Springer Texts in Statistics

Series editors

R. DeVeaux
S.E. Fienberg
I. Olkin

More information about this series at http://www.springer.com/series/417

Angela Dean · Daniel Voss
Danel Draguljić

Design and Analysis of Experiments

Second Edition

Springer

Angela Dean
The Ohio State University
Columbus, OH
USA

Danel Draguljić
Franklin & Marshall College
Lancaster, PA
USA

Daniel Voss
Wright State University
Dayton, OH
USA

ISSN 1431-875X ISSN 2197-4136 (electronic)
Springer Texts in Statistics
ISBN 978-3-319-52248-7 ISBN 978-3-319-52250-0 (eBook)
DOI 10.1007/978-3-319-52250-0

Library of Congress Control Number: 2016963195

1st edition: © Springer-Verlag New York, Inc. 1999
2nd edition: © Springer International Publishing AG 2017
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface to the Second Edition

Since writing the first edition of *Design and Analysis of Experiments*, there have been a number of additions to the research investigator's toolbox. In this second edition, we have incorporated a few of these modern topics.

Small screening designs are now becoming prevalent in industry for aiding the search for a few influential factors from amongst a large pool of factors of potential interest. In Chap. 15, we have expanded the material on saturated designs and introduced the topic of *supersaturated designs* which have fewer observations than the number of factors being investigated. We have illustrated that useful information can be gleaned about influential factors through the use of supersaturated designs even though their contrast estimators are correlated. When curvature is of interest, we have described *definitive screening designs* which have only recently been introduced in the literature, and which allow second order effects to be measured while retaining independence of linear main effects and requiring barely more than twice as many observations as factors.

Another modern set of tools, now used widely in areas such as biomedical and materials engineering, the physical sciences, and the life sciences, is that of *computer experiments*. To give a flavor of this topic, a new Chap. 20 has been added. Computer experiments are typically used when a mathematical description of a physical process is available, but a physical experiment cannot be run for ethical or cost reasons. We have discussed the major issues in both the design and analysis of computer experiments. While the complete treatment of the theoretical background for the analysis is beyond the scope of this book, we have provided enough technical details of the statistical model, as well as an intuitive explanation, to make the analysis accessible to the intended reader. We have also provided computer code needed for both design and analysis.

Chapter 19 has been expanded to include two new experiments involving split-plot designs from the discipline of human factors engineering. In one case, imbalance due to lost data, coupled with a mixed model, motivates introduction of restricted-maximum-likelihood-based methods implemented in the computer software sections, including a comparison of these methods to those based on least squares estimation.

It is now the case that analysis of variance and computation of confidence intervals is almost exclusively done by computer and rarely by hand. However, we have retained the basic material on these topics since it is

fundamental to the understanding of computer output. We have removed some of the more specialized details of least squares estimates from Chaps. 10–12 and canonical analysis details in Chap. 16, relying on the computer software sections to illustrate these.

SAS® software is still used widely in industry, but many university departments now teach the analysis of data using R (R Development Core Team, 2017). This is a command line software for statistical computing and graphics that is freely available on the web. Consequently, we have made a major addition to the book by including sections illustrating the use of R software for each chapter. These sections run parallel to the "Using SAS Software" sections, retained from the first edition.

A few additions have been made to the "Using SAS Software" sections. For example, in Chap. 11, `PROC OPTEX` has been included for generation of efficient block designs. `PROC MIXED` is utilized in Chap. 5 to implement Satterthwaite's method, and also in Chaps. 17–19 to estimate standard errors involving composite variance estimates, and in Chap. 19 to implement *restricted maximum likelihood estimation* given imbalanced data and mixed models.

We have updated the SAS output[1], showing this as reproductions of PC output windows generated by each program. The SAS programs presented can be run on a PC or in a command line environment such as unix, although the latter would use `PROC PLOT` rather than the graphics `PROC SGPLOT`.

Some minor modifications have been made to a few other chapters from the first edition. For example, for assessing which contrasts are non-negligible in single replicate or fractional factorial experiments, we have replaced normal probability plots by *half-normal probability plots* (Chaps. 7, 13 and 15). The reason for this change is that contrast signs are dependent upon which level of the factor is labeled as the high level and which is labeled as the low level. Half-normal plots remove this potential arbitrariness by plotting the absolute values of the contrast estimates against "half-normal scores".

Section 7.6 in the first edition on the control of noise variability and Taguchi experiments has been removed, while the corresponding material in Chap. 15 has been expanded. On teaching the material, we found it preferable to have information on mixed arrays, product arrays, and their analysis, in one location. The selection of multiple comparison methods in Chap. 4 has been shortened to include only those methods that were used constantly throughout the book. Thus, we removed the method of multiple comparisons with the best, which was not illustrated often; however, this method remains appropriate and valid for many situations in practice.

Some of the worked examples in Chap. 10 have been replaced with newer experiments, and new worked examples added to Chaps. 15 and 19. Some new exercises have been added to many chapters. These either replace

[1]The output in our "Using SAS Software" sections was generated using SAS software Version 9.3 of the SAS System for PC. Copyright © SAS 2012 SAS Institute Inc. SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc., Cary, NC, USA.

exercises from the first edition or have been added at the end of the exercise list. All other first edition exercises retain their same numbers in this second edition.

A new website http://www.wright.edu/~dan.voss/DeanVossDraguljic.html has been set up for the second edition. This contains material similar to that on the website for the first edition, including datasets for examples and exercises, SAS and R programs, and any corrections.

We continue to owe a debt of gratitude to many. We extend our thanks to all the many students at The Ohio State University and Wright State University who provided imaginative and interesting experiments and gave us permission to include their projects. We thank all the readers who notified us of errors in the first edition and we hope that we have remembered to include all the corrections. We will be equally grateful to readers of the second edition for notifying us of any newly introduced errors. We are indebted to Russell Lenth for updating the R package `lsmeans` to encompass all the multiple comparisons procedures used in this book. We are grateful to the editorial staff at Springer, especially Rebekah McClure and Hannah Bracken, who were always available to give advice and answer our questions quickly and in detail.

Finally, we extend our love and gratitude to Jeff, Nancy, Tom, Jimmy, Linda, Luka, Nikola, Marija and Anika.

Columbus, USA — Angela Dean
Dayton, USA — Daniel Voss
Lancaster, USA — Danel Draguljić

Preface to the First Edition

The initial motivation for writing this book was the observation from various students that the subject of design and analysis of experiments can seem like "a bunch of miscellaneous topics." We believe that the identification of the objectives of the experiment and the practical considerations governing the design form the heart of the subject matter and serve as the link between the various analytical techniques. We also believe that learning about design and analysis of experiments is best achieved by the planning, running, and analyzing of a simple experiment.

With these considerations in mind, we have included throughout the book the details of the planning stage of several experiments that were run in the course of teaching our classes. The experiments were run by students in statistics and the applied sciences and are sufficiently simple that it is possible to discuss the planning of the entire experiment in a few pages, and the procedures can be reproduced by readers of the book. In each of these experiments, we had access to the investigators' actual report, including the difficulties they came across and how they decided on the treatment factors, the needed number of observations, and the layout of the design. In the later chapters, we have included details of a number of published experiments. The outlines of many other student and published experiments appear as exercises at the ends of the chapters.

Complementing the practical aspects of the design are the statistical aspects of the analysis. We have developed the theory of estimable functions and analysis of variance with some care, but at a low mathematical level. Formulae are provided for almost all analyses so that the statistical methods can be well understood, related design issues can be discussed, and computations can be done by hand in order to check computer output.

We recommend the use of a sophisticated statistical package in conjunction with the book. Use of software helps to focus attention on the statistical issues rather than the calculation. Our particular preference is for the SAS software, and we have included the elementary use of this package at the end of most chapters. Many of the SAS program files and data sets used in the book can be found at www.springer–ny.com. However, the book can equally well be used with any other statistical package. Availability of statistical software has also helped shape the book in that we can discuss more complicated analyses—the analysis of unbalanced designs, for example.

The level of presentation of material is intended to make the book accessible to a wide audience. Standard linear models under normality are used for all analyses. We have avoided using calculus, except in a few optional sections where least squares estimators are obtained. We have also avoided using linear algebra, except in an optional section on the canonical analysis of second-order response surface designs. Contrast coefficients are listed in the form of a vector, but these are interpreted merely as a list of coefficients.

This book reflects a number of personal preferences. First and foremost, we have not put side conditions on the parameters in our models. The reason for this is threefold. Firstly, when side conditions are added to the model, all the parameters appear to be estimable. Consequently, one loses the perspective that in factorial experiments, main effects can be interpreted only as averages over any interactions that happen to be present. Secondly, the side conditions that are the most useful for hand calculation do not coincide with those used by the SAS software. Thirdly, if one feeds a nonestimable parametric function into a computer program such as `PROC GLM` in SAS, the program will declare the function to be "nonestimable," and the user needs to be able to interpret this statement. A consequence is that the traditional solutions to the normal equations do not arise naturally. Since the traditional solutions are for nonestimable parameters, we have tried to avoid giving these, and instead have focused on the estimation of functions of $E[Y]$, all of which are estimable.

We have concentrated on the use of prespecified models and preplanned analyses rather than exploratory data analysis. We have emphasized the experimentwise control of error rates and confidence levels rather than individual error rates and confidence levels.

We rely upon residual plots rather than formal tests to assess model assumptions. This is because of the additional information provided by residual plots when model assumption violations are indicated. For example, plots to check homogeneity of variance also indicate when a variance-stabilizing transformation should be effective. Likewise, nonlinear patterns in a normal probability plot may indicate whether inferences under normality are likely to be liberal or conservative. Except for some tests for lack of fit, we have, in fact, omitted all details of formal testing for model assumptions, even though they are readily available in many computer packages.

The book starts with basic principles and techniques of experimental design and analysis of experiments. It provides a checklist for the planning of experiments, and covers analysis of variance, inferences for treatment contrasts, regression, and analysis of covariance. These basics are then applied in a wide variety of settings. Designs covered include completely randomized designs, complete and incomplete block designs, row-column designs, single replicate designs with confounding, fractional factorial designs, response surface designs, and designs involving nested factors and factors with random effects, including split-plot designs.

In the last few years, "Taguchi methods" have become very popular for industrial experimentation, and we have incorporated some of these ideas. Rather than separating Taguchi methods as special topics, we have interspersed

them throughout the chapters via the notion of including "noise factors" in an experiment and analyzing the variability of the response as the noise factors vary.

We have introduced factorial experiments as early as Chapter 3, but analyzed them as one-way layouts (i.e., using a cell means model). The purpose is to avoid introducing factorial experiments halfway through the book as a totally new topic, and to emphasize that many factorial experiments are run as completely randomized designs. We have analyzed contrasts in a two-factor experiment both via the usual two-way analysis of variance model (where the contrasts are in terms of the main effect and interaction parameters) and also via a cell-means model (where the contrasts are in terms of the treatment combination parameters). The purpose of this is to lay the groundwork for Chapters 13–15, where these contrasts are used in confounding and fractions. It is also the traditional notation used in conjunction with Taguchi methods.

The book is not all-inclusive. For example, we do not cover recovery of interblock information for incomplete block designs with random block effects. We do not provide extensive tables of incomplete block designs. Also, careful coverage of unbalanced models involving random effects is beyond our scope. Finally, inclusion of SAS graphics is limited to low-resolution plots.

The book has been classroom tested successfully over the past five years at The Ohio State University, Wright State University, and Kenyon College, for junior and senior undergraduate students majoring in a variety of fields, first-year graduate students in statistics, and senior graduate students in the applied sciences. These three institutions are somewhat different. The Ohio State University is a large land-grant university offering degrees through the Ph.D., Wright State University is a mid-sized university with few Ph.D. programs, and Kenyon College is a liberal arts undergraduate college. Below we describe typical syllabi that have been used.

At OSU, classes meet for five hours per week for ten weeks. A typical class is composed of 35 students, about a third of whom are graduate students in the applied statistics master's program. The remaining students are undergraduates in the mathematical sciences or graduate students in industrial engineering, biomedical engineering, and various applied sciences. The somewhat ambitious syllabus covers Chapters 1–7 and 10, Sections 11.1–11.4, and Chapters 13, 15, and 17. Students taking these classes plan, run, and analyze their own experiments, usually in a team of four or five students from several different departments. This project serves the function of giving statisticians the opportunity of working with scientists and of seeing the experimental procedure firsthand, and gives the scientists access to colleagues with a broader statistical training. The experience is usually highly rated by the student participants.

Classes at WSU meet four hours per week for ten weeks. A typical class involves about 10 students who are either in the applied statistics master's degree program or who are undergraduates majoring in mathematics with a statistics concentration. Originally, two quarters (20 weeks) of probability and statistics formed the prerequisite, and the course covered much of Chapters 1–4, 6, 7, 10, 11, and 13, with Chapters 3 and 4 being primarily

review material. Currently, students enter with two additional quarters in applied linear models, including regression, analysis of variance, and methods of multiple comparisons, and the course covers Chapters 1 and 2, Sections 3.2, 6.7, and 7.5, Chapters 10, 11, and 13, Sections 15.1–15.2, and perhaps Chapter 16. As at OSU, both of these syllabi are ambitious. During the second half of the course, the students plan, run, and analyze their own experiments, working in groups of one to three. The students provide written and oral reports on the projects, and the discussions during the oral reports are of mutual enjoyment and benefit. A leisurely topics course has also been offered as a sequel, covering the rest of Chapters 14–17.

At Kenyon College, classes meet for three hours a week for 15 weeks. A typical class is composed of about 10 junior and senior undergraduates majoring in various fields. The syllabus covers Chapters 1–7, 10, and 17.

For some areas of application, random effects, nested models, and split-plot designs, which are covered in Chapters 17–19, are important topics. It is possible to design a syllabus that reaches these chapters fairly rapidly by covering Chapters 1–4, 6, 7, 17, 18, 10, 19.

We owe a debt of gratitude to many. For reading of, and comments on, prior drafts, we thank Bradley Hartlaub, Jeffrey Nunemacher, Mark Irwin, an anonymous reviewer, and the many students who suffered through the early drafts. We thank Baoshe An, James Clark, and Dionne Pratt for checking a large number of exercises, and Paul Burte, Kathryn Collins, Yuming Deng, Joseph Mesaros, Dionne Pratt, Joseph Whitmore, and many others for catching numerous typing errors. We are grateful to Peg Steigerwald, Terry England, Dolores Wills, Jill McClane, and Brian J. Williams for supplying hours of typing skills. We extend our thanks to all the many students in classes at The Ohio State University, Wright State University, and the University of Wisconsin at Madison whose imagination and diligence produced so many wonderful experiments; also to Brian H. Williams and Bob Wardrop for supplying data sets; to Nathan Buurma, Colleen Brensinger, and James Colton for library searches; and to the publishers and journal editors who gave us permission to use data and descriptions of experiments. We are especially grateful to the SAS Institute for permission to reproduce portions of SAS programs and corresponding output, and to John Kimmel for his enduring patience and encouragement throughout this endeavor.

This book has been ten years in the making. In the view of the authors, it is “a work in progress temporarily cast in stone”—or in print, as it were. We are wholly responsible for any errors and omissions, and we would be most grateful for comments, corrections, and suggestions from readers so that we can improve any future editions.

Finally, we extend our love and gratitude to Jeff, Nancy, Tommy, and Jimmy, often neglected during this endeavor, for their enduring patience, love, and support.

Columbus, Ohio — Angela Dean
Dayton, Ohio — Daniel Voss

Contents

1 Principles and Techniques 1
1.1 Design: Basic Principles and Techniques 1
1.1.1 The Art of Experimentation 1
1.1.2 Replication 2
1.1.3 Blocking 2
1.1.4 Randomization 3
1.2 Analysis: Basic Principles and Techniques 4

2 Planning Experiments 7
2.1 Introduction 7
2.2 A Checklist for Planning Experiments 7
2.3 A Real Experiment—Cotton-Spinning Experiment 13
2.4 Some Standard Experimental Designs 16
2.4.1 Completely Randomized Designs 17
2.4.2 Block Designs 17
2.4.3 Designs with Two or More Blocking Factors 17
2.4.4 Split-Plot Designs 19
2.5 More Real Experiments 20
2.5.1 Soap Experiment 20
2.5.2 Battery Experiment 24
2.5.3 Cake-Baking Experiment 27
Exercises 29

3 Designs with One Source of Variation 31
3.1 Introduction 31
3.2 Randomization 31
3.3 Model for a Completely Randomized Design 32
3.4 Estimation of Parameters 34
3.4.1 Estimable Functions of Parameters 34
3.4.2 Notation 34
3.4.3 Obtaining Least Squares Estimates 35
3.4.4 Properties of Least Squares Estimators 37
3.4.5 Estimation of σ^2 39
3.4.6 Confidence Bound for σ^2 39
3.5 One-Way Analysis of Variance 41
3.5.1 Testing Equality of Treatment Effects 41
3.5.2 Use of p-Values 45

3.6 Sample Sizes . . . 45
3.6.1 Expected Mean Squares for Treatments . . . 46
3.6.2 Sample Sizes Using Power of a Test . . . 47
3.7 A Real Experiment—Soap Experiment, Continued . . . 49
3.7.1 Checklist, Continued . . . 50
3.7.2 Data Collection and Analysis . . . 50
3.7.3 Discussion by the Experimenter. . . . 52
3.7.4 Further Observations by the Experimenter . . . 52
3.8 Using SAS Software . . . 52
3.8.1 Randomization . . . 52
3.8.2 Analysis of Variance . . . 54
3.8.3 Calculating Sample Size Using Power of a Test. . . . 56
3.9 Using R Software . . . 57
3.9.1 Randomization . . . 59
3.9.2 Reading and Plotting Data . . . 60
3.9.3 Analysis of Variance . . . 62
3.9.4 Calculating Sample Size Using Power of a Test. . . . 64
Exercises. . . . 65

4 Inferences for Contrasts and Treatment Means . . . 69
4.1 Introduction . . . 69
4.2 Contrasts . . . 69
4.2.1 Pairwise Comparisons . . . 70
4.2.2 Treatment Versus Control . . . 71
4.2.3 Difference of Averages. . . . 72
4.2.4 Trends . . . 72
4.3 Individual Contrasts and Treatment Means . . . 74
4.3.1 Confidence Interval for a Single Contrast . . . 74
4.3.2 Confidence Interval for a Single Treatment Mean . . . 76
4.3.3 Hypothesis Test for a Single Contrast or Treatment Mean . . . 77
4.3.4 Equivalence of Tests and Confidence Intervals (Optional) . . . 79
4.4 Methods of Multiple Comparisons. . . . 81
4.4.1 Multiple Confidence Intervals . . . 81
4.4.2 Bonferroni Method for Preplanned Comparisons . . . 83
4.4.3 Scheffé Method of Multiple Comparisons . . . 85
4.4.4 Tukey Method for All Pairwise Comparisons . . . 87
4.4.5 Dunnett Method for Treatment-Versus-Control Comparisons . . . 90
4.4.6 Combination of Methods . . . 92
4.4.7 Methods Not Controlling Experimentwise Error Rate . . . 92
4.5 Sample Sizes . . . 92

4.6 Using SAS Software 94
4.6.1 Inferences on Individual Contrasts 94
4.6.2 Multiple Comparisons 95
4.7 Using R Software 96
4.7.1 Inferences on Individual Contrasts 97
4.7.2 Multiple Comparisons 99
Exercises 100

5 Checking Model Assumptions 103
5.1 Introduction 103
5.2 Strategy for Checking Model Assumptions 103
5.2.1 Residuals 104
5.2.2 Residual Plots 104
5.3 Checking the Fit of the Model 106
5.4 Checking for Outliers 107
5.5 Checking Independence of the Error Terms 108
5.6 Checking the Equal Variance Assumption 110
5.6.1 Detection of Unequal Variances 110
5.6.2 Data Transformations to Equalize Variances 112
5.6.3 Analysis with Unequal Error Variances 115
5.7 Checking the Normality Assumption 117
5.8 Using SAS Software 119
5.8.1 Residual Plots 119
5.8.2 Transforming the Data 123
5.8.3 Implementing Satterthwaite's Method 124
5.9 Using R Software 125
5.9.1 Residual Plots 125
5.9.2 Transforming the Data 129
5.9.3 Implementing Satterthwaite's Method 130
Exercises 132

6 Experiments with Two Crossed Treatment Factors 139
6.1 Introduction 139
6.2 Models and Factorial Effects 139
6.2.1 The Meaning of Interaction 139
6.2.2 Models for Two Treatment Factors 142
6.2.3 Checking the Assumptions on the Model 143
6.3 Contrasts 144
6.3.1 Contrasts for Main Effects and Interactions . . . 144
6.3.2 Writing Contrasts as Coefficient Lists 146
6.4 Analysis of the Two-Way Complete Model 149
6.4.1 Least Squares Estimators for the Two-Way Complete Model 149
6.4.2 Estimation of σ^2 for the Two-Way Complete Model 151
6.4.3 Multiple Comparisons for the Complete Model 152
6.4.4 Analysis of Variance for the Complete Model 155

6.5 Analysis of the Two-Way Main-Effects Model 161
6.5.1 Least Squares Estimators for the Main-Effects Model 161
6.5.2 Estimation of σ^2 in the Main-Effects Model 165
6.5.3 Multiple Comparisons for the Main-Effects Model 166
6.5.4 Unequal Variances 168
6.5.5 Analysis of Variance for Equal Sample Sizes 168
6.5.6 Model Building 170
6.6 Calculating Sample Sizes 171
6.7 Small Experiments 171
6.7.1 One Observation Per Cell 171
6.7.2 Analysis Based on Orthogonal Contrasts 172
6.7.3 Tukey's Test for Additivity 175
6.7.4 A Real Experiment—Air Velocity Experiment 176
6.8 Using SAS Software 177
6.8.1 Analysis of Variance 177
6.8.2 Contrasts and Multiple Comparisons 180
6.8.3 Plots 182
6.8.4 One Observation Per Cell 183
6.9 Using R Software 184
6.9.1 Analysis of Variance 186
6.9.2 Contrasts and Multiple Comparisons 187
6.9.3 Plots 191
6.9.4 One Observation Per Cell 192
Exercises 193

7 Several Crossed Treatment Factors 201
7.1 Introduction 201
7.2 Models and Factorial Effects 201
7.2.1 Models 201
7.2.2 The Meaning of Interaction 202
7.2.3 Separability of Factorial Effects 205
7.2.4 Estimation of Factorial Contrasts 206
7.3 Analysis—Equal Sample Sizes 209
7.4 A Real Experiment—Popcorn–Microwave Experiment 213
7.5 One Observation per Cell 219
7.5.1 Analysis Assuming that Certain Interaction Effects are Negligible 219
7.5.2 Analysis Using Half-Normal Probability Plot of Effect Estimates 221
7.5.3 Analysis Using Confidence Intervals 223

7.6 Using SAS Software . . . 225
7.6.1 Half-Normal Probability Plots of Contrast Estimates . . . 225
7.6.2 Voss–Wang Confidence Interval Method . . . 227
7.6.3 Experiments with Empty Cells . . . 228
7.7 Using R Software . . . 231
7.7.1 Half-Normal Probability Plots of Contrast Estimates . . . 231
7.7.2 Voss–Wang Confidence Interval Method . . . 232
7.7.3 Experiments with Empty Cells . . . 234
Exercises . . . 237

8 Polynomial Regression . . . 249
8.1 Introduction . . . 249
8.2 Models . . . 250
8.3 Least Squares Estimation (Optional) . . . 253
8.3.1 Normal Equations . . . 253
8.3.2 Least Squares Estimates for Simple Linear Regression . . . 254
8.4 Test for Lack of Fit . . . 254
8.5 Analysis of the Simple Linear Regression Model . . . 257
8.6 Analysis of Polynomial Regression Models . . . 260
8.6.1 Analysis of Variance . . . 260
8.6.2 Confidence Intervals . . . 262
8.7 Orthogonal Polynomials and Trend Contrasts (Optional) . . . 263
8.7.1 Simple Linear Regression . . . 263
8.7.2 Quadratic Regression . . . 265
8.7.3 Comments . . . 266
8.8 A Real Experiment—Bean-Soaking Experiment . . . 266
8.8.1 Checklist . . . 267
8.8.2 One-Way Analysis of Variance and Multiple Comparisons . . . 270
8.8.3 Regression Analysis . . . 271
8.9 Using SAS Software . . . 273
8.10 Using R Software . . . 276
Exercises . . . 281

9 Analysis of Covariance . . . 285
9.1 Introduction . . . 285
9.2 Models . . . 285
9.2.1 Checking Model Assumptions and Equality of Slopes . . . 287
9.2.2 Model Extensions . . . 287
9.3 Least Squares Estimates . . . 287
9.3.1 Normal Equations (Optional) . . . 287
9.3.2 Least Squares Estimates and Adjusted Treatment Means . . . 288
9.4 Analysis of Covariance . . . 289

9.5 Treatment Contrasts and Confidence Intervals 293
9.5.1 Individual Confidence Intervals 293
9.5.2 Multiple Comparisons 294
9.6 Using SAS Software 296
9.7 Using R Software 299
Exercises 301

10 Complete Block Designs 305
10.1 Introduction 305
10.2 Blocks, Noise Factors or Covariates? 305
10.3 Design Issues 306
10.3.1 Block Sizes 306
10.3.2 Complete Block Design Definitions 307
10.3.3 The Randomized Complete Block Design 308
10.3.4 The General Complete Block Design 309
10.3.5 How Many Observations? 309
10.4 Analysis of Randomized Complete Block Designs 310
10.4.1 Model and Analysis of Variance 310
10.4.2 Multiple Comparisons 313
10.5 A Real Experiment—Cotton-Spinning Experiment 314
10.5.1 Design Details 314
10.5.2 Sample-Size Calculation 315
10.5.3 Analysis of the Cotton-Spinning Experiment 315
10.6 Analysis of General Complete Block Designs 318
10.6.1 Model and Analysis of Variance 318
10.6.2 Multiple Comparisons for the General Complete Block Design 320
10.6.3 Sample-Size Calculations 322
10.7 Checking Model Assumptions 323
10.8 Factorial Experiments 324
10.9 Using SAS Software 327
10.10 Using R Software 331
Exercises 336

11 Incomplete Block Designs 349
11.1 Introduction 349
11.2 Design Issues 349
11.2.1 Block Sizes 349
11.2.2 Design Plans and Randomization 350
11.2.3 Estimation of Contrasts 351
11.3 Some Special Incomplete Block Designs 352
11.3.1 Balanced Incomplete Block Designs 352
11.3.2 Group Divisible Designs 354
11.3.3 Cyclic Designs 355
11.4 Analysis of General Incomplete Block Designs 356
11.4.1 Contrast Estimators and Multiple Comparisons 356
11.4.2 Analysis of Variance 358

11.4.3 Analysis of Balanced Incomplete Block Designs . . . 359
11.4.4 A Real Experiment—Detergent Experiment . . . 360
11.4.5 Analysis of Group Divisible Designs . . . 365
11.4.6 Analysis of Cyclic Designs . . . 366
11.5 A Real Experiment—Plasma Experiment . . . 366
11.6 Sample Sizes . . . 371
11.7 Factorial Experiments . . . 372
11.7.1 Factorial Structure . . . 372
11.8 Using SAS Software . . . 376
11.8.1 Generation of Efficient Block Designs . . . 376
11.8.2 Analysis of Variance, Contrasts, and Multiple Comparisons . . . 378
11.8.3 Plots . . . 381
11.9 Using R Software . . . 382
11.9.1 Generating Efficient Incomplete Block Designs . . . 382
11.9.2 Analysis of Variance, Contrasts, and Multiple Comparisons . . . 384
11.9.3 Plots . . . 387
Exercises . . . 389

12 Designs with Two Blocking Factors . . . 399
12.1 Introduction . . . 399
12.2 Design Issues . . . 400
12.2.1 Selection and Randomization of Row–Column Designs . . . 400
12.2.2 Latin Square Designs . . . 401
12.2.3 Youden Designs . . . 403
12.2.4 Cyclic and Other Row–Column Designs . . . 404
12.3 Analysis of Row–Column Designs . . . 405
12.3.1 Model for a Row–Column Design . . . 405
12.3.2 Analysis of Variance for a Row–Column Design . . . 406
12.3.3 Confidence Intervals and Multiple Comparisons . . . 407
12.4 Analysis of Latin Square Designs . . . 408
12.4.1 Analysis of Variance for Latin Square Designs . . . 408
12.4.2 Confidence Intervals for Latin Square Designs . . . 409
12.4.3 How Many Observations? . . . 411
12.5 Analysis of Youden Designs . . . 412
12.5.1 Analysis of Variance for Youden Designs . . . 412
12.5.2 Confidence Intervals for Youden Designs . . . 413
12.5.3 How Many Observations? . . . 413
12.6 Checking the Assumptions on the Model . . . 414
12.7 Factorial Experiments in Row–Column Designs . . . 416

12.8 Using SAS Software 416
12.8.1 Factorial Model 417
12.8.2 Plots 419
12.9 Using R Software 420
12.9.1 Factorial Model 423
12.9.2 Plots 424
Exercises 425

13 Confounded Two-Level Factorial Experiments 433
13.1 Introduction 433
13.2 Single Replicate Factorial Experiments 433
13.2.1 Coding and Notation 433
13.2.2 Confounding 434
13.2.3 Analysis 434
13.3 Confounding Using Contrasts 435
13.3.1 Contrasts 435
13.3.2 Experiments in Two Blocks 436
13.3.3 Experiments in Four Blocks 441
13.3.4 Experiments in Eight Blocks 443
13.3.5 Experiments in More Than Eight Blocks 443
13.4 Confounding Using Equations 444
13.4.1 Experiments in Two Blocks 444
13.4.2 Experiments in More Than Two Blocks 445
13.5 A Real Experiment—Mangold Experiment 447
13.6 Plans for Confounded 2^p Experiments 451
13.7 Multireplicate Designs 451
13.8 Complete Confounding: Repeated Single-Replicate Designs 452
13.8.1 A Real Experiment—Decontamination Experiment 452
13.9 Partial Confounding 455
13.10 Comparing the Multireplicate Designs 458
13.11 Using SAS Software 461
13.12 Using R Software 463
Exercises 465

14 Confounding in General Factorial Experiments 473
14.1 Introduction 473
14.2 Confounding with Factors at Three Levels 473
14.2.1 Contrasts 473
14.2.2 Confounding Using Contrasts 474
14.2.3 Confounding Using Equations 475
14.2.4 A Real Experiment—Dye Experiment 478
14.2.5 Plans for Confounded 3^p Experiments 481
14.3 Designing Using Pseudofactors 482
14.3.1 Confounding in 4^p Experiments 482
14.3.2 Confounding in $2^p \times 4^q$ Experiments 483
14.4 Designing Confounded Asymmetric Experiments 483

14.5 Using SAS Software 486
14.6 Using R Software 489
Exercises. 491

15 Fractional Factorial Experiments 495
15.1 Introduction 495
15.2 Fractions from Block Designs; Factors with 2 Levels 495
15.2.1 Half-Fractions of 2^p Experiments; 2^{p-1} Experiments. 495
15.2.2 Resolution and Notation 498
15.2.3 A Real Experiment—Soup Experiment. 499
15.2.4 Quarter-Fractions of 2^p Experiments; 2^{p-2} Experiments. 501
15.2.5 Smaller Fractions of 2^p Experiments 505
15.3 Fractions from Block Designs; Factors with 3 Levels 507
15.3.1 One-Third Fractions of 3^p Experiments; 3^{p-1} Experiments. 507
15.3.2 One-Ninth Fractions of 3^p Experiments; 3^{p-2} Experiments. 511
15.4 Fractions from Block Designs; Other Experiments. 511
15.4.1 $2^p \times 4^q$ Experiments 511
15.4.2 $2^p \times 3^q$ Experiments 512
15.5 Blocked Fractional Factorial Experiments. 513
15.6 Fractions from Orthogonal Arrays 516
15.6.1 2^p Orthogonal Arrays. 516
15.6.2 $2^p \times 4^q$ Orthogonal Arrays 521
15.6.3 3^p Orthogonal Arrays. 522
15.7 Design for the Control of Noise Variability 523
15.7.1 A Real Experiment—Inclinometer Experiment 525
15.8 Small Screening Designs: Orthogonal Main Effect Plans 529
15.8.1 Saturated Designs 529
15.8.2 Supersaturated Designs. 533
15.8.3 Saturated Orthogonal Main Effect Plans Plus Interactions 536
15.8.4 Definitive Screening Designs 537
15.9 Using SAS Software 538
15.9.1 Fractional Factorials. 538
15.9.2 Design for the Control of Noise Variability. . . 539
15.9.3 Analysis of Small Screening Designs 542
15.10 Using R Software 543
15.10.1 Fractional Factorials. 543
15.10.2 Design for the Control of Noise Variability. . . 546
15.10.3 Analysis of Small Screening Designs 547
Exercises. 549

16 Response Surface Methodology 565
16.1 Introduction 565
16.2 First-Order Designs and Analysis 567
16.2.1 Models 567
16.2.2 Standard First-Order Designs 568
16.2.3 Least Squares Estimation 569
16.2.4 Checking Model Assumptions 570
16.2.5 Analysis of Variance 570
16.2.6 Tests for Lack of Fit 572
16.2.7 Path of Steepest Ascent 576
16.3 Second-Order Designs and Analysis 577
16.3.1 Models and Designs 577
16.3.2 Central Composite Designs 578
16.3.3 Generic Test for Lack of Fit of the Second-Order Model 581
16.3.4 Analysis of Variance for a Second-Order Model 581
16.3.5 Canonical Analysis of a Second-Order Model 583
16.4 Properties of Second-Order Designs: CCDs 585
16.4.1 Rotatability 585
16.4.2 Orthogonality 586
16.4.3 Orthogonal Blocking 587
16.5 A Real Experiment: Flour Production Experiment, Continued 589
16.6 Box–Behnken Designs 592
16.7 Using SAS Software 594
16.7.1 Analysis of a Standard First-Order Design . . . 594
16.7.2 Analysis of a Second-Order Design 597
16.8 Using R Software 599
16.8.1 Analysis of a Standard First-Order Design . . . 599
16.8.2 Analysis of a Second-Order Design 602
16.8.3 Generating Designs 606
Exercises 608

17 Random Effects and Variance Components 615
17.1 Introduction 615
17.2 Some Examples 615
17.3 One Random Effect 618
17.3.1 The Random-Effects One-Way Model 618
17.3.2 Estimation of σ^2 619
17.3.3 Estimation of σ_T^2 620
17.3.4 Testing Equality of Treatment Effects 622
17.3.5 Confidence Intervals for Variance Components 625
17.4 Sample Sizes for an Experiment with One Random Effect 628
17.5 Checking Assumptions on the Model 631

17.6 Two or More Random Effects 632
17.6.1 Models and Examples 632
17.6.2 Checking Model Assumptions 634
17.6.3 Estimation of σ^2 634
17.6.4 Estimation of Variance Components 635
17.6.5 Confidence Intervals for Variance Components 637
17.6.6 Hypothesis Tests for Variance Components 640
17.6.7 Sample Sizes 642
17.7 Mixed Models 642
17.7.1 Expected Mean Squares and Hypothesis Tests 643
17.7.2 Confidence Intervals in Mixed Models 645
17.8 Rules for Analysis of Random-Effects and Mixed Models 647
17.8.1 Rules—Equal Sample Sizes 647
17.8.2 Controversy (Optional) 648
17.9 Block Designs and Random Block Effects 649
17.10 Using SAS Software 652
17.10.1 Checking Assumptions on the Model 652
17.10.2 Estimation and Hypothesis Testing 654
17.10.3 Sample Size Calculations 657
17.11 Using R Software 659
17.11.1 Checking Assumptions on the Model 659
17.11.2 Estimation and Hypothesis Testing 661
17.11.3 Sample Size Calculations 664
Exercises 665

18 Nested Models 671
18.1 Introduction 671
18.2 Examples and Models 671
18.3 Analysis of Nested Fixed Effects 674
18.3.1 Least Squares Estimates 674
18.3.2 Estimation of σ^2 675
18.3.3 Confidence Intervals 675
18.3.4 Hypothesis Testing 676
18.4 Analysis of Nested Random Effects 679
18.4.1 Expected Mean Squares 679
18.4.2 Estimation of Variance Components 682
18.4.3 Hypothesis Testing 683
18.4.4 Some Examples 683
18.5 Using SAS Software 687
18.5.1 Voltage Experiment 687

18.6 Using R Software 691
18.6.1 Voltage Experiment 692
18.6.2 Analysis Using Least Squares Estimates and aov 694
18.6.3 Analysis Using Restricted Maximum Likelihood Estimation 695
Exercises 695

19 Split-Plot Designs 703
19.1 Introduction 703
19.2 Designs and Models 703
19.3 Analysis of a Split-Plot Design with Complete Blocks 705
19.3.1 Split-Plot Analysis 706
19.3.2 Whole-Plot Analysis 707
19.3.3 Contrasts Within and Between Whole Plots 708
19.3.4 A Real Experiment—Oats Experiment 709
19.4 Split-Split-Plot Designs 711
19.5 Split-Plot Confounding 713
19.6 A Real Experiment—UAV Experiment 713
19.6.1 Analysis of Variance 715
19.6.2 Multiple Comparisons 716
19.7 A Real Experiment—Mobile Computing Field Study . . . 717
19.7.1 Analysis of Variance 719
19.7.2 Multiple Comparisons 721
19.7.3 Analysis as a Split-Split-Plot Design 721
19.7.4 Design Construction 724
19.8 Using SAS Software 727
19.8.1 The Analysis of Variance Approach 727
19.8.2 Simple Contrasts 732
19.8.3 The Restricted Maximum Likelihood Approach 733
19.8.4 Recovery of Inter-block Information 736
19.8.5 ReML and Unbalanced Data 742
19.9 Using R Software 745
19.9.1 The Analysis of Variance Approach 745
19.9.2 Simple Contrasts 746
19.9.3 The Restricted Maximum Likelihood Approach 747
19.9.4 Recovery of Inter-block Information 753
19.9.5 ReML and Unbalanced Data 756
Exercises 758

20 Computer Experiments 765
20.1 Introduction 765
20.2 Models for Computer Experiments 767
20.3 Gaussian Stochastic Process Model 768

20.4 Design 772
20.4.1 Space-Filling and Non-collapsing Designs . . . 772
20.4.2 Construction of Latin Hypercube Designs 774
20.5 A Real Experiment—Neuron Experiment 776
20.6 Using SAS Software 778
20.6.1 Maximin Latin Hypercube Designs 779
20.6.2 Fitting the GaSP Model 781
20.7 Using R Software 784
20.7.1 Maximin Latin Hypercube Designs 784
20.7.2 Fitting the GaSP Model 785
Exercises 786

Appendix A: Tables 793

Bibliography 821

Index of Authors 825

Index of Experiments 829

Index of Subjects 833