Appendix A

This appendix covers some mathematical details of relativistic cosmology, and is suitable for readers who are familiar with calculus.

The Friedmann Equation

In Chap. 7 we introduced the concept of a scale factor $a(t)$, which is defined as the factor by which distances between commoving objects in a homogeneous and isotropic universe at cosmic time t differ from their distances at the present time t_0. Thus, by definition, $a(t_0) = 1$. We also showed that the expansion rate of the universe, as characterized by the Hubble parameter, is related to the scale factor as:

$$H = \frac{\dot{a}(t)}{a(t)}, \tag{A.1}$$

where an overdot denotes the rate of change, or the time derivative, $\dot{a} = \frac{da}{dt}$.

The magnitude of H is related to the energy density by the Friedmann equation, which we shall now derive.

Let us consider a comoving spherical region of radius $R = a(t)R_0$, as we did in Sect. 8.1. The total energy of a test particle that lies on the boundary of the sphere is given by (see Eq. 8.1)

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mR^2\left(H^2 - \frac{8\pi}{3}G\rho\right). \tag{A.2}$$

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Here, $v = HR$ is the particle's velocity, $M = \frac{4\pi}{3}R^3\rho$ is the mass of matter enclosed by the sphere, and ρ is the mass density. We shall assume that the universe is dominated by matter, so the force due to pressure is negligible; then M does not change during the course of expansion.

Since energy is conserved, the magnitude of the right-hand side of Eq. (A.2) at any time is the same as it is at present; hence we can write

$$R^2\left(H^2 - \frac{8\pi}{3}G\rho\right) = R_0^2\left(H_0^2 - \frac{8\pi}{3}G\rho_0\right),$$

where zero subscripts indicate quantities evaluated at the present time. Dividing by R^2 (where $R = a(t)R_0$) and factoring out H_0^2 on the right-hand side, we can rewrite this equation as

$$H^2 - \frac{8\pi G\rho}{3} = \frac{H_0^2}{a^2}(1 - \Omega_0). \tag{A.3}$$

Here, $\Omega_0 = \frac{\rho_0}{\rho_{c0}}$ is the present value of the density parameter $\Omega = \rho/\rho_c$, where

$$\rho_c = \frac{3H^2}{8\pi G} \tag{A.4}$$

is the critical density.

Equation (A.3) is the famous Friedmann equation, which is a statement of energy conservation. Note that although we derived it here assuming that pressure is negligible (which is not so at early and late times), it is valid in general. Another useful form of the Friedmann equation is obtained by multiplying Eq. (A.3) with a^2. This gives

$$\dot{a}^2 = \frac{8\pi G\rho}{3}a^2 + H_0^2(1 - \Omega_0). \tag{A.5}$$

Solutions in Different Cosmic Epochs

The observed density of the universe is very close to the critical density. Hence, to a good approximation we can set $\Omega_0 = 1$. Then the Friedmann equation simplifies to

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2. \tag{A.6}$$

In general, the mass density ρ includes contributions from matter (atomic and dark matter), radiation, and the vacuum; thus

$$\rho = \rho_m + \rho_r + \rho_v \tag{A.7}$$

The three contributions have different a-dependence,[1]

$$\rho_m = \frac{\rho_{m0}}{a^3}, \quad \rho_r = \frac{\rho_{r0}}{a^4}, \quad \rho_v = \text{const}, \tag{A.8}$$

and come to dominate the universe at different epochs, as we discussed in Sect. 11.7. Their present magnitudes are

$$\rho_{m0} = 2.7 \times 10^{-27}\text{kg/m}^3, \quad \rho_{r0} = 7.9 \times 10^{-31}\text{kg/m}^3,$$
$$\rho_v = 6.0 \times 10^{-27}\text{kg/m}^3$$

We can use Eq. (A.8) to estimate the redshift z_{eq} at the end of the radiation era, when radiation and matter densities are equal:

$$\rho_m(t_{eq}) = \rho_r(t_{eq}) \Rightarrow \frac{\rho_{m0}}{a^3(t_{eq})} = \frac{\rho_{r0}}{a^4(t_{eq})}. \tag{A.9}$$

Simplifying,

$$\frac{\rho_{m0}}{\rho_{r0}} = \frac{1}{a(t_{eq})} = z_{eq} + 1, \tag{A.10}$$

where we have used $\frac{1}{a(t)} = z + 1$ (this is Eq. 7.8). Using the measured values of ρ_{m0} and ρ_{r0}, we find $z_{eq} \approx 3400$.

We can also use Eq. (A.8) to calculate the redshift z_v at the end of the matter era, when the vacuum energy density begins to dominate, $\rho_v = \rho_m$. Using Eqs. (A.8) and (7.8), we can write

$$\rho_v = \rho_{m0}(1 + z_v)^3 \tag{A.11}$$

Solving this for z_v we obtain $z_v = 0.30$.

[1] Neutrinos are nearly massless particles, and their density scales in the same way as the density of photons, $\rho \propto a^{-4}$. The neutrino density is therefore included in the radiation density ρ_r.

Radiation Era

When the universe is dominated by radiation, the Friedmann equation (A.6) takes the form

$$\dot{a}^2 = \frac{8\pi G\rho_{r0}}{3a^2} \tag{A.12}$$

where we have used $\rho \approx \rho_r$ with ρ_r from Eq. (A.8). Taking the square root, we have

$$a\frac{\mathrm{d}a}{\mathrm{d}t} = \left(\frac{8\pi G\rho_{r0}}{3}\right)^{1/2} \tag{A.13}$$

This has the solution

$$a(t) = C_r t^{1/2} \tag{A.14}$$

where the constant coefficient C_r is given by

$$C_r = \left(\frac{32\pi G\rho_{r0}}{3}\right)^{1/4} \tag{A.15}$$

Substituting this solution in Eq. (A.1), we find the Hubble parameter

$$H = \frac{1}{2t} \tag{A.16}$$

The density can now be found from

$$\rho = \frac{3H^2}{8\pi G} = \frac{3}{32\pi Gt^2} \tag{A.17}$$

The temperature at time t is

$$T = \frac{T_0}{a(t)} = \frac{T_0}{C_r t^{1/2}} \tag{A.18}$$

where $T_0 = 2.7\,\mathrm{K}$ is the present CMB temperature (the first part of Eq. (A.18) is Eq. 11.8).

You can now use Eqs. (A.17) and (A.18) to calculate the mass density and the temperature of the universe at any time during the radiation era. If t is in seconds, then

$$\rho = \frac{4.5 \times 10^8}{t^2}\,\mathrm{kg/m^3} \tag{A.19}$$

$$T = \frac{10^{10}}{t^{1/2}}\,\mathrm{K} \tag{A.20}$$

These equations were introduced without derivation in Chap. 14. (In deriving the last relation, we substituted ρ_{r0} into Eq. (A.15) to find $C_r \approx 2 \times 10^{-10}$).

We can approximate the time of matter-radiation equality t_{eq} by substituting Eqs. (A.14) and (A.15) into Eq. (A.10) to get

$$t_{eq} = \left(\frac{\rho_{r0}}{\rho_{m0}}\right)^2 \frac{1}{C_r^2} \approx 68,000\,\mathrm{yrs}. \tag{A.21}$$

This estimate is not very accurate because our expression for $a(t)$ was found by assuming the radiation density is much bigger than the matter density (and all other densities). But at the time of equality t_{eq}, the two densities are equal, so the actual behaviour of $a(t)$ is more complex during the "cross-over" period. A more accurate numerical calculation gives $t_{eq} \approx 51,000\,\mathrm{yrs}$.

Matter Era

When the universe is dominated by matter, the Friedmann equation is

$$\dot{a}^2 = \frac{8\pi G \rho_{m0}}{3a} \tag{A.22}$$

Following the same steps as above, we find the solution

$$a(t) = C_m t^{2/3} \tag{A.23}$$

where

$$C_m = (6\pi G\rho_{m0})^{1/3} \tag{A.24}$$

The Hubble parameter and the mass density are now given by

$$H = \frac{2}{3t} \tag{A.25}$$

and

$$\rho = \frac{1}{6\pi G t^2} \tag{A.26}$$

Substituting the solutions Eqs. (A.23) and (A.24) into Eq. (A.11) we can approximate the time of vacuum domination $t_v \approx 11.5\,\text{Gyr}$. Once again, this estimate is not very precise because our expression for $a(t)$ is not accurate near t_v. A more detailed calculation gives $t_v \approx 10\,\text{Gyr}$.

Note: Here we have defined vacuum domination to start at the time when the energy density of matter becomes equal to the energy density of the vacuum. However, we showed (in question 14 of Chap. 9) that the condition for accelerated expansion to begin is that $\rho_v > \rho_m/2$. This means that accelerated expansion actually begins sooner than the time t_v that we calculated here. This is why we have mentioned several times in the book that acceleration began about 5 billion years ago.

Vacuum Dominated Era

During the vacuum dominated phase, which only began recently, the energy density is given by $\rho_v = \text{const}$, and the Friedmann equation becomes

$$\dot{a}^2 = \frac{8\pi G\rho_v}{3}a^2 \tag{A.27}$$

Taking a square root, we have

$$\dot{a} = H_v a \tag{A.28}$$

where

$$H_v = \sqrt{\frac{8\pi G\rho_v}{3}} \tag{A.29}$$

The solution is

$$a(t) = C_v e^{H_v t} \tag{A.30}$$

where $e \approx 2.72$ is the base of natural logarithms, and $C_v = \text{const}$.

Inflation

During inflation the universe is also dominated by vacuum energy. So the Friedmann equation is the same as (A.27), except the inflationary vacuum energy density ρ_v is much greater than it is at present. The scale factor is still given by (A.30). This tells us that in a time interval Δt the universe expands by a factor $e^{H_v \Delta t}$.

In Chap. 16 we defined the doubling time t_D as the time it takes the universe to double in size. We can find this from

$$e^{H_v t_D} = 2 \tag{A.31}$$

which gives

$$t_D = H_v^{-1} \ln 2 = 0.69 H_v^{-1} \tag{A.32}$$

Flatness Problem

In a universe filled with ordinary matter or radiation, the density ρ is rapidly driven away from the critical value ρ_c, so in order to have $\Omega = \frac{\rho}{\rho_c} \approx 1$ at present, the universe must have started with Ω extremely close to 1 at some early time. We discussed this fact, known as the flatness problem, in Chap. 15; now we will show how it follows from the Friedmann equation Eq. (A.3).

Dividing both sides of Eq. (A.3) by H^2 and using the definition of the critical density Eq. (A.4), we have

$$1 - \Omega = \frac{H_0^2}{H^2 a^2}(1 - \Omega_0). \tag{A.33}$$

Now, from Eq. (A.1), $Ha = \dot{a}$, and thus

$$1 - \Omega \propto \frac{1}{\dot{a}^2}, \tag{A.34}$$

where $\propto$ means "proportional to."

In a universe filled with ordinary matter or radiation, the speed of expansion $\dot{a}$ decreases with time, due to the attractive gravitational force. Then it follows from Eq. (A.34) that $|1-\Omega|$ grows with time. In other words, the universe deviates more and more from the critical density. Using the solutions (A.14) and (A.23), we find that $(1-\Omega) \propto t$ in the radiation era, and $(1-\Omega) \propto t^{2/3}$ in the matter era. For example, from cosmic time $t = 1s$ till the end of the radiation era at $t_{eq} \approx 2 \times 10^{12}$ s, $(1-\Omega)$ grew by a factor of 2×10^{12}, and from t_{eq} to the present time t_0 it grew approximately by a factor $(\frac{t_0}{t_{eq}})^{\frac{2}{3}} \approx 2 \times 10^3$. Overall, $(1-\Omega)$ increased by a factor 4×10^{15} between $t = 1$ s and now.[2] This means that for $|1-\Omega| < 0.1$ now, we must fine-tune $|1-\Omega|$ to be less than 2×10^{-17} at $t = 1s$.

Equation (A.34) also explains how the flatness problem is solved by cosmic inflation. During inflation the expansion of the universe accelerates, so $\dot{a}$ grows and $|1-\Omega|$ decreases with time. From (A.30), $\dot{a} \propto e^{H_v t}$ and

$$(1-\Omega) \propto e^{-2H_v t} \propto a^{-2}. \tag{A.35}$$

This shows that the density approaches the critical density exponentially fast. If, for example, inflation expanded the universe by a factor of 10^{50}, then Ω was driven closer to 1 by a factor of 10^{100}.

Note: By the definition of the doubling time n, (here we denote the doubling time by "n" to relate to Fig. 16.4), we can write the scale factor as

$$a(t) \propto 2^n \tag{A.36}$$

allowing us to recast Eq. (A.35) in terms of the doubling time as,

$$(1-\Omega) \propto 2^{-2n} \tag{A.37}$$

[2] Here we disregard the relatively small change in $(1-\Omega)$ during the vacuum dominated era, which started only recently.

Further Reading

Here is a list of books that may be helpful for further exploration of some of the topics that we have covered. We have grouped them according to their main focus, although most of these books cover other topics as well. In the last group we included some books which are critical of the ideas of inflation, string theory, and the multiverse. The "Further viewing" list includes some excellent web courses on various aspects of cosmology.

Relativity and Quantum Physics

Deutsch, David. *The Fabric of Reality.* New York: Viking Adult, 1997.

Einstein, Albert. *The Meaning of Relativity (5th edition).* Princeton University Press, 2004.

Greene, Brian. *The Fabric of the Cosmos*. New York: Knopf, 2004.

Thorne, Kip S. *Black Holes and Time Warps: Einstein's outrageous legacy.* New York: W. W. Norton & Company, 1995.

Unification of Forces

Greene, Brian. *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*. New York: W. W. Norton and Company, 1999.

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Randall, Lisa. *Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions.* Harper Perennial, 2006.
Weinberg, Steven. *Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature.* Pantheon, 1992.
Wilczek, Frank. *The Lightness of Being: Mass, Ether, and the Unification of Forces.* Basic Books, 2008.
Zee, Anthony. *Fearful Symmetry: The search for Beauty in Modern Physics*. Macmillian, 1986.

Big Bang Cosmology

Coles, Peter. *Cosmology: A Very Short Introduction.* Oxford University Press, 2001.
Harrison, Edward. *Cosmology: The Science of the Universe.* Cambridge University Press, 2000.
Kirshner, Robert P. *The Extravagant Universe: Exploding Stars, Dark Energy, and the Accelerating Cosmos. Princeton University Press, 2002.*
Livio, Mario. *The Accelerating Universe: Infinite Expansion, the Cosmological Constant, and the Beauty of the Cosmos*. Wiley, 2000.
Rees, Martin. *Just Six Numbers: The Deep Forces That Shape the Universe.* Basic Books, 2001.
Silk, Joseph. *The Big Bang.* W. H Freeman & Co., 1988.
Weinberg, Steven. *The First Three Minutes: A Modern View of the Origin of the Universe.* New York: Basic Books, 1993.

Cosmic Inflation

Guth, Alan. *The inflationary Universe.* New York: Perseus Books Group, 1997.

The Multiverse

Davies, Paul. *Cosmic Jackpot.*
Greene, Brian. *The Hidden Reality*. New York: Knopf, 2011.
Kaku, Michio. *Parallel Worlds.*
Susskind, Leonard. *The Cosmic Landscape: String Theory and the Illusion of Intelligent Design.* New York: Little, Brown and Company, 2005.
Vilenkin, Alex. *Many Worlds in One: The Search for Other Universes.* New York Hill and Wang, 2006.

Quantum Origin of the Universe

Krauss, Lawrence. *A Universe from Nothing: Why there is something rather than nothing.* New York: Free Press 2012.
Hawking, Stephen. A Brief History of Time (10th anniversary edition). Bantam, 1998.

Life in the Universe

Davies, Paul. *The Eerie Silence: Renewing Our Search for Alien Intelligence (2nd edition).* Houghton Mifflin, 2010.
Gribbin, John. *Alone in the Universe: Why Our Planet is Unique.* Wiley, 2011.

The Big Picture

Carroll, Sean. *The Big Picture: On the Origins of Life, Meaning, and the Universe Itself.* Dutton, 2016.
Hawking, Stephen and Mlodinov, Leonard. *The Grand Design*. Random House Publishing Group, 2012.
Tegmark, Max. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality.* New York: Knopf, 2014.

Alternative Views on Inflation, String Theory and the Multiverse

Smolin, Lee. *The Trouble with Physics: The Rise of String Theory, The Fall of a Science, and What Comes Next.* Houghton Mifflin Harcourt, 2007.
Steinhardt, Paul and Turok, Neil. *Endless Universe.* Doubleday (5th or Later Edition), 2007.

Further viewing

Whittle, Mark. Cosmology: The History and Nature of Our Universe. Virginia: The Teaching Company, 2008.
Carroll, Sean. Dark Matter, Dark Energy: The Dark Side of the Universe. Virginia: The Teaching Company, 2007.
Alex Filippenko. Understanding the Universe: An Introduction to Astronomy, 2nd Edition. Virginia, The Teaching Company, 2007.

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