frequency
  - analyzer, 50, 252, 641, 642
  - comb, 246, 254, 255, 257–259, 262, 642
  - doubler, 637, 638
  - gap, 505
  - locking, 166
  - modulation, 320, 322, 323, 365, 391, 602
  - multiplication, 543
  - of laser oscillation, 35, 36
  - tripling, 640
- Fresnel
  - coefficients, 41, 42
  - number, 187, 189, 223
  - range, 307
- Frustrated total reflection, 493
- Füchtbauer–Ladenburg relation, 117
- FWHM, *see* Halfwidth
  
- G**
- GaAlAs mixed crystal, 477
- GaAs, 482
  - crystal, 477
  - crystal lattice, 478
  - monolayer, 478
  - quantum well, 458
  - quantum well laser, 457–472
- GaAs/AlAs heterostructure, 479
- GaAs/GaAlAs heterostructure, 478, 479
- Gain, 32, 52, 103, 534
  - bandwidth, 110
  - characteristic, 110, 113
  - coefficient, 21, 100–103, 108, 113, 131, 156, 336–339, 364, 437, 438, 446, 535
  - cross section, 103–106, 395, 438, 469, 470
  - factor, 21, 33–36
  - mediated by a quantum well, 443–448, 464
  - mediated by a quantum wire, 525
  - profile, 110, 111, 469, 516
  - saturation, 117, 131
  - units, 315
- GaN, 487
- Gap energy, 416, 517
- GaSe, 264
- Gaussian
  - beam, 195–225, 228, 230, 624
  - distribution of energy levels, 331
  - line, 68, 69, 105
  - pulse, 251
  - wave, 195–230
- Generalized conductivity, 141
- Generalized dielectric constant, 141
- General Lorentz function, 67, 170
- Germanium, 283
- Giant pulse laser, 236
- Glass, 306
  - fiber, 625
  - fiber laser, 343
  - laser, 298, 300
  - structure model, 328
- Gold vapor laser, 278
- Gouy
  - frequency, 214, 221, 222, 255
  - phase, 35, 202–206, 208, 209, 213, 214, 216, 219, 221–223, 254
- Gravitational wave detector, 243
- Green laser diode, 488
- Group
  - II–VI semiconductors, 475, 476, 488
  - III–V semiconductors, 475, 487
- Group velocity, 199, 223, 259, 626
- Growth
  - coefficient, 100–103
  - rate constant, 100, 155
- Gunn oscillator, 543, 569
  
- H**
- Halfwidth, 6, 49, 67
- HCN laser, 284
- Heat, 60, 82, 87
- Heavy hole band, 515
- Heisenberg uncertainty principle, 232, 267
- Helium–neon laser, 5, 36, 40, 69, 105, 109, 126, 133, 135, 159, 161, 210, 273, 275–277
- Helmholtz equation, 198–200
- Hermite
  - Gaussian modes, 216
  - polynomials, 215
- Hertzian dipole, 70, 264

- Heterostructure, 479
- High frequency
  - current, 138, 142, 362, 586
  - mobility, 362
  - polarization, 138
- High temperature superconductor, 176
- High-power semiconductor laser, 471
- High-pressure CO<sub>2</sub> laser, 284
- History of the laser, 11–14, 172–175
- Hohlraum resonator, 181
- Hole subband, 458, 515, 516
- Holmium-doped fiber laser, 302
- Holography, 277
- Homogeneous line broadening, 63–64
- Homojunction laser, 533
- Homostructure laser, 533
- Host crystals, 303
- Huygens' principle, 223
- H wave, 190
- Hybrid states, 515
  
- I**
- Ideal conductor, 182, 183
- Ideal mirror, 44
- Idler frequency, 640
- Impedance of free space, 191
- Impurity ions in solids, 302–306
- InAs, 487
- Index of refraction, *see* Refractive index
- Indirect gap semiconductor, 482
- Industrial lasers, 7, 8
- Inelastic scattering of electrons, 434, 438–441, 465, 525
- Inhomogeneous line broadening, 64, 69, 111, 273, 288, 298, 307, 517, 526, 527
- Injection locking, 166
- InN, 487
- Intensity, 19, 230
- Interband relaxation, 62
- Intermediate coupling, 277
- Intraband relaxation, 62, 327, 335, 336, 438
- Intraminiband relaxation, 599
- Inversionless laser, 74
- Ions in solids, 304
  
- J**
- Jahn-Teller effect, 321
- Jamin interferometer, 174
- Joint density of states, 436
- Junction, 533
- Junction laser, 423, 533, 534, 536, 538
  
- K**
- KDP, 260, 637, 640
- Kerr lens, 641
- Kerr lens mode locking, 252
- Kirchhoff's diffraction theory, 223
- Kirchhoff's rules, 573, 585
- Kramers degeneracy, 305
- Kramers–Kronig relations, 170
- Kronig-Penney potential, 563
- Krypton ion laser, 279
- K-space bunching, 612
  
- L**
- Laguerre–Gaussian mode, 218
- Lamb dip, 286
- Lambert-Beer law, 102
- Large-signal gain coefficient, 131
- Large-signal gain factor, 34
- Laser
  - amplifier, 313, 314, 326, 623
  - equations, 120, 121, 130, 132, 157–159, 161, 164, 165, 167, 168, 440–442, 448–450
  - linewidth, 133–135
  - market, 7
  - mirror, 44, 47, 491, 493, 494, 496, 497
  - oscillator, 32, 155–161
  - resonator, 18, 26–36, 38–40, 43, 45–53, 196–198, 206–223, 225, 349, 352, 358, 422, 441, 442, 446, 469, 472, 492, 493, 496
  - van der Pol equation, 168
- Lattice
  - constants, 477
  - vibrations, 82, 110, 292, 294, 317, 319, 619
- Lead, 176, 181
- Lead salt laser, 538
- LED, 629
- Level, *see* Density of states
  - broadening, 441
- Light guide, 468
- Light hole band, 515
- LiNbO<sub>3</sub>, 637, 640
- Line
  - broadening, 63, 65, 67–71, 271–274, 306
- Linear response function, 138
- Lineshape function, 63
- Linewidth, 69
- Littrow arrangement, 237, 493
- Longitudinal
  - mode, 197, 198, 216, 223

- pumping, 79
- relaxation, 164
- relaxation time, 72, 164
- Lorentz
  - dispersion function, 65, 147, 153, 405, 602
  - factor, 349, 409
  - functions, 64, 66, 67, 170
  - model of an atom, 70
  - resonance function, 52, 64–67, 145, 147, 153, 273, 335, 405, 433, 616
- Lorentzian
  - function, 52, 64
  - line, 104, 145, 571, 575
  - lineshape, 306
- Lorenz-Haken equations, 168
- Loss, 32
- Low-dimensional
  - active medium, 74
  - semiconductor, 426
- Luminescence, 60, 439, 442, 473, 537
- Luminous efficiency, 630
  
- M**
- Magnetic
  - field constant, 138
  - wave, 190
- Manley–Rowe rule, 639
- Maser, 4
- Master oscillator, 166
- Material
  - equation, 138, 568
  - gain coefficient, 463
  - processing, 245, 281, 291, 296, 300, 316
- Matrix element, 95
- Maxwellian velocity distribution, 271
- Maxwell's equations, 182
- Medicine, 9, 235, 261, 278, 281, 291, 296, 300, 310, 545
- Metal
  - mirror, 44
  - oxide chemical vapor deposition (MOCVD), 479, 485, 630
  - vapor laser, 277, 278
- M factor, 225
- Michelson interferometer, 243
- Microbunches, 347, 368, 390, 391
- Microwave
  - cavity, 182
  - oscillator, 569, 570
- Miniband, 509–511, 546, 548, 561–563
- transport, 549, 576
  - width, 549
- Mini-Brillouin zone, 548
- Mirrorless laser, 312, 352
- Mirror parameters, 211
- Mobility of an electron, 365
- MOCVD, *see* Metal oxide chemical vapor deposition (MOCVD)
- Modal gain coefficient, 113
- Mode
  - density, 93, 188, 189
  - locking, 246–248, 250–255, 257–259, 299
  - selection, 237
  - volume, 210
- Modulation current, 362
- Modulation degree, 601
- Modulation index, 323, 391, 602
- Molecular beam epitaxy, 479, 485, 618, 630
- Monolayer, 478
- Monopole oscillator, 73, 74, 403
- Monte-Carlo technique, 618
- Multi mode laser, 284
- Multi quantum well laser, 471
- Multilayer
  - coating, 37, 494
  - mirror, 44, 494
  
- N**
- NaCl, 283
- Natural linewidth, 72
- Nd:YAG laser, 5, 69, 105, 109, 296
- Nd:YLF laser, 298
- Nd:YVO<sub>4</sub> laser, 298, 299
- Near-field range, 205
- Near-planar resonator, 213
- Negative
  - absorption, 173
  - gain coefficient, 22
  - resistance oscillator, 572
- Neoclassical laser equations, 165
- Neodymium
  - doped glass laser, 298
  - YAG laser, 37
- Nitrogen laser, 280
- Noise, 53, 576
- Nonlinear
  - dispersion, 175, 259
  - polarization, 636, 637
- Nonradiative
  - relaxation, 60, 78, 79, 82, 275, 282, 292, 293, 309, 328, 335, 439, 538, 546
  - transition, 241

**O**

- Occupation number, 24, 25
  - difference, 24–26, 100, 126, 135, 327, 330, 335, 431, 441, 613
  - of photon modes, 46, 47, 92, 93, 135
- Offset frequency, 255, 259, 642
- OLED, 631, *see* Organic LED (OLED)
- One-dimensional
  - active medium, 75
  - density of states, 419, 523
- One-quantum dot laser, 531
- Onset
  - of oscillation, 34, 38, 124, 161, 246, 610
  - time, 34, 124–126, 159–161, 610
- Open resonator, 26, 197
- Optical
  - communications, 8, 487, 528, 623–628
  - constants, 141, 264
  - damage, 314
  - fiber, 623
  - frequency analyzer, 50, 262, 642, 643
  - frequency comb, 246, 254, 255, 257–259, 262, 642
  - parametric oscillator, 639
  - phonon, 439
  - pumping, 18, 78
  - radar, 287
  - ray, 225
  - rectification, 264, 638
  - spectrum analyzer, 642
  - thickness, 112
- Optically pumped gas laser, 284
- Optimum output coupling, 127–130
- Optocoupler, 626
- Organic
  - laser, 633, 634
  - LED, 631
- Oscillation frequency, 36, 574
- Oscillation onset time, 34, 39, 40, 124, 126, 159–161, 610
- Oscillator equations, 575
- Oscillator strength, 116, 170
- Output coupling, 18, 127–130, 358, 571–573, 586
- Output power, 18, 130, 161

**P**

- Parallel beam, 18
- Paraxial
  - electromagnetic wave, 198
  - ray, 226
  - wave, 207
- Partial reflector, 18
- Paschen notation, 277
- Pauli principle, 331, 432
- Perfect conductor, 176, 508
- Periodic boundary conditions, 522, 557, 561
- P germanium laser, 543
- Phase
  - locking, 166, 575
  - portrait, 178
  - relaxation, 73, 173
  - velocity, 199, 259
- Phonon assisted energy transfer, 302, 307, 325, 331, 334
- Phonon Raman scattering, 306
- Phonons, 62, 82, 87, 264, 294, 303, 306, 320, 322, 329, 434, 439, 464, 538, 543, 546, 612, 618, 619, 625, 643
- Photodiode, 627, 643
- Photodynamic therapy, 278
- Photoluminescence, 60
- Photon
  - density, 19, 122
  - flux, 230
  - lifetime, 31
  - number, 46
  - occupation number, 47
- Photonic bandgap, 505
- Photonic crystal, 495, 496
- Planck's radiation law, 86, 431
- Plane
  - wave, 199
  - wave transfer matrix method, 497–499, 559–561
  - space bunching, 612
- Poisson equation, 581
- Polar optic phonon, 439
- Polarization, 80, 138, 139, 142–635
  - current, 151–404
- Polarization conductivity, 152, 161
- Polymer laser, 633
- Population, 22
  - difference, 22, 24
  - inversion, 23, 24
- Potential well, 317, 513, 554, 564
- Poynting vector, 507
- Pr:YAG laser, 297
- Pressure broadening, 273
- Probability
  - current density, 560
  - density, 560
- Propagating dipole domain mode, 570
- Propagation
  - matrix, 499

of a Gaussian beam, 228  
 Pulse distortion due to dispersion, 626  
 Pulse repetition rate, 248  
 Pump  
   band, 81  
   -probe method, 261  
   rate, 59  
 Pumping a laser, 18  
 Pure charge accumulation mode, 569

## Q

Q factor, 45, 51  
 Quantum  
   cascade laser, 6, 105, 109, 424, 545–550  
   dot, 528, 529  
   dot laser, 529  
   efficiency, 60, 321  
   layer, 511  
   well, 443, 448, 458–465, 511–517, 630  
   well laser, 424, 448, 450, 457–473, 511–518  
   wire, 522–524  
   wire laser, 424, 521–528  
 Quarter-wavelength film, 507  
 Quartz glass, 624  
 Quasi-Fermi energy, 75, 332, 429, 430, 432, 443, 459–462, 468, 516, 534, 633  
 Quasiband, 63, 307, 327–333  
   laser, 63, 327, 633  
 Quasiclassical oscillator, 53, 568, 573, 575  
 Quasiequilibrium, 429  
 Quasiparticle, 63, 331, 450  
 Quasiplane standing wave, 27  
 Quasiplane wave, 18  
 Quasithermal equilibrium, 336, 439

## R

Rabi oscillation, 176  
 Racah notation, 277  
 Radiance, 230  
 Radiation pressure, 316  
 Radiationless transition, 60  
 Radiative  
   electron-hole pair, 451, 452  
   pair level, 435  
   transition, 333, 429  
 Random laser, 313  
 Rare earth ions, 304  
 Rate equations, 119–121  
 Ray optics, 225–228, 230  
 Rayleigh  
   range, 205

  scattering, 625  
 Recombination, 533  
 Reduced  
   density of states, 434, 436, 534  
   mass, 435, 523  
 Reflector, 18, 491, 493–505  
 Refractive index, 38, 42, 91, 141, 146, 467  
 Regenerative amplifier, 575  
 Relative occupation number, 24, 330  
 Relativistic Doppler effect, 354  
 Relaxation, 72  
   oscillation, 131–133  
   time, 59, 78, 120  
 Resolving power, 50  
 Resonance frequency, 28, 36, 71, 192, 592  
 Resonant energy transfer, 301, 302  
 Resonant tunneling diode, 543, 583, 584  
 Resonator, 10, 26–32, 36, 43–55, 181, 192, 196–198, 207–225, 492  
   boundary conditions, 28, 209  
   eigenvalue problem, 27, 35, 208, 216, 221  
   mode, 29, 184–197, 207  
   stability diagram, 211, 227  
 Response function, 138, 264, 604  
 Rod laser, 8, 304  
 Round trip transit time, 28, 246, 249  
 Ruby  
   laser, 61, 342  
   type, 60, 297, 342  
 Ruby laser, 307  
 Russel-Saunders coupling, 278

## S

Safety, 6  
 Saturation field, 371–375, 594, 606–608  
 Saturation of absorption, 117  
 Schawlow–Townes formula, 135  
 Schrödinger equation, 554  
 Schrödinger equation, 317, 511, 512, 553  
 Seed laser, 166  
 Self  
   absorption, 118  
   -amplified spontaneous emission, 352  
   -excited oscillator, 32, 173, 568, 584  
   focusing, 252, 641  
   -terminating laser, 278  
 Semiclassical  
   equation of motion, 597  
   laser equations, 165  
 Semiconductor superlattice, 548, 549  
 oscillator, 543, 569–572, 576, 578, 580–582

- SESAM, 299
  - Single mode laser, 236
  - Slave oscillator, 166
  - Slowly varying envelope approximation (SVEA), 143, 156, 165, 166, 575, 587
  - Small-signal gain
    - coefficient, 101
    - factor, 34
  - Snell's law, 38, 227
  - Solid state
    - lasers, 291–304, 306, 307
    - oscillator, 569, 572–576
  - Space charge domains, 619
  - Spatial
    - frequency, 14
    - hole burning, 287
  - Spatial frequency, 15
  - Spectral
    - energy density, 86
    - hole burning, 238, 330
  - Spectrum analyzer, 571
  - Speed of light, 4, 199, 223
  - Spherical wave, 199
  - Spin
    - lattice relaxation, 328, 331
    - orbit interaction, 276, 516
  - Split-off band, 515
  - Spontaneous
    - emission, 71, 87, 90, 98, 172, 439, 440, 444
    - lifetime, 71, 78, 88, 440
  - Square well potential, 513, 554, 563
  - Stability diagram of resonators, 211, 227
  - Stable resonator, 225
  - Standing wave, 27, 29, 185, 208
  - Stark effect, 305, 307
  - Stimulated
    - emission, 23, 89, 93, 98
    - Raman scattering, 285, 643
  - Stratified periodic medium, 501
  - Subband, 511–516
  - Superconductor, 176
  - Superlattice, 548
    - Bloch laser, 11, 59, 424, 591, 592, 594–606, 610, 612, 613, 615–619
    - oscillator, *see* Bloch laser
  - Survey of
    - lasers, 5, 6, 105, 109, 425
    - semiconductor lasers, 423–425, 538–543
  - Susceptibility, 146, 147, 635
  - SVEA, *see* Slowly varying envelope approximation (SVEA)
  - Synchronization, 144, 150, 151, 612
- T**
- Tailoring of semiconductors, 425
  - Tandem, 313
  - Tapered wiggler, 353
  - TE waves, 190
  - TEA laser, 280, 281, 284, 311
  - Telecommunication, 8
  - Terahertz
    - gap, 542
    - radiation, 263–265
  - Thermal equilibrium, 86, 430
  - Thin film dye laser, 311
  - Three-dimensional active medium, 75
  - Three-level laser, 60, 342
  - Threshold
    - condition, 34, 46, 122–124, 441, 448, 469, 470, 472, 571, 573
    - current, 442, 449, 470, 536
    - gain factor, 46
    - pump rate, 122, 123
    - resistance, 573
  - Thulium fiber laser, 302, 343
  - Tight binding model, 556, 563
  - Time domain spectroscopy, 263, 264
  - Titanium-sapphire, 148
    - laser, 5, 77–81, 105, 109–111, 292–295, 319–321, 323, 324, 643
  - TM waves, 190
  - Total reflection, 493, 624
  - Transfer matrix method, 497–499, 559–562
  - Transition
    - energy, 22, 67, 121, 393
    - frequency, 58, 70, 393
    - metal ions, 304
    - probability, 98
  - Transparency
    - condition, 23, 25
    - density, 25, 60, 107, 338, 340, 419, 432, 434, 442, 462, 535
    - frequency, 22, 337, 338, 466, 603
  - Transverse
    - electric wave, 190
    - magnetic wave, 190
    - mode, 198, 214–217, 223
    - pumping, 79
    - relaxation, 73, 163, 164
  - Tunable
    - laser, 78–80, 292–296, 352
    - semiconductor laser, 493, 518
  - Two-band laser, 61
  - Two-dimensional
    - active medium, 75, 112, 114
    - density of states, 419

gain characteristic, 113, 445, 462, 463  
 reduced density of states, 444  
 semiconductor, 418

Two-level  
 atomic system, 22, 401, 402  
 based laser, 58  
 laser, 61

Two-quasiband laser, 63, 633

**U**

Ultrashort snapshot, 261  
 Undulator, *see* Wiggler  
 Unipolar semiconductor laser, 421, 543  
 Unstable resonator, 225  
 Upconversion, 302  
 UV laser diode, 487

**V**

V factor, 30–32  
 Valence band, 417  
 Van der Pol  
 equation, 186, 586  
 oscillator, 584–588  
 VCSEL, 472  
 Vertical-cavity surface-emitting laser, 8, 44, 472, 473, 494  
 Vibronic  
 band, 295  
 energy levels, 77, 294, 302, 319–321, 323  
 laser, 77, 78, 280, 294–296, 304, 317–324  
 states, 294, 319–321, 323  
 Voigt profile, 112

**W**

Wannier  
 function, 617

Wave  
 equation, 183, 198, 554  
 function, 417, 511, 512, 522, 554  
 packet, 248, 249  
 vector, 18, 19, 199, 219

Waveguide, 193  
 Fabry-Perot resonator, 422

Wavelength, 4, 191, 199, 220  
 Whispering gallery mode, 493  
 White laser light, 259  
 Wiggler, 350  
 Work function, 632

**X**

X-ray  
 free-electron laser, 352, 353  
 laser, 312  
 SASE FEL, *see* SASE free-electron laser  
 SASE free-electron laser, 352

**Y**

YAG lasers, 296–298  
 Ytterbium-doped fiber laser, 300, 343

**Z**

Zero  
 dimensional active medium, 75  
 phonon line, 323  
 point energy, 513  
 ZnSe, 488  
 ZnTe, 264