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Table A.1 Random numbers* (Section 1)

261801	405393	795629	984038	937262	246802
455002	022803	942084	282366	862592	208908
401740	342531	110624	556808	929467	890422
588997	239885	861372	174674	802976	282967
878442	246174	934034	742133	788160	605104
158548	322915	195521	983592	287197	830518
419143	997169	938007	082445	423073	519993
468385	248119	799893	063443	821373	704116
352182	301533	866569	449214	257435	604647
881577	417401	914692	217629	342337	450742
761539	294275	904508	663363	198103	529062
648011	303897	815210	185000	035892	480611
596382	818454	382630	113271	042611	207228
821102	165656	622192	468490	255562	823977
472026	149443	564800	889812	196473	653488
681894	132802	471119	153007	617730	685888
133778	808504	097707	851472	057572	050069
830760	701419	536410	808673	426637	399767
366782	135519	726629	862649	075541	397422
321998	149556	680438	422543	848364	306968
642738	281911	765212	135961	427055	902029
066663	851516	403807	602870	105581	909955
056204	979791	258619	779554	427618	575144
033241	108783	671985	058186	077758	054072
403903	074870	873724	674234	619433	307203
991027	677059	586841	917044	894829	379409
836509	522824	988627	383546	584577	618169
722169	041552	050665	023694	081899	589699
854044	734187	033408	962651	546079	509095
425201	043159	215327	508295	931671	964890
915378	082157	907379	037189	732182	432281
934288	871662	971697	666687	561764	676994
113504	564494	700751	839657	275097	926682
961588	702512	550995	312095	033745	730169
940589	655496	568514	252098	670421	378339
957832	004510	407427	178467	096529	896791

*Random numbers were generated using the SAS statements “retain seed 1613126064;” and “rn = floor(10*ranuni(seed));”

Table A.1 Random numbers (Section 2)

772357	993916	529577	678243	561863	847933
432529	970061	557500	798091	179138	481792
307505	525070	258159	554655	575046	678559
466350	487907	299729	248259	480128	346994
617303	468101	318577	923429	038402	757219
593585	243209	814395	862822	746040	791440
001642	339004	160458	600772	662567	368585
149273	042555	774276	484976	819195	397235
650084	170717	096777	646438	049121	435982
291696	199860	255968	977965	704832	152207
380204	676820	172541	838781	875496	866057
601789	268945	637114	763853	972534	321807
034840	046770	352998	091564	836769	679665
021222	269557	705784	227686	602822	502110
657269	645233	803339	576422	510363	650292
496756	485525	641125	398402	351104	116450
837444	628196	558438	029761	119038	624675
488075	739348	349563	070527	096075	150099
704658	615314	955986	559666	475521	954801
102178	988809	827881	636004	439186	939951
693252	078895	586050	545691	461858	159346
252611	441081	786315	281005	010070	205751
280273	117608	080447	202151	108901	325238
453702	029011	657141	620700	604050	013203
327097	317434	109367	377563	036065	203413
786141	861409	263375	518852	087913	861715
194910	643115	993964	022892	046775	061654
875988	516714	699665	303752	334663	145398
370225	083754	123844	056596	275830	997716
060627	822010	449146	256578	444509	074713
536122	366557	689934	795977	120966	293430
413614	370683	263343	215245	320757	951902
722209	875974	091346	411939	116215	304007
661019	124710	939043	865547	430354	627979
631837	027762	468015	102214	231263	950303
751081	600240	089651	253141	586782	632880

Table A.1 Random numbers (Section 3)

059546	019867	891131	219608	173193	734618
176484	135942	054815	297442	998872	645538
052799	283293	624836	780633	998025	994382
490320	135340	917771	861186	092763	140293
978406	693078	035305	548903	669275	913423
503664	229298	561181	108615	391336	315399
978589	171347	250219	115588	116702	928926
308761	190680	538063	250996	045474	183835
393619	650606	523668	256623	023861	001087
894218	530435	255864	410171	294324	298222
087945	008260	939499	504474	432769	457539
864409	902698	565732	934650	688277	927769
492768	775393	750111	405919	025468	318104
843672	633027	110801	875047	295640	159752
961195	008740	286149	342458	354060	395475
153985	621691	205500	784869	590596	216745
532725	257818	410867	430907	397363	823657
066251	237385	491172	852331	749317	870908
849872	417854	375970	970009	258362	689175
605204	959747	687531	185277	704122	455741
139687	548165	188942	191439	020545	008885
665434	415021	014415	363450	845716	338980
747965	348796	602064	736557	173122	088897
070671	978664	679967	551926	584739	875137
367207	150356	985292	225591	614974	546526
133364	130583	171077	591404	458326	989837
750462	837858	752783	034800	224890	303605
661831	942203	640775	553730	172198	613256
571491	742343	546985	291036	129295	092863
733891	915752	993268	161851	581437	310503
860024	372853	655098	860970	835755	851476
693411	850277	659850	450238	891661	833743
496459	267113	803320	951153	296721	724280
041583	582606	563804	291119	571246	779088
820676	609521	563473	685897	392666	640840
235342	259362	833616	965799	738929	309229

Table A.1 Random numbers (Section4)

016142	555203	337411	794826	713966	709974
978894	911393	618079	204585	836997	998908
678441	751010	977003	501509	309106	120588
620001	359523	273064	751323	180015	270379
948285	975435	696866	279117	039232	576484
927964	446542	181604	237955	204176	239554
051454	087543	736477	327127	875598	152154
293224	763344	831295	678062	350044	697498
921180	741707	429758	879961	077184	796016
198788	997136	659392	029750	290365	343074
568833	057583	981327	556771	948875	122583
530095	245050	419676	273606	426779	587147
916245	783959	537085	612017	322930	588669
298122	300842	211852	293561	364121	122979
734366	970620	945898	471468	699734	435229
456614	268986	327941	303186	728708	507817
254953	280996	188769	520142	922840	785908
724665	374545	680830	659533	902847	232275
097212	307241	239959	688514	751627	247604
672881	324642	750191	371545	448195	277834
281900	769322	174336	545677	112313	933411
615009	687396	574876	813695	737331	089523
722940	863951	870968	179200	803442	700204
263346	922254	075810	083638	427701	558054
970212	185873	338039	716699	873664	140895
474212	828410	407079	839120	428749	942084
180898	123880	750659	550722	340545	552695
189104	806512	971589	794609	977397	597013
281079	947929	814252	381829	963815	052642
101405	749903	786162	715052	078825	599255
483562	696164	660208	304095	640899	681489
616631	612340	689906	434929	864908	485266
695033	292608	472863	401903	962354	609183
924279	180637	627669	740082	626024	669425
153076	294647	891462	034556	163029	112465
604409	335384	672094	246758	202371	516723

Table A.1 Random numbers (Section 5)

178620	984332	234149	062802	203306	409996
857451	696719	245360	219945	554014	514398
363613	427959	500852	392452	867274	977098
934682	279698	307888	039229	143251	417387
355775	341604	728003	783298	021655	069731
665699	212463	461254	662458	822056	868018
490532	385220	245113	034522	661336	735326
177715	678060	444410	984458	394908	203395
247042	584562	285965	331489	536429	150486
247037	647390	958757	889429	320390	733635
037684	319850	319453	708172	600706	116880
163936	682211	276583	695090	141138	775518
936059	110744	230502	313261	037153	962437
723498	531743	968071	809740	728333	510996
260338	879915	375157	296812	522971	126148
224336	392250	500589	802202	461732	596494
983601	286360	962976	318965	952288	371597
950090	003498	944526	890667	177182	557799
517339	436412	783049	641166	652511	767894
192027	132666	760861	620786	271242	132728
503888	846019	856424	233882	942325	511328
796439	106427	930753	519367	261714	969157
354978	019946	325140	390753	674999	265593
881853	487399	093950	592023	463922	642370
834658	788932	959353	528824	971467	890240
429335	115816	387007	698632	332875	685767
502244	063360	540982	294989	737673	576578
230094	859807	408288	251055	307629	402631
108970	046489	583823	027768	600042	788227
985730	336414	654852	476272	011614	049166
797389	447994	431552	294961	293555	180306
967876	584097	346493	644527	708534	540623
488214	736136	666810	990017	550198	332353
594677	673425	481734	749014	380123	200747
216666	893249	173621	966579	934953	101352
829921	075112	978105	427576	707853	535755

Table A.1 Random numbers (Section 6)

797468	484252	023853	192278	937112	102503
869406	106834	234802	302737	222508	069132
084703	456756	742761	693385	713106	704798
953014	558897	076970	286515	902394	008147
068334	355648	814540	713770	474093	866359
291019	695796	692442	099917	354198	854868
829045	784428	553248	413099	160541	531210
347351	078917	067538	591284	925325	240708
961281	639335	808637	097914	370752	132493
237408	602057	784998	569469	812566	754613
949564	679405	303591	345128	074802	919895
611262	590822	844714	210655	727420	947239
382111	838410	235673	240015	226367	794195
017136	100540	728145	640255	869932	716829
832572	001164	362585	877812	958998	300684
559820	106016	954619	384380	539268	309304
567125	265846	256533	543082	341655	423495
135117	247924	017309	392054	724319	938577
491976	446939	602958	446781	684067	223566
031030	690179	893112	500077	640800	683948
821529	204137	910310	756711	114949	579204
889160	294936	024169	683546	518509	569679
089569	665239	349069	158437	093839	659940
713873	707840	086598	612305	446369	210679
436460	508406	967487	781618	226710	547386
613674	173160	898727	687279	638647	536756
366699	338577	843373	151717	430829	944266
828010	825546	202510	695671	505800	680682
340755	280452	630101	428665	695421	280009
358767	365611	845098	195830	471533	344402
495054	142723	141577	566053	231325	044934
495165	641454	169446	498379	947861	680617
326075	085211	962907	515048	074505	256236
897734	334967	269056	297336	570543	835922
039744	115260	301407	795887	731089	488919
279055	728952	717435	600372	561076	481948

Table A.2 Coefficients c_i for orthogonal polynomial trend contrasts

$v = 3$				$v = 4$				
Trend	c_1	c_2	c_3	Trend	c_1	c_2	c_3	c_4
Linear	-1	0	1	Linear	-3	-1	1	3
Quadratic	1	-2	1	Quadratic	1	-1	-1	1
				Cubic	-1	3	-3	1

$v = 5$					
Trend	c_1	c_2	c_3	c_4	c_5
Linear	-2	-1	0	1	2
Quadratic	2	-1	-2	-1	2
Cubic	-1	2	0	-2	1
Quartic	1	-4	6	-4	1

$v = 6$						
Trend	c_1	c_2	c_3	c_4	c_5	c_6
Linear	-5	-3	-1	1	3	5
Quadratic	5	-1	-4	-4	-1	5
Cubic	-5	7	4	-4	-7	5
Quartic	1	-3	2	2	-3	1
Quintic	-1	5	-10	10	-5	1

$v = 7$							
Trend	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Linear	-3	-2	-1	0	1	2	3
Quadratic	5	0	-3	-4	-3	0	5
Cubic	-1	1	1	0	-1	-1	1
Quartic	3	-7	1	6	1	-7	3
Quintic	-1	4	-5	0	5	-4	1
Sextic	1	-6	15	-20	15	-6	1

Table A.3 Standard normal distribution:* upper α critical coefficients, z_α , and upper-tail probabilities, $\alpha = P(Z > z_\alpha)$

α	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005	0.00025	0.0001
z_α	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	3.481	3.719
z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005

*Values were generated using the SAS statements “z_alpha = probit(1-alpha);” and “alpha = 1 - probnorm(z_alpha);”

Table A.4 Student's t-distribution:* upper α critical coefficients, $t_{df,\alpha}$, where $\alpha = P(t_{df} > t_{df,\alpha})$

df	α								
	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005	0.0001
1	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6	3183
2	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60	70.70
3	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92	22.20
4	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	13.03
5	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	9.678
6	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	8.025
7	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408	7.063
8	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	6.442
9	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	6.010
10	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	5.694
11	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	5.453
12	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	5.263
13	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	5.111
14	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	4.985
15	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	4.880
16	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	4.791
17	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965	4.714
18	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	4.648
19	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	4.590
20	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	4.539
21	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	4.493
22	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	4.452
23	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768	4.415
24	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745	4.382
25	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	4.352
26	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	4.324
27	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	4.299
28	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	4.275
29	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	4.254
30	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	4.234
35	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591	4.153
40	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	4.094
45	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520	4.049
50	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496	4.014
55	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476	3.986
60	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	3.962
70	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435	3.926
80	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416	3.899
90	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402	3.878
100	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390	3.862
110	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381	3.848
120	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	3.837
∞	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	3.719

*Values $t_{df,\alpha}$ were generated using the SAS statements "t = tinv(1-alpha,df);" for $df < \infty$ and "t = probit(1-alpha)" for $df = \infty$

Table A.5 Chi-squared distribution:* upper α critical coefficients, $\chi^2_{df,\alpha}$, where $\alpha = P(\chi^2_{df} > \chi^2_{df,\alpha})$

df	α									
	0.999	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.001
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	10.83
2	0.002	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	13.82
3	0.024	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	16.27
4	0.091	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	18.47
5	0.210	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	20.52
6	0.381	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	22.46
7	0.598	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	24.32
8	0.857	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	26.12
9	1.152	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	27.88
10	1.479	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	29.59
11	1.834	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	31.26
12	2.214	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	32.91
13	2.617	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	34.53
14	3.041	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	36.12
15	3.483	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	37.70
16	3.942	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	39.25
17	4.416	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	40.79
18	4.905	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	42.31
19	5.407	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	43.82
20	5.921	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	45.31
21	6.447	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	46.80
22	6.983	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	48.27
23	7.529	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	49.73
24	8.085	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	51.18
25	8.649	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	52.62
26	9.222	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	54.05
27	9.803	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	55.48
28	10.39	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	56.89
29	10.99	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	58.30
30	11.59	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	59.70
35	14.69	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	66.62
40	17.92	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	73.40
45	21.25	25.90	28.37	30.61	33.35	57.51	61.66	65.41	69.96	80.08
50	24.67	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	86.66
55	28.17	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29	93.17
60	31.74	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	99.61
70	39.04	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	112.3
80	46.52	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	124.8
90	54.16	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	137.2
100	61.92	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	149.4
120	77.76	86.92	91.57	95.70	100.6	140.2	146.6	152.2	159.0	173.6

*Values were generated using the SAS statement “chi2 = cinv(1-alpha,df);”

Table A.6 F-distribution:* upper α critical coefficients, $F_{\nu_1, \nu_2, \alpha}$, where $\alpha = P(F_{\nu_1, \nu_2} > F_{\nu_1, \nu_2, \alpha})$

ν_2	α	ν_1								
		1	2	3	4	5	6	7	8	9
1	0.100	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
	0.050	161	200	216	225	230	234	237	239	241
	0.010	4052	5000	5403	5625	5764	5859	5928	5981	6022
2	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.050	18.5	19.0	19.2	19.3	19.3	19.3	19.4	19.4	19.4
	0.010	98.5	99.0	99.2	99.3	99.3	99.3	99.4	99.4	99.4
	0.001	999	999	999	999	999	999	999	999	999
3	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.050	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	0.010	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.4
	0.001	167	149	141	137	135	133	132	131	130
4	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	0.010	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
	0.001	74.1	61.3	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5	0.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	0.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	0.010	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
	0.001	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.7	27.2
6	0.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	0.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	0.010	13.8	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	0.001	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7	0.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	0.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	0.010	12.3	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	0.001	29.3	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3
8	0.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	0.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	0.010	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	0.001	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.1	11.8
9	0.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	0.010	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	0.001	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1
10	0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	0.010	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	0.001	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96

(continued)

Table A.6 (F-distribution, continued)

ν_2	α	ν_1								
		1	2	3	4	5	6	7	8	9
11	0.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	0.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	0.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	0.001	19.7	13.8	11.6	10.4	9.58	9.05	8.66	8.35	8.12
12	0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	0.001	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48
13	0.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	0.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	0.001	17.8	12.3	10.2	9.07	8.35	7.86	7.49	7.21	6.98
14	0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	0.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	0.001	17.1	11.8	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	0.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	0.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	0.001	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	0.001	16.1	11.0	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	0.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	0.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	0.010	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
	0.001	15.7	10.7	8.73	7.68	7.02	6.56	6.22	5.96	5.75
18	0.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	0.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	0.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	0.001	15.4	10.4	8.49	7.46	6.81	6.35	6.02	5.76	5.56
19	0.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	0.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	0.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	0.001	15.1	10.2	8.28	7.27	6.62	6.18	5.85	5.59	5.39
20	0.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	0.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	0.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	0.001	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24

(continued)

Table A.6 (F-distribution, continued)

ν_2	α	ν_1								
		1	2	3	4	5	6	7	8	9
22	0.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	0.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	0.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	0.001	14.4	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
25	0.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	0.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	0.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	0.001	13.9	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
30	0.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	0.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	0.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	0.001	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
35	0.100	2.85	2.46	2.25	2.11	2.02	1.95	1.90	1.85	1.82
	0.050	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16
	0.010	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96
	0.001	12.9	8.47	6.79	5.88	5.30	4.89	4.59	4.36	4.18
40	0.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	0.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	0.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	0.001	12.6	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
60	0.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	0.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	0.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	0.001	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
80	0.100	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71
	0.050	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00
	0.010	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64
	0.001	11.7	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53
100	0.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	0.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	0.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	0.001	11.5	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
120	0.100	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
	0.050	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	0.010	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
	0.001	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
1000	0.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	0.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	0.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	0.001	10.9	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

(continued)

Table A.6 (F-distribution, continued)

ν_2	α	ν_1									
		10	12	15	20	25	30	40	60	120	1000
1	0.100	60.2	60.7	61.2	61.7	62.1	62.3	62.5	62.8	63.1	63.3
	0.050	242	244	246	248	249	250	251	252	253	254
	0.010	6056	6106	6157	6209	6240	6261	6287	6313	6339	6363
2	0.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
	0.050	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5
	0.010	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5
	0.001	999	999	999	999	999	999	999	999	999	1000
3	0.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.14	5.13
	0.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.53
	0.010	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
	0.001	129	128	127	126	126	125	125	124	124	124
4	0.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
	0.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
	0.010	14.6	14.4	14.2	14.0	13.9	13.8	13.8	13.7	13.6	13.5
	0.001	48.1	47.4	46.8	46.1	45.7	45.4	45.1	44.8	44.4	44.1
5	0.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.11
	0.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43	4.40	4.37
	0.010	10.1	9.89	9.72	9.55	9.45	9.38	9.29	9.20	9.11	9.03
	0.001	26.9	26.4	25.9	25.4	25.1	24.9	24.6	24.3	24.1	23.8
6	0.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.76	2.74	2.72
	0.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74	3.70	3.67
	0.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.06	6.97	6.89
	0.001	18.4	18.0	17.6	17.1	16.9	16.7	16.4	16.2	16.0	15.8
7	0.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.51	2.49	2.47
	0.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.23
	0.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.82	5.74	5.66
	0.001	14.1	13.7	13.3	12.9	12.7	12.5	12.3	12.1	11.9	11.7
8	0.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.30
	0.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.93
	0.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.03	4.95	4.87
	0.001	11.5	11.2	10.8	10.5	10.3	10.1	9.92	9.73	9.53	9.36
9	0.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.21	2.18	2.16
	0.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79	2.75	2.71
	0.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.48	4.40	4.32
	0.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.19	8.00	7.84
10	0.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.11	2.08	2.06
	0.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62	2.58	2.54
	0.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.08	4.00	3.92
	0.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.12	6.94	6.78

(continued)

Table A.6 (F-distribution, continued)

ν_2	α	ν_1									
		10	12	15	20	25	30	40	60	120	1000
11	0.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.98
	0.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49	2.45	2.41
	0.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.78	3.69	3.61
	0.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.35	6.18	6.02
12	0.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.96	1.93	1.91
	0.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.38	2.34	2.30
	0.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.54	3.45	3.37
	0.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.76	5.59	5.44
13	0.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
	0.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30	2.25	2.21
	0.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.34	3.25	3.18
	0.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.30	5.14	4.99
14	0.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.86	1.83	1.80
	0.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22	2.18	2.14
	0.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.18	3.09	3.02
	0.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	4.94	4.77	4.62
15	0.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.82	1.79	1.76
	0.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16	2.11	2.07
	0.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.05	2.96	2.88
	0.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.64	4.47	4.33
16	0.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78	1.75	1.72
	0.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11	2.06	2.02
	0.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.93	2.84	2.76
	0.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.39	4.23	4.08
17	0.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.75	1.72	1.69
	0.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06	2.01	1.97
	0.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.83	2.75	2.66
	0.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.18	4.02	3.87
18	0.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.72	1.69	1.66
	0.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02	1.97	1.92
	0.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.75	2.66	2.58
	0.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.00	3.84	3.69
19	0.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.70	1.67	1.64
	0.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
	0.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.67	2.58	2.50
	0.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.84	3.68	3.53
20	0.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.68	1.64	1.61
	0.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95	1.90	1.85
	0.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.61	2.52	2.43
	0.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.70	3.54	3.40

(continued)

Table A.6 (F-distribution, continued)

ν_2	α	ν_1									
		10	12	15	20	25	30	40	60	120	1000
22	0.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
	0.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89	1.84	1.79
	0.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.50	2.40	2.32
	0.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.48	3.32	3.17
25	0.100	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.59	1.56	1.52
	0.050	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.72
	0.010	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.36	2.27	2.18
	0.001	4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.22	3.06	2.91
30	0.100	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.54	1.50	1.46
	0.050	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74	1.68	1.63
	0.010	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.21	2.11	2.02
	0.001	4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.92	2.76	2.61
35	0.100	1.79	1.74	1.69	1.63	1.60	1.57	1.53	1.50	1.46	1.42
	0.050	2.11	2.04	1.96	1.88	1.82	1.79	1.74	1.68	1.62	1.57
	0.010	2.88	2.74	2.60	2.44	2.35	2.28	2.19	2.10	2.00	1.90
	0.001	4.03	3.79	3.55	3.29	3.13	3.02	2.87	2.72	2.56	2.40
40	0.100	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
	0.050	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64	1.58	1.52
	0.010	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.02	1.92	1.82
	0.001	3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.57	2.41	2.25
60	0.100	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.40	1.35	1.30
	0.050	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53	1.47	1.40
	0.010	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.84	1.73	1.62
	0.001	3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.25	2.08	1.92
80	0.100	1.68	1.63	1.57	1.51	1.47	1.44	1.40	1.36	1.31	1.25
	0.050	1.95	1.88	1.79	1.70	1.64	1.60	1.54	1.48	1.41	1.34
	0.010	2.55	2.42	2.27	2.12	2.01	1.94	1.85	1.75	1.63	1.51
	0.001	3.39	3.16	2.93	2.68	2.52	2.41	2.26	2.10	1.92	1.75
100	0.100	1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.34	1.28	1.22
	0.050	1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.45	1.38	1.30
	0.010	2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.69	1.57	1.45
	0.001	3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.01	1.83	1.64
120	0.100	1.65	1.60	1.55	1.48	1.44	1.41	1.37	1.32	1.26	1.20
	0.050	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43	1.35	1.27
	0.010	2.47	2.34	2.19	2.03	1.93	1.86	1.76	1.66	1.53	1.40
	0.001	3.24	3.02	2.78	2.53	2.37	2.26	2.11	1.95	1.77	1.57
1000	0.100	1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.25	1.18	1.08
	0.050	1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.33	1.24	1.11
	0.010	2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.50	1.35	1.16
	0.001	2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.69	1.49	1.22

*Values $F_{\nu_1, \nu_2, \alpha}$ were generated using the SAS statement “f=round(finvc(1-alpha,df1,df2),0.01);”

Table A.7 Power of the F -test: $\pi(\phi) = P(F_{v_1, v_2, \phi} > F_{v_1, v_2, \alpha})$

$v_1 = 1, \alpha = 0.05$											
v_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.211	0.301	0.405	0.515	0.623	0.721	0.805	0.870	0.919	0.952	0.973
6	0.223	0.320	0.430	0.546	0.657	0.755	0.836	0.897	0.939	0.966	0.982
7	0.232	0.333	0.449	0.568	0.681	0.779	0.856	0.913	0.951	0.974	0.987
8	0.239	0.344	0.463	0.585	0.698	0.795	0.871	0.924	0.959	0.979	0.990
9	0.245	0.352	0.474	0.598	0.712	0.808	0.881	0.932	0.964	0.982	0.992
10	0.249	0.359	0.483	0.608	0.722	0.817	0.889	0.938	0.968	0.985	0.993
12	0.256	0.370	0.496	0.623	0.738	0.831	0.900	0.946	0.973	0.988	0.995
15	0.263	0.380	0.510	0.639	0.753	0.844	0.910	0.953	0.977	0.990	0.996
20	0.270	0.391	0.524	0.654	0.767	0.857	0.919	0.959	0.981	0.992	0.997
30	0.278	0.402	0.537	0.668	0.781	0.868	0.928	0.964	0.984	0.994	0.998
60	0.285	0.413	0.551	0.683	0.795	0.879	0.936	0.969	0.987	0.995	0.998
120	0.289	0.418	0.557	0.690	0.801	0.884	0.939	0.971	0.988	0.995	0.998
1000	0.293	0.423	0.563	0.696	0.807	0.889	0.942	0.973	0.989	0.996	0.999
$v_1 = 2, \alpha = 0.05$											
v_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.196	0.283	0.386	0.499	0.611	0.714	0.802	0.870	0.920	0.954	0.975
6	0.211	0.307	0.421	0.543	0.660	0.764	0.847	0.907	0.948	0.973	0.987
7	0.223	0.327	0.449	0.576	0.696	0.798	0.876	0.930	0.963	0.983	0.992
8	0.233	0.343	0.470	0.602	0.723	0.822	0.896	0.944	0.973	0.988	0.995
9	0.241	0.356	0.487	0.622	0.743	0.840	0.910	0.954	0.979	0.991	0.997
10	0.248	0.366	0.502	0.638	0.759	0.854	0.920	0.961	0.983	0.993	0.998
12	0.258	0.383	0.523	0.662	0.783	0.874	0.934	0.969	0.987	0.995	0.998
15	0.270	0.400	0.546	0.687	0.805	0.892	0.947	0.977	0.991	0.997	0.999
20	0.282	0.418	0.569	0.711	0.827	0.908	0.957	0.982	0.994	0.998	0.999
30	0.294	0.437	0.592	0.735	0.847	0.923	0.966	0.987	0.996	0.999	1.000
60	0.308	0.457	0.615	0.758	0.866	0.935	0.973	0.990	0.997	0.999	1.000
120	0.314	0.467	0.627	0.769	0.875	0.941	0.976	0.992	0.998	0.999	1.000
1000	0.321	0.476	0.637	0.778	0.882	0.946	0.979	0.993	0.998	1.000	1.000
$v_1 = 3, \alpha = 0.05$											
v_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.191	0.279	0.384	0.499	0.614	0.719	0.808	0.876	0.925	0.958	0.978
6	0.209	0.308	0.426	0.552	0.674	0.779	0.861	0.919	0.956	0.978	0.990
7	0.224	0.332	0.460	0.593	0.717	0.819	0.894	0.944	0.973	0.988	0.995
8	0.236	0.352	0.487	0.625	0.750	0.848	0.916	0.959	0.982	0.993	0.997
9	0.246	0.368	0.509	0.651	0.775	0.869	0.932	0.968	0.987	0.995	0.998
10	0.255	0.382	0.528	0.671	0.794	0.885	0.943	0.975	0.990	0.997	0.999
12	0.269	0.404	0.556	0.703	0.822	0.907	0.957	0.983	0.994	0.998	1.000
15	0.284	0.428	0.586	0.734	0.849	0.926	0.968	0.988	0.996	0.999	1.000
20	0.300	0.453	0.617	0.764	0.874	0.942	0.978	0.993	0.998	1.000	1.000
30	0.318	0.479	0.648	0.794	0.897	0.956	0.985	0.995	0.999	1.000	1.000
60	0.338	0.507	0.680	0.822	0.917	0.968	0.990	0.997	0.999	1.000	1.000
120	0.348	0.522	0.696	0.835	0.926	0.972	0.992	0.998	1.000	1.000	1.000
1000	0.357	0.535	0.709	0.847	0.933	0.976	0.993	0.998	1.000	1.000	1.000

(continued)

Table A.7 (Power, continued)

$\nu_1 = 4, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.190	0.278	0.385	0.502	0.619	0.726	0.814	0.882	0.930	0.961	0.980
6	0.209	0.311	0.433	0.562	0.686	0.791	0.872	0.928	0.962	0.982	0.992
7	0.226	0.338	0.471	0.609	0.735	0.836	0.908	0.953	0.978	0.991	0.997
8	0.240	0.361	0.503	0.646	0.771	0.867	0.931	0.968	0.987	0.995	0.998
9	0.252	0.381	0.529	0.675	0.799	0.890	0.946	0.977	0.991	0.997	0.999
10	0.262	0.398	0.551	0.699	0.821	0.906	0.957	0.983	0.994	0.998	1.000
12	0.279	0.424	0.586	0.736	0.852	0.929	0.970	0.990	0.997	0.999	1.000
15	0.298	0.454	0.622	0.771	0.881	0.948	0.981	0.994	0.998	1.000	1.000
20	0.319	0.485	0.659	0.806	0.907	0.963	0.988	0.997	0.999	1.000	1.000
30	0.342	0.519	0.697	0.840	0.930	0.975	0.993	0.998	1.000	1.000	1.000
60	0.368	0.555	0.735	0.870	0.949	0.984	0.996	0.999	1.000	1.000	1.000
120	0.382	0.574	0.754	0.885	0.957	0.987	0.997	1.000	1.000	1.000	1.000
1000	0.394	0.591	0.771	0.896	0.963	0.990	0.998	1.000	1.000	1.000	1.000
$\nu_1 = 5, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.278	0.387	0.506	0.624	0.731	0.820	0.887	0.934	0.964	0.981
6	0.210	0.314	0.438	0.571	0.696	0.801	0.880	0.934	0.967	0.985	0.994
7	0.228	0.344	0.481	0.622	0.749	0.849	0.918	0.960	0.982	0.993	0.998
8	0.244	0.370	0.517	0.663	0.789	0.882	0.941	0.974	0.990	0.997	0.999
9	0.257	0.392	0.546	0.696	0.819	0.905	0.956	0.983	0.994	0.998	1.000
10	0.269	0.411	0.571	0.722	0.842	0.922	0.967	0.988	0.996	0.999	1.000
12	0.289	0.442	0.610	0.762	0.875	0.944	0.979	0.993	0.998	1.000	1.000
15	0.311	0.477	0.652	0.802	0.905	0.962	0.988	0.997	0.999	1.000	1.000
20	0.336	0.514	0.694	0.839	0.931	0.976	0.993	0.999	1.000	1.000	1.000
30	0.365	0.555	0.738	0.874	0.952	0.986	0.997	0.999	1.000	1.000	1.000
60	0.397	0.598	0.781	0.906	0.969	0.992	0.999	1.000	1.000	1.000	1.000
120	0.414	0.621	0.802	0.920	0.975	0.994	0.999	1.000	1.000	1.000	1.000
1000	0.431	0.642	0.821	0.931	0.980	0.996	0.999	1.000	1.000	1.000	1.000
$\nu_1 = 6, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.279	0.388	0.509	0.628	0.736	0.824	0.891	0.936	0.966	0.983
6	0.211	0.317	0.443	0.578	0.704	0.809	0.887	0.939	0.970	0.986	0.994
7	0.231	0.349	0.490	0.633	0.760	0.859	0.925	0.965	0.985	0.994	0.998
8	0.248	0.378	0.528	0.677	0.802	0.893	0.949	0.978	0.992	0.997	0.999
9	0.262	0.402	0.560	0.712	0.834	0.916	0.964	0.986	0.996	0.999	1.000
10	0.276	0.423	0.588	0.741	0.858	0.933	0.973	0.991	0.997	0.999	1.000
12	0.298	0.458	0.631	0.784	0.892	0.955	0.984	0.996	0.999	1.000	1.000
15	0.323	0.497	0.677	0.826	0.922	0.972	0.992	0.998	1.000	1.000	1.000
20	0.352	0.540	0.724	0.865	0.947	0.984	0.996	0.999	1.000	1.000	1.000
30	0.385	0.587	0.772	0.901	0.967	0.992	0.998	1.000	1.000	1.000	1.000
60	0.424	0.637	0.819	0.931	0.981	0.996	0.999	1.000	1.000	1.000	1.000
120	0.445	0.663	0.842	0.944	0.986	0.997	1.000	1.000	1.000	1.000	1.000
1000	0.465	0.687	0.860	0.954	0.989	0.998	1.000	1.000	1.000	1.000	1.000

(continued)

Table A.7 (Power, continued)

$\nu_1 = 7, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.280	0.390	0.511	0.631	0.739	0.828	0.894	0.939	0.967	0.984
6	0.212	0.319	0.448	0.584	0.710	0.815	0.892	0.943	0.972	0.988	0.995
7	0.233	0.354	0.497	0.642	0.769	0.867	0.931	0.968	0.987	0.995	0.998
8	0.251	0.384	0.538	0.688	0.813	0.901	0.954	0.982	0.993	0.998	0.999
9	0.267	0.410	0.573	0.726	0.846	0.925	0.969	0.989	0.997	0.999	1.000
10	0.281	0.434	0.602	0.756	0.871	0.942	0.978	0.993	0.998	1.000	1.000
12	0.305	0.472	0.649	0.801	0.906	0.963	0.988	0.997	0.999	1.000	1.000
15	0.333	0.515	0.699	0.845	0.935	0.978	0.994	0.999	1.000	1.000	1.000
20	0.366	0.562	0.750	0.886	0.959	0.989	0.998	1.000	1.000	1.000	1.000
30	0.405	0.615	0.801	0.921	0.977	0.995	0.999	1.000	1.000	1.000	1.000
60	0.450	0.672	0.850	0.950	0.988	0.998	1.000	1.000	1.000	1.000	1.000
120	0.475	0.701	0.873	0.962	0.992	0.999	1.000	1.000	1.000	1.000	1.000
1000	0.498	0.728	0.892	0.970	0.994	0.999	1.000	1.000	1.000	1.000	1.000
$\nu_1 = 8, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.280	0.391	0.513	0.634	0.742	0.830	0.896	0.941	0.968	0.984
6	0.213	0.321	0.452	0.589	0.716	0.821	0.897	0.946	0.974	0.989	0.996
7	0.234	0.358	0.503	0.649	0.777	0.873	0.936	0.971	0.988	0.996	0.999
8	0.254	0.390	0.546	0.698	0.822	0.908	0.959	0.984	0.994	0.998	1.000
9	0.271	0.418	0.583	0.737	0.856	0.932	0.973	0.991	0.997	0.999	1.000
10	0.286	0.443	0.614	0.769	0.882	0.949	0.981	0.994	0.999	1.000	1.000
12	0.312	0.484	0.664	0.816	0.916	0.969	0.991	0.998	1.000	1.000	1.000
15	0.343	0.530	0.717	0.861	0.945	0.983	0.996	0.999	1.000	1.000	1.000
20	0.379	0.583	0.772	0.902	0.968	0.992	0.999	1.000	1.000	1.000	1.000
30	0.423	0.640	0.825	0.936	0.983	0.997	1.000	1.000	1.000	1.000	1.000
60	0.474	0.703	0.876	0.963	0.993	0.999	1.000	1.000	1.000	1.000	1.000
120	0.502	0.735	0.898	0.973	0.995	0.999	1.000	1.000	1.000	1.000	1.000
1000	0.530	0.764	0.917	0.981	0.997	1.000	1.000	1.000	1.000	1.000	1.000

(continued)

Table A.7 (Power, continued)

$\nu_1 = 9, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.281	0.393	0.515	0.637	0.745	0.833	0.898	0.942	0.969	0.985
6	0.214	0.323	0.455	0.593	0.721	0.825	0.900	0.948	0.976	0.990	0.996
7	0.236	0.361	0.508	0.656	0.783	0.878	0.939	0.973	0.989	0.996	0.999
8	0.256	0.395	0.553	0.706	0.830	0.914	0.962	0.986	0.995	0.999	1.000
9	0.274	0.424	0.592	0.747	0.864	0.938	0.976	0.992	0.998	0.999	1.000
10	0.290	0.450	0.625	0.780	0.890	0.954	0.984	0.995	0.999	1.000	1.000
12	0.318	0.494	0.678	0.828	0.925	0.973	0.992	0.998	1.000	1.000	1.000
15	0.352	0.544	0.733	0.874	0.953	0.986	0.997	0.999	1.000	1.000	1.000
20	0.391	0.601	0.790	0.915	0.974	0.994	0.999	1.000	1.000	1.000	1.000
30	0.439	0.663	0.846	0.948	0.988	0.998	1.000	1.000	1.000	1.000	1.000
60	0.496	0.730	0.896	0.973	0.995	0.999	1.000	1.000	1.000	1.000	1.000
120	0.529	0.765	0.919	0.982	0.997	1.000	1.000	1.000	1.000	1.000	1.000
1000	0.560	0.795	0.936	0.988	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\nu_1 = 10, \alpha = 0.05$											
ν_2	ϕ										
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5	0.189	0.281	0.394	0.517	0.639	0.747	0.835	0.900	0.943	0.970	0.985
6	0.215	0.325	0.458	0.597	0.725	0.829	0.903	0.950	0.977	0.990	0.996
7	0.238	0.364	0.513	0.661	0.789	0.883	0.942	0.975	0.990	0.997	0.999
8	0.258	0.399	0.560	0.713	0.836	0.919	0.965	0.987	0.996	0.999	1.000
9	0.277	0.430	0.600	0.755	0.871	0.942	0.978	0.993	0.998	1.000	1.000
10	0.294	0.457	0.634	0.789	0.897	0.958	0.986	0.996	0.999	1.000	1.000
12	0.324	0.504	0.689	0.839	0.932	0.977	0.994	0.999	1.000	1.000	1.000
15	0.359	0.557	0.747	0.885	0.959	0.989	0.998	1.000	1.000	1.000	1.000
20	0.402	0.617	0.806	0.926	0.979	0.996	0.999	1.000	1.000	1.000	1.000
30	0.454	0.683	0.863	0.958	0.991	0.999	1.000	1.000	1.000	1.000	1.000
60	0.517	0.755	0.913	0.980	0.997	1.000	1.000	1.000	1.000	1.000	1.000
120	0.553	0.791	0.935	0.987	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1000	0.588	0.823	0.951	0.992	0.999	1.000	1.000	1.000	1.000	1.000	1.000

*Power was computed using the SAS statements “nc = v*phi**2;”, “Falpha = finv(1-alpha,nu1,nu2);”, “power = 1 - probf(Falpha,nu1,nu2,nc);”, and “power = round(power,.01);” for values of the parameters “v”, “phi”, “alpha”, “nu1 = v-1” and “nu2”
 “Falpha” is the upper- α critical value of the F -distribution with “nu1” and “nu2” degrees of freedom, and “nc” is the noncentrality parameter for the corresponding noncentral F -distribution

Table A.8 Tukey's method:* upper α critical values, $q_{\nu,df,\alpha}$, of the studentized range distribution

df	α	ν													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
2	0.01	14.0	19.0	22.3	24.7	26.6	28.2	29.5	30.7	31.7	33.4	34.8	36.0	37.0	37.9
	0.05	6.08	8.33	9.80	10.9	11.7	12.4	13.0	13.5	14.0	14.7	15.4	15.9	16.4	16.8
	0.10	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.72	10.3	10.7	11.1	11.4	11.7
3	0.01	8.26	10.6	12.2	13.3	14.2	15.0	15.6	16.2	16.7	17.5	18.2	18.8	19.3	19.8
	0.05	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.95	10.3	10.7	11.0	11.2
	0.10	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29	7.67	7.98	8.25	8.48	8.68
4	0.01	6.51	8.12	9.17	9.96	10.6	11.1	11.5	11.9	12.3	12.8	13.3	13.7	14.1	14.4
	0.05	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.21	8.52	8.79	9.03	9.23
	0.10	3.01	3.98	4.59	5.03	5.39	5.68	5.93	6.14	6.33	6.65	6.91	7.13	7.33	7.50
5	0.01	5.70	6.98	7.81	8.42	8.91	9.32	9.67	9.97	10.2	10.7	11.1	11.4	11.7	11.9
	0.05	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.32	7.60	7.83	8.03	8.21
	0.10	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82	6.10	6.34	6.54	6.71	6.86
6	0.01	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.48	9.81	10.1	10.3	10.5
	0.05	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.79	7.03	7.24	7.43	7.59
	0.10	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50	5.76	5.98	6.16	6.32	6.47
7	0.01	4.95	5.92	6.55	7.02	7.39	7.70	7.98	8.21	8.43	8.80	9.11	9.38	9.62	9.84
	0.05	3.34	4.16	4.68	5.06	5.36	5.61	5.81	5.99	6.15	6.42	6.65	6.84	7.00	7.15
	0.10	2.68	3.45	3.93	4.28	4.55	4.78	4.97	5.14	5.28	5.53	5.74	5.91	6.06	6.20
8	0.01	4.74	5.64	6.21	6.63	6.97	7.24	7.48	7.69	7.88	8.20	8.46	8.70	8.90	9.08
	0.05	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.17	6.39	6.57	6.72	6.86
	0.10	2.63	3.37	3.83	4.17	4.43	4.65	4.83	4.99	5.13	5.36	5.56	5.73	5.87	6.00
9	0.01	4.60	5.43	5.96	6.35	6.66	6.92	7.14	7.33	7.50	7.79	8.03	8.23	8.41	8.57
	0.05	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.98	6.19	6.36	6.51	6.64
	0.10	2.59	3.32	3.76	4.08	4.34	4.54	4.72	4.87	5.01	5.23	5.42	5.58	5.72	5.85
10	0.01	4.48	5.27	5.77	6.14	6.43	6.67	6.88	7.05	7.21	7.48	7.71	7.90	8.07	8.22
	0.05	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.83	6.03	6.19	6.34	6.47
	0.10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91	5.13	5.32	5.47	5.61	5.73
11	0.01	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.25	7.46	7.64	7.80	7.94
	0.05	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.71	5.90	6.06	6.20	6.33
	0.10	2.54	3.23	3.66	3.96	4.20	4.40	4.57	4.71	4.84	5.05	5.23	5.38	5.51	5.63

(continued)

Table A.8 (Tukey’s method, continued)

df	α	ν													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
12	0.01	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.68	6.82	7.07	7.28	7.46	7.62	7.76
	0.05	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.26	5.39	5.61	5.80	5.95	6.08	6.20
	0.10	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.65	4.78	4.99	5.16	5.31	5.44	5.55
14	0.01	4.21	4.89	5.32	5.63	5.88	6.09	6.26	6.41	6.55	6.77	6.97	7.13	7.27	7.40
	0.05	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.46	5.64	5.78	5.91	6.03
	0.10	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68	4.88	5.05	5.19	5.32	5.43
16	0.01	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.56	6.74	6.90	7.03	7.15
	0.05	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.35	5.52	5.66	5.79	5.90
	0.10	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61	4.80	4.97	5.11	5.23	5.33
18	0.01	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.41	6.58	6.73	6.85	6.97
	0.05	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.27	5.43	5.57	5.69	5.79
	0.10	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55	4.75	4.90	5.04	5.16	5.26
20	0.01	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.28	6.45	6.59	6.71	6.82
	0.05	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.20	5.36	5.49	5.61	5.71
	0.10	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51	4.70	4.85	4.99	5.10	5.20
24	0.01	3.96	4.55	4.91	5.17	5.37	5.54	5.68	5.81	5.92	6.11	6.26	6.39	6.51	6.61
	0.05	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.10	5.25	5.38	5.49	5.59
	0.10	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.34	4.44	4.63	4.78	4.91	5.02	5.12
30	0.01	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.93	6.08	6.20	6.31	6.41
	0.05	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	5.00	5.15	5.27	5.38	5.47
	0.10	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38	4.56	4.71	4.83	4.94	5.03
40	0.01	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60	5.76	5.90	6.02	6.12	6.21
	0.05	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.90	5.04	5.16	5.27	5.36
	0.10	2.38	2.99	3.35	3.60	3.80	3.96	4.10	4.21	4.32	4.49	4.63	4.75	4.86	4.95
60	0.01	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45	5.60	5.73	5.84	5.93	6.01
	0.05	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.81	4.94	5.06	5.15	5.24
	0.10	2.36	2.96	3.31	3.56	3.75	3.91	4.04	4.16	4.25	4.42	4.56	4.67	4.78	4.86
120	0.01	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.44	5.56	5.66	5.75	5.83
	0.05	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.71	4.84	4.95	5.04	5.13
	0.10	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.10	4.19	4.35	4.48	4.60	4.69	4.78
∞	0.01	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.29	5.40	5.49	5.57	5.65
	0.05	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.62	4.74	4.85	4.93	5.01
	0.10	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13	4.28	4.41	4.52	4.61	4.69

*Values $q_{\nu,df,\alpha}$ were generated using the SAS statement “qT=probmcc(‘range’,,prob,df,v);”, where “prob” = $1 - \alpha$, and “df=.” for $df = \infty$

Table A.9 Dunnett's one-sided method:* upper α critical coefficients $w_{D1} = t_{v-1,df,\alpha}^{(0.5)}$

df	α	v - 1													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
2	0.01	8.88	10.0	10.9	11.5	12.0	12.5	12.8	13.2	13.5	14.0	14.4	14.7	15.1	15.3
	0.05	3.80	4.34	4.71	5.00	5.24	5.43	5.60	5.75	5.88	6.11	6.29	6.45	6.59	6.72
	0.10	2.54	2.92	3.19	3.40	3.57	3.71	3.83	3.94	4.03	4.19	4.32	4.44	4.54	4.62
3	0.01	5.48	6.04	6.44	6.74	6.99	7.20	7.38	7.53	7.67	7.91	8.11	8.28	8.43	8.56
	0.05	2.94	3.28	3.52	3.70	3.85	3.97	4.08	4.17	4.25	4.39	4.51	4.61	4.70	4.78
	0.10	2.13	2.41	2.61	2.75	2.87	2.97	3.06	3.13	3.20	3.31	3.41	3.49	3.56	3.62
4	0.01	4.41	4.80	5.07	5.28	5.45	5.59	5.71	5.82	5.92	6.08	6.22	6.34	6.44	6.53
	0.05	2.61	2.88	3.08	3.22	3.34	3.44	3.52	3.59	3.66	3.77	3.86	3.94	4.01	4.07
	0.10	1.96	2.20	2.37	2.50	2.60	2.68	2.75	2.82	2.87	2.97	3.05	3.11	3.17	3.22
5	0.01	3.90	4.21	4.43	4.60	4.73	4.85	4.94	5.03	5.11	5.24	5.34	5.44	5.52	5.59
	0.05	2.44	2.68	2.85	2.98	3.08	3.16	3.24	3.30	3.36	3.45	3.53	3.60	3.66	3.71
	0.10	1.87	2.09	2.24	2.36	2.45	2.53	2.59	2.65	2.70	2.78	2.86	2.92	2.97	3.02
6	0.01	3.61	3.88	4.06	4.21	4.32	4.42	4.51	4.58	4.64	4.76	4.85	4.93	5.00	5.06
	0.05	2.34	2.56	2.71	2.83	2.92	3.00	3.06	3.12	3.17	3.26	3.33	3.40	3.45	3.50
	0.10	1.82	2.02	2.17	2.27	2.36	2.43	2.49	2.54	2.59	2.67	2.74	2.79	2.84	2.89
7	0.01	3.42	3.66	3.83	3.96	4.06	4.15	4.22	4.29	4.35	4.45	4.53	4.60	4.67	4.72
	0.05	2.27	2.48	2.62	2.73	2.82	2.89	2.95	3.00	3.05	3.13	3.20	3.26	3.31	3.36
	0.10	1.78	1.98	2.11	2.22	2.30	2.37	2.42	2.47	2.52	2.59	2.66	2.71	2.76	2.80
8	0.01	3.29	3.51	3.66	3.78	3.88	3.96	4.03	4.09	4.14	4.23	4.31	4.38	4.43	4.49
	0.05	2.22	2.42	2.55	2.66	2.74	2.81	2.87	2.92	2.96	3.04	3.11	3.16	3.21	3.25
	0.10	1.75	1.94	2.08	2.17	2.25	2.32	2.38	2.42	2.47	2.54	2.60	2.65	2.70	2.74
9	0.01	3.19	3.40	3.54	3.66	3.75	3.82	3.89	3.94	3.99	4.08	4.15	4.21	4.26	4.31
	0.05	2.18	2.37	2.50	2.60	2.68	2.75	2.81	2.86	2.90	2.97	3.04	3.09	3.14	3.18
	0.10	1.73	1.92	2.05	2.14	2.22	2.28	2.34	2.39	2.43	2.50	2.56	2.61	2.65	2.69
10	0.01	3.11	3.31	3.45	3.56	3.64	3.72	3.78	3.83	3.88	3.96	4.03	4.08	4.14	4.18
	0.05	2.15	2.34	2.47	2.56	2.64	2.70	2.76	2.81	2.85	2.92	2.98	3.03	3.08	3.12
	0.10	1.71	1.90	2.02	2.12	2.19	2.26	2.31	2.35	2.40	2.46	2.52	2.57	2.61	2.65
11	0.01	3.06	3.25	3.38	3.48	3.56	3.63	3.69	3.74	3.79	3.86	3.93	3.99	4.03	4.08
	0.05	2.13	2.31	2.43	2.53	2.60	2.67	2.72	2.77	2.81	2.88	2.94	2.99	3.03	3.07
	0.10	1.70	1.88	2.01	2.10	2.17	2.23	2.29	2.33	2.37	2.44	2.49	2.54	2.58	2.62

(continued)

Table A.9 (Dunnett’s one-sided method, continued)

df	α	v - 1													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
12	0.01	3.01	3.19	3.32	3.42	3.50	3.56	3.62	3.67	3.71	3.79	3.85	3.91	3.95	3.99
	0.05	2.11	2.29	2.41	2.50	2.58	2.64	2.69	2.74	2.78	2.84	2.90	2.95	2.99	3.03
	0.10	1.69	1.87	1.99	2.08	2.16	2.22	2.27	2.31	2.35	2.42	2.47	2.52	2.56	2.60
14	0.01	2.94	3.11	3.23	3.33	3.40	3.46	3.51	3.56	3.60	3.67	3.73	3.78	3.83	3.87
	0.05	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.69	2.73	2.79	2.85	2.89	2.93	2.97
	0.10	1.67	1.85	1.97	2.06	2.13	2.19	2.24	2.28	2.32	2.38	2.44	2.48	2.52	2.56
16	0.01	2.88	3.05	3.17	3.26	3.33	3.39	3.44	3.48	3.52	3.59	3.65	3.70	3.74	3.77
	0.05	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65	2.69	2.75	2.81	2.85	2.89	2.93
	0.10	1.66	1.83	1.95	2.04	2.11	2.17	2.22	2.26	2.30	2.36	2.41	2.46	2.50	2.53
18	0.01	2.84	3.01	3.12	3.21	3.27	3.33	3.38	3.42	3.46	3.53	3.58	3.63	3.67	3.71
	0.05	2.04	2.21	2.32	2.41	2.48	2.53	2.58	2.62	2.66	2.72	2.78	2.82	2.86	2.89
	0.10	1.65	1.82	1.94	2.02	2.09	2.15	2.20	2.24	2.28	2.34	2.39	2.44	2.48	2.51
20	0.01	2.81	2.97	3.08	3.17	3.23	3.29	3.34	3.38	3.42	3.48	3.53	3.58	3.62	3.65
	0.05	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60	2.64	2.70	2.75	2.80	2.83	2.87
	0.10	1.64	1.81	1.93	2.01	2.08	2.14	2.19	2.23	2.26	2.33	2.38	2.42	2.46	2.49
24	0.01	2.77	2.92	3.03	3.11	3.17	3.22	3.27	3.31	3.35	3.41	3.46	3.50	3.54	3.57
	0.05	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57	2.60	2.66	2.72	2.76	2.80	2.83
	0.10	1.63	1.80	1.91	2.00	2.06	2.12	2.17	2.21	2.24	2.30	2.35	2.40	2.43	2.47
30	0.01	2.72	2.87	2.97	3.05	3.11	3.16	3.21	3.25	3.28	3.34	3.39	3.43	3.46	3.50
	0.05	1.99	2.15	2.25	2.34	2.40	2.45	2.50	2.54	2.57	2.63	2.68	2.72	2.76	2.79
	0.10	1.62	1.79	1.90	1.98	2.05	2.10	2.15	2.19	2.22	2.28	2.33	2.37	2.41	2.44
40	0.01	2.68	2.82	2.92	2.99	3.05	3.10	3.14	3.18	3.21	3.27	3.32	3.36	3.39	3.42
	0.05	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51	2.54	2.60	2.65	2.69	2.72	2.75
	0.10	1.61	1.77	1.88	1.96	2.03	2.08	2.13	2.17	2.20	2.26	2.31	2.35	2.39	2.42
60	0.01	2.64	2.78	2.87	2.94	3.00	3.04	3.08	3.12	3.15	3.20	3.25	3.29	3.32	3.35
	0.05	1.95	2.10	2.21	2.28	2.34	2.40	2.44	2.48	2.51	2.57	2.61	2.65	2.69	2.72
	0.10	1.60	1.76	1.87	1.95	2.01	2.06	2.11	2.15	2.18	2.24	2.29	2.33	2.36	2.39
120	0.01	2.60	2.73	2.82	2.89	2.94	2.99	3.03	3.06	3.09	3.14	3.18	3.22	3.25	3.28
	0.05	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45	2.48	2.53	2.58	2.62	2.65	2.68
	0.10	1.59	1.75	1.85	1.93	1.99	2.05	2.09	2.13	2.16	2.22	2.27	2.31	2.34	2.37
∞	0.01	2.56	2.69	2.77	2.84	2.89	2.93	2.97	3.00	3.03	3.08	3.12	3.15	3.18	3.21
	0.05	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42	2.45	2.50	2.55	2.58	2.62	2.64
	0.10	1.58	1.73	1.84	1.92	1.98	2.03	2.07	2.11	2.14	2.20	2.24	2.28	2.32	2.35

*Values $w_{D1} = t_{v-1,df,\alpha}^{(0.5)}$ were generated using the SAS statement

“wD1 = probmc(‘dunnett1’,,prob,df,vm1);”, where “prob” = $1 - \alpha$, “vm1” = $v - 1$, $t_{v-1,df,\alpha}^{(0.5)}$ is the upper α critical value for the maximum of a $(v - 1)$ -variate t -distribution with common correlation $\rho = 0.5$ and degrees of freedom df , and “df=.” for $df = \infty$

Table A.10 Dunnett's two-sided method:* upper α critical coefficients $w_{D2} = |t|_{v-1,df,\alpha}^{(0.5)}$

df	α	$v - 1$													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
2	0.01	12.4	13.8	14.8	15.6	16.2	16.7	17.1	17.5	17.8	18.4	18.8	19.2	19.6	19.9
	0.05	5.42	6.06	6.51	6.85	7.12	7.35	7.54	7.71	7.85	8.10	8.31	8.49	8.64	8.77
	0.10	3.72	4.18	4.50	4.74	4.93	5.09	5.23	5.34	5.45	5.62	5.77	5.89	6.00	6.09
3	0.01	6.97	7.64	8.10	8.46	8.74	8.98	9.19	9.37	9.52	9.79	10.0	10.2	10.4	10.5
	0.05	3.87	4.26	4.54	4.75	4.92	5.06	5.18	5.28	5.37	5.53	5.66	5.77	5.87	5.95
	0.10	2.91	3.23	3.45	3.62	3.75	3.87	3.96	4.04	4.12	4.24	4.34	4.43	4.51	4.58
4	0.01	5.36	5.81	6.12	6.36	6.55	6.72	6.85	6.98	7.08	7.27	7.42	7.55	7.66	7.77
	0.05	3.31	3.62	3.83	3.99	4.13	4.23	4.33	4.41	4.48	4.60	4.71	4.79	4.87	4.94
	0.10	2.60	2.86	3.05	3.18	3.30	3.39	3.47	3.54	3.60	3.70	3.79	3.86	3.92	3.98
5	0.01	4.63	4.97	5.22	5.41	5.56	5.68	5.79	5.89	5.97	6.11	6.24	6.34	6.43	6.51
	0.05	3.03	3.29	3.48	3.62	3.73	3.82	3.90	3.97	4.03	4.14	4.23	4.30	4.37	4.42
	0.10	2.43	2.67	2.83	2.96	3.05	3.14	3.21	3.27	3.32	3.41	3.49	3.56	3.61	3.66
6	0.01	4.21	4.51	4.71	4.87	5.00	5.10	5.20	5.28	5.35	5.47	5.57	5.66	5.74	5.80
	0.05	2.86	3.10	3.26	3.39	3.49	3.57	3.64	3.71	3.76	3.86	3.94	4.00	4.06	4.11
	0.10	2.33	2.55	2.70	2.81	2.91	2.98	3.05	3.10	3.15	3.24	3.31	3.37	3.42	3.47
7	0.01	3.95	4.21	4.39	4.53	4.64	4.74	4.82	4.89	4.96	5.07	5.16	5.24	5.31	5.37
	0.05	2.75	2.97	3.12	3.24	3.33	3.41	3.48	3.53	3.58	3.67	3.75	3.81	3.86	3.91
	0.10	2.26	2.47	2.61	2.72	2.81	2.88	2.94	2.99	3.04	3.12	3.18	3.24	3.29	3.33
8	0.01	3.77	4.00	4.17	4.29	4.40	4.48	4.56	4.62	4.68	4.78	4.86	4.93	5.00	5.05
	0.05	2.67	2.88	3.02	3.13	3.22	3.29	3.35	3.41	3.46	3.54	3.61	3.67	3.72	3.76
	0.10	2.22	2.41	2.55	2.65	2.73	2.80	2.86	2.91	2.96	3.03	3.10	3.15	3.20	3.24
9	0.01	3.63	3.85	4.01	4.12	4.22	4.30	4.37	4.43	4.48	4.57	4.65	4.71	4.77	4.82
	0.05	2.61	2.81	2.95	3.05	3.14	3.20	3.26	3.32	3.36	3.44	3.51	3.56	3.61	3.65
	0.10	2.18	2.37	2.50	2.60	2.68	2.74	2.80	2.85	2.89	2.97	3.03	3.08	3.13	3.17
10	0.01	3.53	3.74	3.88	3.99	4.08	4.16	4.22	4.28	4.33	4.42	4.49	4.55	4.60	4.65
	0.05	2.57	2.76	2.89	2.99	3.07	3.14	3.19	3.24	3.29	3.36	3.43	3.48	3.53	3.57
	0.10	2.15	2.34	2.46	2.56	2.64	2.70	2.75	2.80	2.84	2.92	2.98	3.03	3.07	3.11
11	0.01	3.45	3.65	3.79	3.89	3.98	4.05	4.11	4.16	4.21	4.29	4.36	4.42	4.47	4.52
	0.05	2.53	2.72	2.84	2.94	3.02	3.08	3.14	3.19	3.23	3.30	3.36	3.42	3.46	3.50
	0.10	2.13	2.31	2.43	2.53	2.60	2.66	2.72	2.76	2.80	2.87	2.93	2.98	3.03	3.06

(continued)

Table A.10 (Dunnett’s two-sided method, continued)

df	α	$v - 1$													
		2	3	4	5	6	7	8	9	10	12	14	16	18	20
12	0.01	3.39	3.58	3.71	3.81	3.89	3.96	4.02	4.07	4.12	4.19	4.26	4.32	4.37	4.41
	0.05	2.50	2.68	2.81	2.90	2.98	3.04	3.09	3.14	3.18	3.25	3.31	3.36	3.41	3.45
	0.10	2.11	2.29	2.41	2.50	2.57	2.64	2.69	2.73	2.77	2.84	2.90	2.95	2.99	3.03
14	0.01	3.29	3.47	3.59	3.69	3.76	3.83	3.88	3.93	3.97	4.05	4.11	4.16	4.20	4.25
	0.05	2.46	2.63	2.75	2.84	2.91	2.97	3.02	3.07	3.11	3.18	3.23	3.28	3.32	3.36
	0.10	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.68	2.72	2.79	2.84	2.89	2.93	2.97
16	0.01	3.22	3.39	3.51	3.60	3.67	3.73	3.78	3.83	3.87	3.94	4.00	4.05	4.09	4.13
	0.05	2.42	2.59	2.71	2.80	2.87	2.92	2.97	3.02	3.06	3.12	3.18	3.22	3.26	3.30
	0.10	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65	2.69	2.75	2.80	2.85	2.89	2.93
18	0.01	3.17	3.33	3.44	3.53	3.60	3.66	3.71	3.75	3.79	3.86	3.91	3.96	4.00	4.04
	0.05	2.40	2.56	2.68	2.76	2.83	2.89	2.94	2.98	3.01	3.08	3.13	3.18	3.22	3.25
	0.10	2.04	2.21	2.32	2.41	2.47	2.53	2.58	2.62	2.66	2.72	2.77	2.82	2.86	2.89
20	0.01	3.13	3.29	3.40	3.48	3.55	3.60	3.65	3.69	3.73	3.80	3.85	3.90	3.94	3.97
	0.05	2.38	2.54	2.65	2.73	2.80	2.86	2.90	2.95	2.98	3.05	3.10	3.14	3.18	3.22
	0.10	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60	2.64	2.70	2.75	2.79	2.83	2.87
24	0.01	3.07	3.22	3.32	3.40	3.47	3.52	3.57	3.61	3.64	3.70	3.76	3.80	3.84	3.87
	0.05	2.35	2.51	2.61	2.70	2.76	2.81	2.86	2.90	2.94	3.00	3.05	3.09	3.13	3.16
	0.10	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57	2.60	2.66	2.71	2.76	2.79	2.83
30	0.01	3.01	3.15	3.25	3.33	3.39	3.44	3.49	3.52	3.56	3.62	3.66	3.71	3.74	3.77
	0.05	2.32	2.47	2.58	2.66	2.72	2.77	2.82	2.86	2.89	2.95	3.00	3.04	3.08	3.11
	0.10	1.99	2.15	2.25	2.33	2.40	2.45	2.50	2.54	2.57	2.63	2.68	2.72	2.76	2.79
40	0.01	2.95	3.09	3.19	3.26	3.32	3.37	3.41	3.44	3.48	3.53	3.58	3.62	3.65	3.68
	0.05	2.29	2.44	2.54	2.62	2.68	2.73	2.77	2.81	2.85	2.90	2.95	2.99	3.02	3.06
	0.10	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51	2.54	2.60	2.65	2.69	2.72	2.75
60	0.01	2.90	3.03	3.12	3.19	3.25	3.29	3.33	3.37	3.40	3.45	3.49	3.53	3.56	3.59
	0.05	2.27	2.41	2.51	2.58	2.64	2.69	2.73	2.77	2.80	2.86	2.90	2.94	2.97	3.00
	0.10	1.95	2.10	2.21	2.28	2.34	2.40	2.44	2.48	2.51	2.57	2.61	2.65	2.69	2.72
120	0.01	2.85	2.97	3.06	3.12	3.18	3.22	3.26	3.29	3.32	3.37	3.41	3.45	3.48	3.50
	0.05	2.24	2.38	2.47	2.55	2.60	2.65	2.69	2.73	2.76	2.81	2.86	2.89	2.93	2.95
	0.10	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45	2.48	2.53	2.58	2.62	2.65	2.68
∞	0.01	2.79	2.91	3.00	3.06	3.11	3.15	3.19	3.22	3.25	3.29	3.33	3.37	3.40	3.42
	0.05	2.21	2.35	2.44	2.51	2.57	2.61	2.65	2.69	2.72	2.77	2.81	2.85	2.88	2.91
	0.10	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42	2.45	2.50	2.55	2.58	2.62	2.64

*Values $w_{D2} = t_{v-1,df,\alpha}^{(0.5)}$ generated using the SAS statement “wD2 = probmc(‘dunnett2’,,prob, df,vm1);”, where “prob” = $1 - \alpha$, “vm1” = $v - 1$, $|t_{v-1,df,\alpha}^{(0.5)}|$ is the upper α critical value for the maximum absolute value of a $(v - 1)$ -variate t -distribution with common correlation $\rho = 0.5$ and degrees of freedom df , and “df=.” for $df = \infty$

Table A.11 Voss–Wang method: upper α critical coefficients $w_V = v_{m,d,\alpha}$ for m orthogonal contrasts and d sums of squares pooled into each quasi mean squared error

even		α				odd		α			
m	d	0.10	0.05	0.01	0.001	m	d	0.10	0.05	0.01	0.001
2	1	12.4	25.0	126.	1384.	3	2	5.31	7.61	17.0	54.4
4	2	9.08	13.1	29.9	105.	5	3	6.35	8.27	14.7	32.0
6	3	8.57	11.2	19.9	50.0	7	4	6.82	8.56	13.8	26.8
8	4	8.37	10.5	16.6	33.2	9	5	7.18	8.73	13.3	24.7
10	5	8.39	10.3	16.0	29.5	11	6	7.42	8.89	13.0	21.4
12	6	8.37	10.1	14.7	24.1	13	7	7.64	8.95	12.4	20.4
14	7	8.43	9.89	14.0	22.5	15	8	7.76	9.04	12.4	19.2
16	8	8.45	9.88	13.5	21.8	17	9	7.85	9.10	12.4	19.1
18	9	8.51	9.77	13.5	19.9	19	10	7.98	9.10	12.2	17.8
20	10	8.50	9.70	12.9	19.0	21	11	8.03	9.17	12.1	16.8
22	11	8.53	9.78	12.9	18.2	23	12	8.12	9.19	11.9	17.2
24	12	8.55	9.76	12.7	18.6	25	13	8.15	9.28	11.9	17.2
26	13	8.60	9.80	12.5	18.2	27	14	8.22	9.33	11.8	16.5
28	14	8.61	9.78	12.6	18.3	29	15	8.27	9.36	11.8	16.7
30	15	8.63	9.78	12.4	17.5	31	16	8.32	9.38	11.8	16.3
32	16	8.70	9.76	12.4	16.9	33	17	8.35	9.38	11.8	15.9
34	17	8.66	9.70	12.2	16.5	35	18	8.38	9.35	11.7	15.8
36	18	8.69	9.71	12.1	16.3	37	19	8.41	9.40	11.6	15.6
38	19	8.70	9.67	12.0	16.5	39	20	8.43	9.37	11.6	16.0
40	20	8.71	9.69	12.0	16.5	41	21	8.45	9.36	11.5	15.5
42	21	8.72	9.64	12.0	15.8	43	22	8.48	9.39	11.6	15.5
44	22	8.73	9.68	12.0	15.7	45	23	8.52	9.40	11.6	15.2
46	23	8.76	9.69	11.9	15.7	47	24	8.53	9.42	11.5	15.1
48	24	8.75	9.65	11.8	15.8	49	25	8.55	9.44	11.5	15.1
50	25	8.76	9.68	11.8	15.7	51	26	8.57	9.42	11.5	15.0
52	26	8.78	9.66	11.8	15.6	53	27	8.60	9.45	11.5	15.0
54	27	8.80	9.69	11.8	15.5	55	28	8.61	9.46	11.4	15.0
56	28	8.82	9.70	11.8	15.7	57	29	8.64	9.51	11.5	15.1
58	29	8.81	9.71	11.7	15.3	59	30	8.63	9.49	11.4	15.0
60	30	8.83	9.70	11.7	15.3	61	31	8.66	9.49	11.4	15.1
62	31	8.84	9.68	11.7	15.9	63	32	8.67	9.50	11.4	15.4

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