Appendix A: Math for the Brewer

A.1 Introduction

This is a reference section for designing and estimating parameters of a brew. Keep in mind that while there is considerable science in brewing, there is also room for the art of brewing. There are a lot of things we can control, and there are a lot of things we understand, through science, that the brewers of yesteryear did not understand. But there is still enough variability in the overall process that even the brewer's mood might influence the final product—the "art".

A.2 Designing Your Brew

One of the best ways to start designing a beer is to try to create a particular style. A beer style will define things such as the starting, original gravity (OG), and the relative percentage of each ingredient's contribution to the gravity. The style might also specify the overall hop character and bitterness in the product. So, in this section, we will start with a simple style and use it as an example to illustrate how to calculate the needed materials. Let us assume our example style is defined as the following:

- Grain bill:
    - 85 % Pale 2-row
    - 10 % crystal 20L
    - 5 % Honey

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- OG 1.085
- Hops:
  - Flowery, spicy, and mildly citrusy, e.g., cascade or centennial
- Bitterness 30IBU

A.1.1 Volume

Since all calculations depend on the volume, we must first decide how much beer we are making. Let us assume we are interested in making a small test batch similar to what a homebrewer might make. A typical homebrew batch size might be 5 gallons, or about 19 L. When considering our batch volume, we also need to account for system losses after the boil, such as the volume of bitter wort left at the bottom of the boil which cannot be separated from the trub—the residual hops and hot-break proteins. So, we define the following variables:

- V_{Target} = the target volume of beer we want to put into the fermenter. For our example, we will use 19 L.
- L_{HH} = the volume of post boil, bitter wort which cannot be easily separated from the spent hops and hot break. With care, one should be able to keep this to less than 1 L.

So, for the example at hand, we will use 20 L when calculating expected gravities and bitterness concentrations.

This does not mean that we will only need 20 L of water on brew day as there are other losses for which we must account, such as evaporation of water during the boil. You never want to cover a wort boil to minimize this loss. (DMS!) Typical losses for 20 L batch are about 2 L per hour of boil (i.e., 10 % loss per hour). So,

- L_{evap} = losses to evaporation in the wort boil.

Again, there is room for art here. A faster, more vigorous boil will enhance Maillard reactions but will incur a higher loss rate. So, for a typical one-hour boil, we might need a volume of 22 L into the boil kettle.

Finally, there are losses to the grains after the mash and sparging. This last account of water will depend on the sparging method. Continuous sparging will require more water since there is always a full volume of water in the lauter tun. Batch sparging simply drains, refills, and drains (recall 1st and 2nd runnings). This sparging method is easiest to account for water without knowing the lauter tun volume. The spent grains we toss out are not dry. We might do our best to drain the lauter tun completely, but the grain, like a sponge, will hold back some water. Then, we need to factor in system losses throughout the process. So these water losses in a homebrewing setup using batch sparging will include the following:

- L_{grain} = loss to grain. This is the water that is left in the grain after sparging. We can start with a guess 0.5 of liter per kg of grain used. As we will see in a moment, a round number for the grain bill might be 10 kg.
- L_{DV} = other losses in the mash/sparging due to dead volume, for example, the false bottom in a mash tun as well as hoses. Again, this depends on system, but for discussion purposes, we will assume 1 L.

Accounting for all of these losses, the starting volume of water one might need is determined from

$$V_{\text{Start}} = V_{\text{Target}} + L_{\text{grain}} + L_{\text{evap}} + L_{\text{HH}} + L_{\text{DV}} \tag{A.1}$$

Using the example of volumes and assumptions above, we will need at least

$$V_{\text{Start}} = 19L + (0.5\frac{\text{L}}{\text{kg}} \cdot 10\text{ kg}) + (2\frac{\text{L}}{\text{h}} \cdot 1\text{ h}) + 1L + 1L = 28L$$

on brew day. Again, every system and brewer is different so this is just an estimate using reasonable assumptions. It is generally wise to add another 10–15 % to this volume just to be sure.

A.1.2 Designing the Grain Bill

Recall that we measure sugar concentration by measuring density. Density is defined as

$$\rho = \text{Mass}/\text{Volume} \tag{A.2}$$

In brewing, we measure specific gravity "gravities" for short, which are simply a density measurement relative to the density of water. Since water has a specific gravity of 1.000 g/mL, an OG of 1.085 g/mL for our recipe means that there is 0.085 "gravity points" of fermentable sugar to be dissolved *per liter of water*. We approach the grain bill design through the use of "total gravity points" (Gpts), defined as

$$\text{Gpts} = (\text{OG} - 1) \cdot 1000 \cdot \text{Volume} \tag{A.3}$$

or, using our example

$$\text{Gpts} = 85\frac{\text{Gpts}}{\text{L}} \cdot 20\text{ L} = 1700\text{ Gpts} \tag{A.4}$$

Table A.1 Maximum gravity points per kilogram per liter

Ingredient	Gpts
Sucrose (table sugar), Brown sugar	384
Lactose	384
Dextrose (corn sugar)	350
Belgium candi sugar, Molasses	250–292
Honey	250–292
2-row malt	317
6-row malt	292
Vienna malt, Munich malt	292
Crystal malt	283
Chocolate malt	234
Oatmeal (flaked)	267
Corn (flaked)	325

Note that this is essentially the total mass, in some undefined units, of fermentable sugars to be dissolved in water. We will determine each ingredient's contribution to the gravity, i.e, "extract potential", relative to sucrose. If we added pure sucrose to water, we would expect 384 gravity points per kilogram of sugar per liter of water (see Table A.1).

So, to determine the gravity points that a given ingredient will contribute, all we need to do is multiply by the extract potential of each grain. The extract potential of various malted grains is usually available from the malt vendor. However, most are in the vicinity of 70 %, and this will be a good number to use as an approximation.

The other thing we need to consider is brew house efficiency. This is something that must be determined for each setup by actually using the system. Actual efficiency must be determined several times before one really knows how efficient a particular process or equipment arrangement is. That said, a good starting point is to estimate an efficiency of 70 %. This is where some of the "art" of brewing comes in. There are a myriad of different equipment setup possibilities, compounded by the different approaches and methods that could be used by the brewer. Since we are making an estimate, 70 % will be fine for the moment. So the gravity points contributed by each ingredient are determined by multiplying the mass by the extract potential, the efficiency, and the gravity points.

$$\mathrm{Gpts}_{\mathrm{ingredient}} = \mathrm{mass}\,(\mathrm{kg}) \cdot \mathrm{extractPotential}\% \cdot \mathrm{efficiency}\% \cdot \frac{\mathrm{Gpts}}{\mathrm{kg}} \tag{A.5}$$

Rearranging,

$$\frac{\mathrm{Gpts}_{\mathrm{ingredient}}}{\mathrm{extractPotential}\% \cdot \mathrm{efficiency}\% \cdot \frac{\mathrm{Gpts}}{\mathrm{kg}}} = \mathrm{mass}\;\;(\mathrm{kg}) \tag{A.6}$$

Table A.2 Grain bill assumptions and calculated mass

Ingredient	Style%	Gpts	ExtPot%	Eff%	Gpts per kg	Required mass (kg)
Pale ale	85	1445	70	70	317	9.30
Crystal 10 L	10	170	70	70	283	1.23
Honey	5	85	100	100	270	0.31

We will use the ingredient percentages determined by the style to determine how many gravity points each will contribute to the total. For example, since the pale ale malt is 85 % of the recipe, we expect this ingredient to contribute

$$\text{Gpts}_{\text{PaleAle}} = 0.85 \cdot 1700 = 1445 \text{ Gpts} \tag{A.7}$$

and from this, we can find the needed mass of this recipe component. Table A.2 summarizes the rest of the calculations.

CHECKPOINT A.1
Why was 100 % used for both the extract potential and efficiency of the honey?

A.1.3 Hops

Beer bitterness is measured in International Bitterness units (IBUs). We can estimate this if we know the alpha acid content of the hops, and something called ``utilization'. Utilization basically accounts for the rate at which the alpha acid is isomerized during the boil and thus solubilized into the wort. The formula for predicting the IBU is

$$\text{IBU} = \frac{10 \cdot m \cdot \%\text{AA} \cdot \text{Utl}}{\text{Volume}} \tag{A.8}$$

where

M	is the mass in grams of hops,
%AA	is the percent of alpha acid in the hops, and
Utl	is the (decimal) utilization.

The utilization number depends primarily on boil time, but it also depends on the original gravity as well. The actual value of the utilization number can be calculated; however, given that our IBU calculation is an estimation, it is far easier to look up the utilization from a table or a graph with very little loss of accuracy. Just such a graph is shown in Fig. A.1.

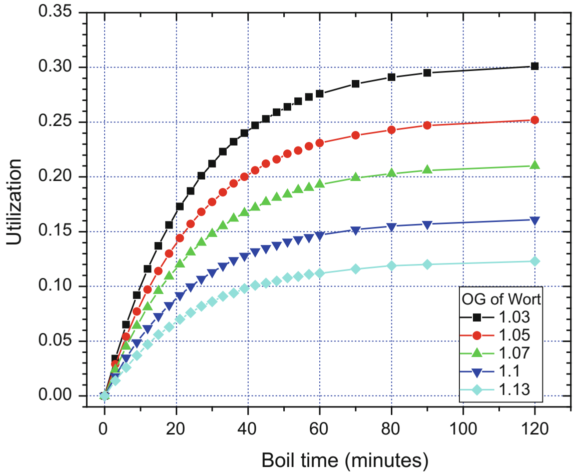

Fig. A.1 Tinseth's hop utilization as a function of boil time and original gravity (OG)

So, continuing with the above example, the question might be: What mass of Cascade hops (at 7 % alpha acid content) do we need to hit the target 30 IBU for our recipe? We are boiling for one hour. Our OG = 1.085 is not on the graph in Fig. A.1, but we can extrapolate between the 1.07 and 1.1 curve. Looking at the 60 min mark, we can estimate a utilization of Utl = 0.17. Inserting this into Eq. A.8, we find that we will need 50.4 g of Cascade hops boiled for 60 min to reach 30 IBU.

$$\begin{aligned} 30\ \mathrm{IBU} &= \frac{10 \cdot m \cdot 7 \cdot 0.17}{20\ \mathrm{L}} \\ m &= 50.4\ \mathrm{g} \end{aligned} \tag{A.9}$$

In practice, we would likely not want to add all of the hops to get the 30 IBU value in one batch. Remember that addition of hops can be done early in the boil to give bitterness, in the middle of the boil to provide flavor, or at the end of the boil to lend the aroma to the beer.

CHECKPOINT A.2

What would be the mass of hops added to a boil such that 80 % of the IBU value was obtained at 60 min, 15 % of the IBU value was generated at 15 min, and 5 % of the IBU value was obtained at 5 min?

A.1.4 Percent Alcohol by Volume (ABV)

Estimating the percent alcohol by volume (ABV) is as simple as measuring your original gravity (OG) (just before fermentation) and final gravity (FG), taken at the end of the fermentation process.

$$\mathrm{ABV}(\%) = (\mathrm{OG} - \mathrm{FG}) \cdot 131 \tag{A.10}$$

One can estimate the FG based on the type of yeast used and the starting OG. Yeasts are characterized by their apparent attenuation. By definition, the apparent attenuation is:

$$\mathrm{AA} = \frac{(\mathrm{OG} - \mathrm{FG})}{(\mathrm{OG} - 1)} \tag{A.11}$$

Actual attenuation is different from apparent attenuation. Attenuation basically refers to how much sugars the yeast ate. An (actual) attenuation of 75 % means that one-quarter of the original sugars remain; 50 %, half. Because the hydrometer responds to *overall* density, and alcohol has a very different density than sugar solution, we define an apparent attenuation based on gravity readings. So, rearranging the above, we can estimate the FG (and therefore the ABV) from

$$\mathrm{OG} - [\mathrm{AA} \cdot (\mathrm{OG} - 1)] = \mathrm{FG} \tag{A.12}$$

Yeast manufacturers will usually state the apparent attenuation on the package. Continuing our example, a typical apparent attenuation might be 65 %, or in the notation above, AA = 0.65. Then, we can estimate the final gravity and ABV

$$\begin{aligned} 1.085 - [0.65 \cdot (0.085)] &= \mathrm{FG} \\ 1.030 &= \mathrm{FG} \end{aligned} \tag{A.13}$$

$$\mathrm{ABV}(\%) = (1.085 - 1.030) \cdot 131 = 7.24\,\% \tag{A.14}$$

Note that this is an estimate to guide the brewer in designing the beer and predicting the ABV. One should always measure the actual OG and FG gravities when computing the final (and actual) ABV.

A.1.5 Color and SRM

Measuring SRM color by the homebrewer is impractical. Measuring SRM color with samples and color cards is extremely subjective and irreproducible.

Furthermore, this depends on the color of the ambient, illuminating light. Actual determination of color must be done analytically using absorption spectroscopy. The currently accepted method of reporting color in beer, as with any color-involved measurement in the industry, is using CIE $L^*a^*b^*$ coordinates. However, this is an unfamiliar measurement system to most homebrewers, and as such, the older SRM color measurement system is still common. Predicting SRM color is problematic for the homebrewer since the final color is very dependent on process and malts used.

That said, the final color can be estimated based on a recipe. The malting industry will publish the SRM color to characterize their malts. In general, the larger the SRM, the darker the malt. There does not exist an ab initio method of translating this information into a SRM color of the beer since it depends on (a) the quantity of grain used, (b) the volume of water, and (c) the overall process.

We still try to estimate the final color by borrowing the gravity unit idea from above; each component of the recipe will contribute to the color. Deemed “malt color units” (MCUs), each ingredient will contribute its share to the total and will depend its “color potential”, amount used, and volume of water. An MCU can be determined using the formula:

$$\mathrm{MCU} = \mathrm{SRM}_{\mathrm{Grain}} \cdot 8.35 \cdot \frac{\mathrm{Mass}_{(\mathrm{kg})}}{\mathrm{Volume}_{(\mathrm{liters})}} \tag{A.15}$$

The unfortunate thing about MCUs is that their contribution to the final color is not always a linear, one-to-one relationship as the gravity calculation was. This part of the calculation relies on the empirical work of others to make the final estimation. Before we illustrate each method, let us start with a recipe, Table A.3, as an example to use as we work through the math for each approach. For the recipe, we assume a volume of 24.6 L as the final volume.

Equivalence model (SRM < 10)—For SRM values less than 10, we simply add the MCUs and say that the final SRM is the MCU total,

$$\mathrm{SRM} = \mathrm{MCU}_{\mathrm{Total}} = \sum_{\mathrm{ingredient},i} \mathrm{MCU}_i \tag{A.16}$$

This approach is really only valid for lighter colored beers (SRM less than 10). Adding the MCUs from the example in Table A.3, we get SRM = 23.3, so we conclude that this approach may not be the best for our example.

Table A.3 Example recipe for calculating color

Ingredient	Mass (kg)	SRM (rating)	MCU
Pale 2-row	9.39	2	6.369
Cyrstal 30L	1.54	30	15.692
Rye	0.86	3	0.877
Corn grits	0.36	3	0.365

Linear model (9 < SRM < 20)—There are a few different linear models for the determination of SRM from MCU. Randy Mosher suggested one such calculation that roughly agrees with a range of SRM values. A modification of this method of calculation, attributed to Ray Daniels, scales and shifts the final SRM, but the model is still linear over the range from 9 to 20. The formula suggested is:

$$\mathrm{SRM} = 0.2 \cdot \mathrm{MCU}_{\mathrm{Total}} + 8 \tag{A.17}$$

and inserting the above recipe example, an SRM = 11.69 is predicted. It cannot be stressed enough that the final color of the beer is very much process dependent. So, these calculations are, at best, good to two significant figures. Thus, the example gives SRM = 12. The issue with this approach is that it gives unrealistic SRM color for small MCU totals, thus limiting the range of validity.

Power law—The power law approach, suggested by Dan Morey, tries to account for the nonlinearity in the observed data. The formula

$$\mathrm{SRM} = 1.49 \cdot \mathrm{MCU}_{\mathrm{total}}^{0.69} \tag{A.18}$$

provides the SRM for the sample based on the total of the ingredient values. Using the example recipe, this formula predicts an SRM = 13.

There is another way in which this approach could be interpreted. Let us assume that the SRM contribution component of each ingredient is linear, but the MCU conversion to SRM for each follows the above power law. So, then the final contributes

$$\mathrm{SRM} = \sum_{\mathrm{ingredient},i} 1.49 \cdot \mathrm{MCU}_i^{0.69} \tag{A.19}$$

Following this approach, the color prediction for the example recipe is SRM = 17.

So which of the above methods are “correct”? Without an absorption spectrometer, it is impossible to verify any of the above calculations. These only serve as estimations to guide the brewer.

A.1.6 Misc—Strike Water Temperature

Chapter 8 introduced us to the science behind the energy balance equations to determine strike water temperature, the temperature of the hot liquor added initially to the grist to form the mash. In summary, we just set the thermal energy gained by the grains and any water that is already present equal to the energy loss by the hot, strike water. In this example, we assume a single infusion mash in which only one addition of hot strike water is added to dry grains. For the example, we will also include the thermal mass of a Styrofoam mash tun such as might be used in our homebrewer example.

The change in heat energy for the system can be described mathematically as follows:

$$\Delta Q_{\text{water}} + \Delta Q_{\text{grain}} + \Delta Q_{\text{tun}} = 0 \tag{A.20}$$

Inserting the specific heats and temperatures (from $\Delta Q = m\, c\, \Delta T$), this becomes:

$$m_{\text{water}} \cdot c_{\text{water}} \cdot \left(T_f - T_i\right)_{\text{water}} + m_{\text{grain}} \cdot c_{\text{grain}} \cdot \left(T_f - T_i\right)_{\text{grain}} + m_{\text{tun}} \cdot c_{\text{tun}} \cdot \left(T_f - T_i\right)_{\text{tun}} = 0 \tag{A.21}$$

The density of water is $\rho = 1000\ \text{kg/m}^3$, or 1 kg/L, and $m_{\text{water}} = V_{\text{water}}\, \rho_{\text{water}}$. Inserting this and the specific heats for all three components (see Table 8.3):

$$\begin{aligned} &V_{\text{water}} \cdot 1\frac{\text{kg}}{\text{L}} \cdot 4184\frac{\text{J}}{\text{kg °C}} \cdot \left(T_f - T_i\right)_{\text{water}} + m_{\text{grain}} \cdot 1674\frac{\text{J}}{\text{kg °C}} \cdot \left(T_f - T_i\right)_{\text{grain}} \\ &\quad + m_{\text{tun}} \cdot 1300\frac{\text{J}}{\text{kg °C}} \cdot \left(T_f - T_i\right)_{\text{tun}} = 0 \end{aligned} \tag{A.22}$$

The variable T_f is the final equilibrium temperature of the system and is the desired mash temperature; $T_f = T_{\text{mash}}$. The variable T_i for the grain and the mash tun is the beginning temperature; let us assume room temperature $T_i = 20$ °C. The variable T_i for the water is the desired strike temperature; we will call it T_{strike}.

$$\begin{aligned} &V_{\text{water}} \cdot 1\frac{\text{kg}}{\text{L}} \cdot 4184\frac{\text{J}}{\text{kg °C}} \cdot \left(T_{\text{mash}} - T_{\text{strike}}\right)_{\text{water}} + m_{\text{grain}} \cdot 1674\frac{\text{J}}{\text{kg °C}} \cdot \left(T_{\text{mash}} - 20\ \text{°C}\right)_{\text{grain}} \\ &\quad + m_{\text{tun}} \cdot 1300\frac{\text{J}}{\text{kg °C}} \cdot \left(T_{\text{mash}} - 20\ \text{°C}\right)_{\text{tun}} = 0 \end{aligned} \tag{A.23}$$

So now let us apply some specific numbers as an example. This example will also take us through some unit conversions. A brewer desires a mash temperature of 155 °F (68 °C) with 8 lbs (3.6 kg) of grain using a water-to-grist ratio of 1.4 qts/lb. The mash tun is a 3 lb (1.4 kg) container. The volume of water expected in the mash tun is:

$$8\ \text{lbs} \cdot 1.4\frac{\text{qts}}{\text{lb}} = 11.2\ \text{qts} \cdot \frac{1\ \text{L}}{1.06\ \text{qts}} = 10.6\ \text{L} \tag{A.24}$$

Putting this information into the equation,

$$10.6l \cdot 1\frac{\text{kg}}{\text{L}} \cdot 4184\frac{\text{J}}{\text{kg }^{\circ}\text{C}} \cdot (68\ ^{\circ}\text{C} - T_{\text{strike}})_{\text{water}} + 3.6\ \text{kg} \cdot 1674\frac{\text{J}}{\text{kg }^{\circ}\text{C}} \cdot (68\ ^{\circ}\text{C} - 20\ ^{\circ}\text{C})_{\text{grain}} \\ + 1.4\ \text{kg} \cdot 1300\frac{\text{J}}{\text{kg }^{\circ}\text{C}} \cdot (68\ ^{\circ}\text{C} - 20\ ^{\circ}\text{C})_{\text{tun}} = 0 \tag{A.25}$$

Then,

$$44{,}350 \cdot (68\ ^{\circ}\text{C} - T_{\text{strike}}) + 289{,}267 + 87{,}360 = 0 \tag{A.26}$$

or

$$\begin{aligned} 3{,}015{,}800 \cdot -44{,}350T_{\text{strike}} &= -376{,}627 \\ T_{\text{strike}} &= 76.5\ ^{\circ}\text{C} \end{aligned} \tag{A.27}$$

The calculations outlined in this appendix are written with the homebrewer in mind. However, by changing the volume of the beer being designed, the same calculations work to arrive at a recipe for any size brewery. The largest variable in the conversion from small scale to large is efficiency. Knowing the exact values for the gravity points due to a specific malt (the values in Table A.1 are approximate and not specific to a particular malt variety) can also help provide a much more accurate calculation of the recipe.

Appendix B: R134a Refrigerant Data

B.1 Introduction

Refrigerant R134a—Tetraflouroethane data including specific volume (density), specific energy, specific enthalpy, and specific entropy. Source of data: NIST Chemistry WebBook—http://webbook.nist.gov/chemistry/fluid/ (Tables B.1, B.2 and B.3).

Table B.1 Saturated, organized by temperature

Pressure (kPa)	Temp (°C)	Volume (m^3/kg)		Energy (kJ/kg)		Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		v_f	v_g	u_f	u_g	h_f	h_g	s_f	s_g
−40	51.2	7.05E−04	0.3611	−0.04	207.37	0.00	225.86	0.0000	0.9687
−36	62.9	7.11E−04	0.2977	4.99	209.66	5.04	228.39	0.0214	0.9632
−32	76.7	7.17E−04	0.2473	10.05	211.96	10.10	230.92	0.0425	0.9582
−28	92.7	7.23E−04	0.2068	15.13	214.26	15.20	233.43	0.0634	0.9536
−26	101.7	7.27E−04	0.1896	17.69	215.41	17.76	234.68	0.0738	0.9515
−24	111.3	7.30E−04	0.1741	20.25	216.55	20.33	235.93	0.0841	0.9495
−22	121.7	7.33E−04	0.1601	22.82	217.70	22.91	237.17	0.0944	0.9476
−20	132.7	7.36E−04	0.1474	25.39	218.85	25.49	238.41	0.1046	0.9457
−18	144.6	7.40E−04	0.1359	27.98	219.99	28.09	239.64	0.1148	0.9440
−16	157.3	7.43E−04	0.1255	30.57	221.13	30.69	240.87	0.1249	0.9423
−14	170.8	7.46E−04	0.1161	33.17	222.27	33.30	242.09	0.1350	0.9407
−12	185.2	7.50E−04	0.1074	35.78	223.41	35.92	243.31	0.1451	0.9392
−10	200.6	7.54E−04	0.0996	38.40	224.54	38.55	244.52	0.1550	0.9377

(continued)

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Table B.1 (continued)

Pressure (kPa)	Temp (°C)	Volume (m^3/kg)		Energy (kJ/kg)		Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		v_f	v_g	u_f	u_g	h_f	h_g	s_f	s_g
−8	216.9	7.57E−04	0.0924	41.03	225.67	41.19	245.72	0.1650	0.9364
−6	234.3	7.61E−04	0.0859	43.67	226.80	43.84	246.92	0.1749	0.9351
−4	252.7	7.65E−04	0.0799	46.31	227.93	46.50	248.11	0.1848	0.9338
−2	272.2	7.68E−04	0.0744	48.97	229.05	49.17	249.29	0.1946	0.9326
0	292.8	7.72E−04	0.0693	51.63	230.17	51.86	250.46	0.2044	0.9315
2	314.6	7.76E−04	0.0647	54.30	231.28	54.55	251.62	0.2142	0.9304
4	337.7	7.80E−04	0.0604	56.99	232.39	57.25	252.78	0.2239	0.9294
6	362.0	7.85E−04	0.0564	59.68	233.49	59.97	253.92	0.2336	0.9284
8	387.6	7.89E−04	0.0528	62.39	234.58	62.69	255.05	0.2432	0.9274
12	443.0	7.97E−04	0.0463	67.83	236.76	68.19	257.29	0.2625	0.9256
16	504.3	8.07E−04	0.0408	73.32	238.91	73.73	259.47	0.2816	0.9240
20	571.7	8.16E−04	0.0360	78.86	241.02	79.32	261.60	0.3006	0.9224
24	645.8	8.26E−04	0.0319	84.45	243.11	84.98	263.68	0.3196	0.9210
26	685.4	8.31E−04	0.0300	87.26	244.13	87.83	264.70	0.3290	0.9203
28	726.9	8.37E−04	0.0283	90.09	245.15	90.70	265.69	0.3385	0.9196
30	770.2	8.42E−04	0.0266	92.93	246.16	93.58	266.67	0.3479	0.9189
32	815.4	8.48E−04	0.0251	95.79	247.15	96.48	267.64	0.3573	0.9182
34	862.6	8.54E−04	0.0237	98.66	248.13	99.40	268.58	0.3667	0.9175
36	911.9	8.60E−04	0.0224	101.55	249.10	102.33	269.50	0.3761	0.9168
38	963.2	8.66E−04	0.0211	104.46	250.05	105.29	270.41	0.3855	0.9162
40	1016.6	8.72E−04	0.0200	107.38	250.99	108.27	271.28	0.3949	0.9155
42	1072.2	8.79E−04	0.0189	110.32	251.91	111.26	272.14	0.4043	0.9147
44	1130.1	8.85E−04	0.0178	113.28	252.81	114.28	272.97	0.4136	0.9140
48	1252.9	9.00E−04	0.0160	119.26	254.56	120.39	274.55	0.4324	0.9125
52	1385.4	9.15E−04	0.0143	125.33	256.23	126.60	276.01	0.4513	0.9108
56	1528.2	9.32E−04	0.0128	131.49	257.79	132.92	277.32	0.4702	0.9089
60	1681.8	9.50E−04	0.0114	137.76	259.24	139.36	278.49	0.4892	0.9068
70	2116.8	1.00E−03	0.0087	154.01	262.19	156.14	280.51	0.5376	0.9000
80	2633.2	1.08E−03	0.0064	171.41	263.69	174.25	280.67	0.5880	0.8894
90	3244.2	1.19E−03	0.0046	190.91	262.30	194.78	277.27	0.6434	0.8706
100	3972.4	1.54E−03	0.0027	219.05	248.89	225.15	259.54	0.7232	0.8153
101.06	4059.1	1.95E−03	0.0020	233.57	233.57	241.49	241.49	0.7665	0.7665

Table B.2 Saturated, organized by pressure

Pressure (kPa)	Temp (°C)	Volume (m^3/kg)		Energy (kJ/kg)		Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		v_f	v_g	u_f	u_g	h_f	h_g	s_f	s_g
60	−36.9	7.10E−04	0.3112	3.8	209.1	3.9	227.8	0.0164	0.9645
80	−31.1	7.19E−04	0.2376	11.2	212.5	11.2	231.5	0.0472	0.9572
100	−26.4	7.26E−04	0.1926	17.2	215.2	17.3	234.5	0.0720	0.9519
120	−22.3	7.32E−04	0.1621	22.4	217.5	22.5	237.0	0.0928	0.9478
140	−18.8	7.38E−04	0.1402	27.0	219.6	27.1	239.2	0.1110	0.9446
160	−15.6	7.44E−04	0.1235	31.1	221.4	31.2	241.1	0.1270	0.9420
180	−12.7	7.49E−04	0.1104	34.9	223.0	35.0	242.9	0.1415	0.9397
200	−10.1	7.53E−04	0.0999	38.3	224.5	38.5	244.5	0.1547	0.9378
220	−7.6	7.58E−04	0.0912	41.5	225.9	41.7	245.9	0.1668	0.9361
240	−5.4	7.62E−04	0.0839	44.5	227.2	44.7	247.3	0.1780	0.9347
260	−3.2	7.66E−04	0.0777	47.3	228.4	47.5	248.6	0.1885	0.9333
280	−1.2	7.70E−04	0.0724	50.0	229.5	50.2	249.7	0.1984	0.9322
300	0.7	7.74E−04	0.0677	52.5	230.5	52.8	250.9	0.2077	0.9311
320	2.5	7.77E−04	0.0636	54.9	231.5	55.2	251.9	0.2165	0.9301
340	4.2	7.81E−04	0.0600	57.3	232.5	57.5	252.9	0.2248	0.9293
360	5.8	7.84E−04	0.0567	59.5	233.4	59.8	253.8	0.2328	0.9284
400	8.9	7.91E−04	0.0512	63.7	235.1	64.0	255.6	0.2477	0.9270
500	15.7	8.06E−04	0.0411	73.0	238.8	73.4	259.3	0.2803	0.9241
600	21.6	8.20E−04	0.0343	81.0	241.9	81.5	262.4	0.3081	0.9219
700	26.7	8.33E−04	0.0294	88.3	244.5	88.8	265.1	0.3324	0.9200
800	31.3	8.46E−04	0.0256	94.8	246.8	95.5	267.3	0.3541	0.9184
900	35.5	8.58E−04	0.0227	100.9	248.9	101.6	269.3	0.3739	0.9170
1000	39.4	8.70E−04	0.0203	106.5	250.7	107.4	271.0	0.3920	0.9157

(continued)

Table B.2 (continued)

Pressure (kPa)	Temp (°C)	Volume (m^3/kg)		Energy (kJ/kg)		Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		v_f	v_g	u_f	u_g	h_f	h_g	s_f	s_g
1200	46.3	8.94E−04	0.0167	116.7	253.8	117.8	273.9	0.4245	0.9131
1400	52.4	9.17E−04	0.0141	126.0	256.4	127.3	276.2	0.4533	0.9106
1600	57.9	9.40E−04	0.0121	134.5	258.5	136.0	277.9	0.4792	0.9080
1800	62.9	9.64E−04	0.0106	142.4	260.2	144.1	279.2	0.5031	0.9051
2000	67.5	9.89E−04	0.0093	149.8	261.6	151.8	280.1	0.5252	0.9020
2500	77.6	1.06E−03	0.0069	167.1	263.5	169.7	280.9	0.5755	0.8925
3000	86.2	1.14E−03	0.0053	183.1	263.4	186.6	279.2	0.6215	0.8792

Table B.3 Superheated vapor temperature in parenthesis is the saturation temperature at the indicated pressure

Temp (°C)	P = 0.06 MPa (−36.9 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
−20	0.3361	220.6	240.8	1.018
−10	0.3505	227.6	248.6	1.048
0	0.3648	234.7	256.5	1.077
10	0.3789	241.9	264.7	1.107
20	0.3930	249.4	272.9	1.135
30	0.4071	257.0	281.4	1.164
40	0.4210	264.7	290.0	1.192
50	0.4350	272.6	298.7	1.219
60	0.4488	280.7	307.7	1.246
70	0.4627	289.0	316.8	1.273
80	0.4765	297.4	326.0	1.300
90	0.4903	306.0	335.4	1.326
100	0.5041	314.8	345.0	1.352
Temp (°C)	P = 0.10 MPa (−26.4 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
−20	0.1984	219.7	239.5	0.972
−10	0.2074	226.8	247.5	1.003
0	0.2163	234.0	255.6	1.033
10	0.2251	241.3	263.8	1.063
20	0.2337	248.8	272.2	1.092
30	0.2423	256.5	280.7	1.120
40	0.2509	264.3	289.3	1.149
50	0.2594	272.2	298.2	1.176
60	0.2678	280.4	307.1	1.204
70	0.2763	288.6	316.3	1.231
80	0.2847	297.1	325.6	1.257
90	0.2930	305.7	335.0	1.284
100	0.3014	314.5	344.6	1.310
Temp (°C)	P = 0.14 MPa (−18.8 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
−20	0.1402	219.6	239.2	0.945
−10	0.1461	225.9	246.4	0.972
0	0.1526	233.2	254.6	1.003
10	0.1591	240.7	262.9	1.033
20	0.1654	248.2	271.4	1.062
30	0.1717	255.9	280.0	1.091

(continued)

Table B.3 (continued)

40	0.1780	263.8	288.7	1.120
50	0.1841	271.8	297.6	1.147
60	0.1903	280.0	306.6	1.175
70	0.1964	288.3	315.8	1.202
80	0.2024	296.8	325.1	1.229
90	0.2085	305.4	334.6	1.255
100	0.2145	314.2	344.2	1.282
Temp (°C)	P = 0.18 MPa (−12.7 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
−10	0.1119	225.0	245.2	0.948
0	0.1172	232.5	253.6	0.980
10	0.1224	240.0	262.0	1.010
20	0.1275	247.6	270.6	1.040
30	0.1325	255.4	279.3	1.069
40	0.1374	263.3	288.1	1.098
50	0.1423	271.4	297.0	1.126
60	0.1472	279.6	306.1	1.153
70	0.1520	287.9	315.3	1.181
80	0.1567	296.4	324.6	1.207
90	0.1615	305.1	334.1	1.234
100	0.1662	313.9	343.8	1.260
Temp (°C)	P = 0.20 MPa (−10.1 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg.K)
−10				
0	0.1048	232.1	253.1	0.970
10	0.1096	239.7	261.6	1.001
20	0.1142	247.4	270.2	1.030
30	0.1187	255.2	278.9	1.060
40	0.1232	263.1	287.7	1.088
50	0.1277	271.2	296.7	1.116
60	0.1321	279.4	305.8	1.144
70	0.1364	287.7	315.0	1.171
80	0.1407	296.3	324.4	1.198
90	0.1451	304.9	333.9	1.225
100	0.1493	313.7	343.6	1.251

(continued)

Table B.3 (continued)

Temp (°C)	P = 0.24 MPa (−5.4 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
−10				
0	0.0862	231.3	252.0	0.952
10	0.0903	239.0	260.7	0.983
20	0.0942	246.8	269.4	1.013
30	0.0981	254.6	278.2	1.043
40	0.1019	262.6	287.1	1.072
50	0.1057	270.7	296.1	1.100
60	0.1094	279.0	305.2	1.128
70	0.1131	287.4	314.5	1.156
80	0.1168	295.9	323.9	1.183
90	0.1204	304.6	333.5	1.209
100	0.1240	313.5	343.2	1.236
Temp (°C)	P = 0.28 MPa (−1.2 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
10	0.0765	238.3	259.7	0.968
20	0.0800	246.1	268.5	0.999
30	0.0834	254.1	277.4	1.029
40	0.0867	262.1	286.4	1.058
50	0.0900	270.3	295.5	1.086
60	0.0932	278.6	304.7	1.114
70	0.0964	287.0	314.0	1.142
80	0.0996	295.6	323.5	1.169
90	0.1028	304.3	333.1	1.196
100	0.1059	313.2	342.8	1.222
110	0.1090	322.2	352.7	1.248
120	0.1121	331.3	362.7	1.274
Temp (°C)	P = 0.32 MPa (2.5 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
10	0.0661	237.5	258.7	0.954
20	0.0693	245.5	267.7	0.986
30	0.0723	253.5	276.7	1.016
40	0.0753	261.6	285.7	1.045
50	0.0782	269.8	294.9	1.074
60	0.0811	278.2	304.1	1.102

(continued)

Table B.3 (continued)

70	0.0839	286.6	313.5	1.130
80	0.0868	295.2	323.0	1.157
90	0.0895	304.0	332.6	1.184
100	0.0923	312.9	342.4	1.211
110	0.0950	321.9	352.3	1.237
120	0.0977	331.1	362.4	1.263
Temp (°C)	P = 0.40 MPa (8.9 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
10	0.0515	236.0	256.6	0.931
20	0.0542	244.2	265.9	0.963
30	0.0568	252.4	275.1	0.994
40	0.0593	260.6	284.3	1.024
50	0.0617	268.9	293.6	1.053
60	0.0641	277.3	303.0	1.081
70	0.0664	285.9	312.4	1.109
80	0.0687	294.5	322.0	1.137
90	0.0710	303.3	331.7	1.164
100	0.0733	312.3	341.6	1.191
110	0.0755	321.3	351.5	1.217
120	0.0777	330.6	361.6	1.243
Temp (°C)	P = 0.50 MPa (15.7 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
20	0.0421	242.4	263.5	0.938
30	0.0443	250.8	273.0	0.970
40	0.0465	259.3	282.5	1.001
50	0.0485	267.7	292.0	1.031
60	0.0505	276.3	301.5	1.060
70	0.0524	284.9	311.1	1.088
80	0.0543	293.6	320.8	1.116
90	0.0562	302.5	330.6	1.144
100	0.0581	311.5	340.5	1.171
110	0.0599	320.6	350.6	1.197
120	0.0617	329.9	360.7	1.223
130	0.0635	339.3	371.0	1.249
140	0.0653	348.8	381.5	1.275

(continued)

Table B.3 (continued)

Temp (°C)	P = 0.60 MPa (21.6 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
20				
30	0.0360	249.2	270.8	0.950
40	0.0379	257.9	280.6	0.982
50	0.0397	266.5	290.3	1.012
60	0.0414	275.2	300.0	1.042
70	0.0431	283.9	309.7	1.071
80	0.0447	292.7	319.6	1.099
90	0.0463	301.7	329.5	1.126
100	0.0479	310.7	339.5	1.154
110	0.0495	319.9	349.6	1.180
120	0.0510	329.2	359.8	1.207
130	0.0525	338.7	370.2	1.233
140	0.0540	348.3	380.7	1.258
Temp (°C)	P = 0.70 MPa (26.7 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
20				
30	0.0300	247.5	268.5	0.931
40	0.0317	256.4	278.6	0.964
50	0.0333	265.2	288.5	0.995
60	0.0349	274.0	298.4	1.026
70	0.0364	282.9	308.3	1.055
80	0.0378	291.8	318.3	1.084
90	0.0393	300.8	328.3	1.111
100	0.0406	310.0	338.4	1.139
110	0.0420	319.2	348.6	1.166
120	0.0434	328.6	358.9	1.192
130	0.0447	338.0	369.3	1.219
140	0.0460	347.7	379.9	1.244
Temp (°C)	P = 0.80 MPa (31.3 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
40	0.0270	254.8	276.5	0.948
50	0.0285	263.9	286.7	0.980
60	0.0300	272.8	296.8	1.011
70	0.0313	281.8	306.9	1.041

(continued)

Table B.3 (continued)

80	0.0327	290.8	317.0	1.070
90	0.0339	300.0	327.1	1.098
100	0.0352	309.2	337.3	1.126
110	0.0364	318.5	347.6	1.153
120	0.0376	327.9	358.0	1.180
130	0.0388	337.4	368.5	1.206
140	0.0400	347.1	379.1	1.232
150	0.0411	356.9	389.8	1.258
160	0.0423	366.8	400.6	1.283
Temp (°C)	P = 0.90 MPa (35.5 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
40	0.0234	253.1	274.2	0.933
50	0.0248	262.4	284.8	0.966
60	0.0261	271.6	295.1	0.998
70	0.0274	280.7	305.4	1.028
80	0.0286	289.9	315.6	1.057
90	0.0298	299.1	325.9	1.086
100	0.0310	308.3	336.2	1.114
110	0.0321	317.7	346.6	1.141
120	0.0332	327.2	357.0	1.168
130	0.0342	336.8	367.6	1.195
140	0.0353	346.5	378.2	1.221
150	0.0363	356.3	389.0	1.247
160	0.0374	366.2	399.9	1.272
Temp (°C)	P = 1.00 MPa (39.4 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
40	0.0204	251.3	271.7	0.918
50	0.0218	260.9	282.7	0.953
60	0.0231	270.3	293.4	0.985
70	0.0243	279.6	303.9	1.016
80	0.0254	288.9	314.3	1.046
90	0.0265	298.2	324.7	1.075
100	0.0276	307.5	335.1	1.103
110	0.0286	316.9	345.5	1.131
120	0.0296	326.5	356.1	1.158
130	0.0306	336.1	366.7	1.185

(continued)

Table B.3 (continued)

140	0.0316	345.9	377.4	1.211
150	0.0325	355.7	388.2	1.237
160	0.0335	365.7	399.2	1.262
Temp (°C)	P = 1.2 MPa (46.3 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
50	0.0172	257.6	278.3	0.927
60	0.0184	267.6	289.6	0.961
70	0.0195	277.2	300.6	0.994
80	0.0205	286.8	311.4	1.025
90	0.0215	296.3	322.1	1.055
100	0.0224	305.8	332.7	1.084
110	0.0233	315.4	343.4	1.112
120	0.0242	325.0	354.1	1.139
130	0.0251	334.8	364.9	1.166
140	0.0259	344.6	375.7	1.193
150	0.0268	354.6	386.7	1.219
160	0.0276	364.6	397.7	1.245
170	0.0284	374.8	408.8	1.270
180	0.0292	385.1	420.1	1.295
Temp (°C)	P = 1.4 MPa (52.4 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
50				
60	0.0150	264.5	285.5	0.939
70	0.0161	274.6	297.1	0.973
80	0.0170	284.5	308.3	1.006
90	0.0179	294.3	319.4	1.036
100	0.0188	304.0	330.3	1.066
110	0.0196	313.8	341.2	1.095
120	0.0204	323.6	352.1	1.123
130	0.0212	333.4	363.0	1.150
140	0.0219	343.4	374.0	1.177
150	0.0226	353.4	385.1	1.204
160	0.0234	363.5	396.2	1.230
170	0.0241	373.8	407.4	1.255
180	0.0248	384.1	418.8	1.281

(continued)

Table B.3 (continued)

Temp (°C)	P = 1.6 MPa (57.9 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
50				
60	0.0124	260.9	280.7	0.916
70	0.0134	271.8	293.3	0.954
80	0.0144	282.1	305.1	0.987
90	0.0152	292.2	316.5	1.019
100	0.0160	302.1	327.8	1.050
110	0.0168	312.1	338.9	1.080
120	0.0175	322.0	350.0	1.108
130	0.0182	332.0	361.1	1.136
140	0.0189	342.1	372.3	1.163
150	0.0195	352.2	383.5	1.190
160	0.0202	362.4	394.7	1.216
170	0.0208	372.7	406.0	1.242
180	0.0215	383.1	417.4	1.268
Temp (°C)	P = 1.8 MPa (62.9 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
70	0.0113	416.7	437.1	1.729
80	0.0123	427.6	449.7	1.766
90	0.0131	438.1	461.6	1.799
100	0.0139	448.3	473.3	1.831
110	0.0146	458.5	484.7	1.861
120	0.0152	468.6	496.0	1.890
130	0.0159	478.7	507.3	1.918
140	0.0165	488.9	518.6	1.946
150	0.0171	499.1	529.9	1.973
160	0.0177	509.4	541.3	2.000
170	0.0183	519.8	552.7	2.026
180	0.0189	530.3	564.3	2.051
Temp (°C)	P = 2.0 MPa (67.5 °C)			
	Volume v (m^3/kg)	Energy u (kJ/kg)	Enthalpy h (kJ/kg)	Entropy s (kJ/kg K)
70	0.0096	412.9	432.1	1.709
80	0.0105	424.7	445.8	1.748
90	0.0114	435.7	458.4	1.783
100	0.0121	446.2	470.5	1.816

(continued)

Table B.3 (continued)

110	0.0128	456.6	482.2	1.847
120	0.0134	466.9	493.8	1.877
130	0.0141	477.2	505.3	1.906
140	0.0146	487.5	516.8	1.934
150	0.0152	497.8	528.2	1.961
160	0.0158	508.2	539.7	1.988
170	0.0163	518.7	551.3	2.015
180	0.0168	529.2	562.9	2.041

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