Fundamentals of Biomechanics

Equilibrium, Motion, and Deformation

Fourth Edition

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Foreword

Biomechanics is a discipline utilized by different groups of professionals. It is a required basic science for orthopedic surgeons, neurosurgeons, osteopaths, physiatrists, rheumatologists, physical and occupational therapists, chiropractors, athletic trainers and beyond. These medical and paramedical specialists usually do not have a strong mathematics and physics background. Biomechanics must be presented to these professionals in a rather nonmathematical way so that they may learn the concepts of mechanics without a rigorous mathematical approach.

On the other hand, many engineers work in fields in which biomechanics plays a significant role. Human factors engineering, ergonomics, biomechanics research, and prosthetic research and development all require that the engineers working in the field have a strong knowledge of biomechanics. They are equipped to learn biomechanics through a rigorous mathematical approach. Classical textbooks in the engineering fields do not approach the biological side of biomechanics.

Fundamentals of Biomechanics (Fourth Edition) approaches biomechanics through a rigorous mathematical standpoint while emphasizing the biological side. This book will be very useful for engineers studying biomechanics and for medical specialists enrolled in courses who desire a more intensive study of biomechanics and are equipped through previous study of mathematics to develop a deeper comprehension of engineering as it applies to the human body.

Significant progress has been made in the field of biomechanics during the last few decades. Solid knowledge and understanding of biomechanical concepts, principles, assessment methods, and tools are essential components of the study for clinicians, researchers, and practitioners in their efforts to prevent musculoskeletal disorders and improve patient care that will reduce related disability when they do occur.

This work was prepared in a combined clinical setting at the New York University Hospital for Joint Diseases Orthopedic Institute and teaching setting within the Program of

Ergonomics and Biomechanics at the Graduate School of Arts and Science, New York University. The authors of this volume have the unique experience of teaching biomechanics in a clinical setting to professionals from diverse backgrounds. This work reflects their many years of classroom teaching, rehabilitation treatment, and practical and research experience.

Fundamentals of Biomechanics has been translated into three languages (Greek, Japanese, and, coming soon, Italian) and has contributed to many discussions in the field to advance biomechanical knowledge.

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Preface

Biomechanics is an exciting and fascinating specialty with the goal of better understanding the musculoskeletal system to enable the development of methods to prevent problems or to improve treatment of patients.

Biomechanics has increasingly become an interdisciplinary field where engineers, physicists, computer scientists, biologists, and material scientists work together to support physicians, sports scientists, ergonomists, and physiotherapists and many other professionals.

This book *Fundamentals of Biomechanics* summarizes the basics of mechanics, both static and dynamics including kinematics and kinetics. The book introduces vectors and moments, applying them with many simple examples, which are essential to determine quantitatively or at least estimate loads acting during different situations or exercises on bones and joints. Joints and bones are mostly stabilized by their associated ligaments and muscles and therefore such calculations also require knowledge of the complex anatomy. Creativity is also needed to simplify these often complicated scenarios to reduce the parameters for the free body diagrams that can be used to develop the equations that can be solved. This book presents the concepts and explains in detail examples for the elbow, the shoulder, the spinal column, the neck, the lumbar spine, the hip and the knee, as well as the ankle joint. The reader however should also be aware that results from such calculations should be validated with available in vivo studies because muscle forces are often not known and the simplifications may be too strong.

The book also explains stress and strain relations, which can cause the failure of structures. The differences between the mechanical properties of hard and soft biological tissues are presented. The beauty of biomechanics is that mechanics can be applied to biological tissues to explain healing or degenerative processes. This knowledge is important to better understand what happens on the cellular level of these tissues and to explain remodeling processes in these structures. In order to move deeper into biological applications other books may also

be recommended; some of these can be found in the suggested readings following specific chapters. This book may also serve as reference when notations or definitions or units are not clear.

One of the most important unique features that should be emphasized is the fact that each chapter contains exercise problems and detailed solutions that help to practice the concepts via many examples. Therefore this book should not only be recommended to students but also to professors who teach biomechanics. People from other disciplines like "normal" engineers or physicists are often asked to teach biomechanics for example to physiotherapists. For these professionals, this book may serve as a valuable source for their own preparation.

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Contents

Chapter 1 Introduction 1
- 1.1 Mechanics / 3
- 1.2 Biomechanics / 5
- 1.3 Basic Concepts / 6
- 1.4 Newton's Laws / 6
- 1.5 Dimensional Analysis / 7
- 1.6 Systems of Units / 9
- 1.7 Conversion of Units / 11
- 1.8 Mathematics / 12
- 1.9 Scalars and Vectors / 13
- 1.10 Modeling and Approximations / 13
- 1.11 Generalized Procedure / 14
- 1.12 Scope of the Text / 14
- 1.13 Notation / 15

References, Suggested Reading, and Other Resources / 16

Chapter 2 Force Vector 21
- 2.1 Definition of Force / 23
- 2.2 Properties of Force as a Vector Quantity / 23
- 2.3 Dimension and Units of Force / 23
- 2.4 Force Systems / 24
- 2.5 External and Internal Forces / 24
- 2.6 Normal and Tangential Forces / 25
- 2.7 Tensile and Compressive Force / 25
- 2.8 Coplanar Forces / 25
- 2.9 Collinear Forces / 26
- 2.10 Concurrent Forces / 26
- 2.11 Parallel Force / 26
- 2.12 Gravitational Force or Weight / 26
- 2.13 Distributed Force Systems and Pressure / 27
- 2.14 Frictional Forces / 29
- 2.15 Exercise Problems / 31

Chapter 3 Moment and Torque Vectors 37
- 3.1 Definitions of Moment and Torque Vectors / 39
- 3.2 Magnitude of Moment / 39
- 3.3 Direction of Moment / 39
- 3.4 Dimension and Units of Moment / 40

3.5 Some Fine Points About the Moment Vector / 41
3.6 The Net or Resultant Moment / 42
3.7 The Couple and Couple-Moment / 47
3.8 Translation of Forces / 47
3.9 Moment as a Vector Product / 48
3.10 Exercise Problems / 53

Chapter 4 Statics: Systems in Equilibrium 61
4.1 Overview / 63
4.2 Newton's Laws of Mechanics / 63
4.3 Conditions for Equilibrium / 65
4.4 Free-Body Diagrams / 67
4.5 Procedure to Analyze Systems in Equilibrium / 68
4.6 Notes Concerning the Equilibrium Equations / 69
4.7 Constraints and Reactions / 71
4.8 Simply Supported Structures / 71
4.9 Cable-Pulley Systems and Traction Devices / 78
4.10 Built-In Structures / 80
4.11 Systems Involving Friction / 86
4.12 Center of Gravity Determination / 88
4.13 Exercise Problems / 93

Chapter 5 Applications of Statics to Biomechanics 101
5.1 Skeletal Joints / 103
5.2 Skeletal Muscles / 104
5.3 Basic Considerations / 105
5.4 Basic Assumptions and Limitations / 106
5.5 Mechanics of the Elbow / 107
5.6 Mechanics of the Shoulder / 112
5.7 Mechanics of the Spinal Column / 116
5.8 Mechanics of the Hip / 121
5.9 Mechanics of the Knee / 128
5.10 Mechanics of the Ankle / 133
5.11 Exercise Problems / 135
References / 139

Chapter 6 Introduction to Dynamics 141
6.1 Dynamics / 143
6.2 Kinematics and Kinetics / 143
6.3 Linear, Angular, and General Motions / 144
6.4 Distance and Displacement / 145
6.5 Speed and Velocity / 145
6.6 Acceleration / 145
6.7 Inertia and Momentum / 146
6.8 Degree of Freedom / 146
6.9 Particle Concept / 146
6.10 Reference Frames and Coordinate Systems / 147
6.11 Prerequisites for Dynamic Analysis / 147
6.12 Topics to Be Covered / 147

Chapter 7 Linear Kinematics 149
7.1 Uniaxial Motion / 151
7.2 Position, Displacement, Velocity, and Acceleration / 151
7.3 Dimensions and Units / 153
7.4 Measured and Derived Quantities / 154
7.5 Uniaxial Motion with Constant Acceleration / 155
7.6 Examples of Uniaxial Motion / 157
7.7 Biaxial Motion / 163
7.8 Position, Velocity, and Acceleration Vectors / 163
7.9 Biaxial Motion with Constant Acceleration / 166
7.10 Projectile Motion / 167
7.11 Applications to Athletics / 170
7.12 Exercise Problems / 175

Chapter 8 Linear Kinetics 179
8.1 Overview / 181
8.2 Equations of Motion / 181
8.3 Special Cases of Translational Motion / 183
8.3.1 Force Is Constant / 183
8.3.2 Force Is a Function of Time / 184
8.3.3 Force Is a Function of Displacement / 184
8.4 Procedure for Problem Solving in Kinetics / 185
8.5 Work and Energy Methods / 187
8.6 Mechanical Work / 188
8.6.1 Work Done by a Constant Force / 188
8.6.2 Work Done by a Varying Force / 189
8.6.3 Work as a Scalar Product / 189
8.7 Mechanical Energy / 190
8.7.1 Potential Energy / 190
8.7.2 Kinetic Energy / 191
8.8 Work–Energy Theorem / 191
8.9 Conservation of Energy Principle / 191
8.10 Dimension and Units of Work and Energy / 192
8.11 Power / 192
8.12 Applications of Energy Methods / 193
8.13 Exercise Problems / 198

Chapter 9 Angular Kinematics 203
9.1 Polar Coordinates / 205
9.2 Angular Position and Displacement / 205
9.3 Angular Velocity / 206
9.4 Angular Acceleration / 206
9.5 Dimensions and Units / 207
9.6 Definitions of Basic Concepts / 208
9.7 Rotational Motion About a Fixed Axis / 217
9.8 Relationships Between Linear and Angular Quantities / 218
9.9 Uniform Circular Motion / 219
9.10 Rotational Motion with Constant Acceleration / 219
9.11 Relative Motion / 220

9.12 Linkage Systems / 222
9.13 Exercise Problems / 226

Chapter 10 Angular Kinetics 231
10.1 Kinetics of Angular Motion / 233
10.2 Torque and Angular Acceleration / 239
10.3 Mass Moment of Inertia / 240
10.4 Parallel-Axis Theorem / 242
10.5 Radius of Gyration / 242
10.6 Segmental Motion Analysis / 243
10.7 Rotational Kinetic Energy / 247
10.8 Angular Work and Power / 248
10.9 Exercise Problems / 250

Chapter 11 Impulse and Momentum 253
11.1 Introduction / 255
11.2 Linear Momentum and Impulse / 255
11.3 Applications of the Impulse-Momentum Method / 257
11.4 Conservation of Linear Momentum / 264
11.5 Impact and Collisions / 264
11.6 One-Dimensional Collisions / 265
11.6.1 Perfectly Inelastic Collision / 266
11.6.2 Perfectly Elastic Collision / 267
11.6.3 Elastoplastic Collision / 268
11.7 Two-Dimensional Collisions / 270
11.8 Angular Impulse and Momentum / 273
11.9 Summary of Basic Equations / 274
11.10 Kinetics of Rigid Bodies in Plane Motion / 275
11.11 Exercise Problems / 276

Chapter 12 Introduction to Deformable Body Mechanics 279
12.1 Overview / 281
12.2 Applied Forces and Deformations / 282
12.3 Internal Forces and Moments / 282
12.4 Stress and Strain / 283
12.5 General Procedure / 284
12.6 Mathematics Involved / 285
12.7 Topics to Be Covered / 285
Suggested Reading / 286

Chapter 13 Stress and Strain 287
13.1 Basic Loading Configurations / 289
13.2 Uniaxial Tension Test / 289
13.3 Load-Elongation Diagrams / 290
13.4 Simple Stress / 291
13.5 Simple Strain / 292
13.6 Stress–Strain Diagrams / 294

13.7 Elastic Deformations / 295
13.8 Hooke's Law / 297
13.9 Plastic Deformations / 297
13.10 Necking / 298
13.11 Work and Strain Energy / 299
13.12 Strain Hardening / 299
13.13 Hysteresis Loop / 299
13.14 Properties Based on Stress–Strain Diagrams / 300
13.15 Idealized Models of Material Behavior / 300
13.16 Mechanical Properties of Materials / 301
13.17 Example Problems / 302
13.18 Exercise Problems / 309

Chapter 14 Multiaxial Deformations and Stress Analyses 317
14.1 Poisson's Ratio / 319
14.2 Biaxial and Triaxial Stresses / 320
14.3 Stress Transformation / 325
14.4 Principal Stresses / 326
14.5 Mohr's Circle / 327
14.6 Failure Theories / 330
14.7 Allowable Stress and Factor of Safety / 332
14.8 Factors Affecting the Strength of Materials / 333
14.9 Fatigue and Endurance / 334
14.10 Stress Concentration / 335
14.11 Torsion / 337
14.12 Bending / 344
14.13 Combined Loading / 354
14.14 Exercise Problems / 356

Chapter 15 Mechanical Properties of Biological Tissues 361
15.1 Viscoelasticity / 363
15.2 Analogies Based on Springs and Dashpots / 364
15.3 Empirical Models of Viscoelasticity / 365
15.3.1 Kelvin-Voight Model / 365
15.3.2 Maxwell Model / 366
15.3.3 Standard Solid Model / 367
15.4 Time-Dependent Material Response / 368
15.5 Comparison of Elasticity and Viscoelasticity / 369
15.6 Common Characteristics of Biological Tissues / 371
15.7 Biomechanics of Bone / 373
15.7.1 Composition of Bone / 373
15.7.2 Mechanical Properties of Bone / 374
15.7.3 Structural Integrity of Bone / 376
15.7.4 Bone Fractures / 377
15.8 Tendons and Ligaments / 378
15.9 Skeletal Muscles / 379
15.10 Articular Cartilage / 381
15.11 Discussion / 382
15.12 Exercise Problems / 383

Erratum to: Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation E1

Appendix A: Plane Geometry 389
A.1 Angles / 391
A.2 Triangles / 391
A.3 Law of Sines / 392
A.4 Law of Cosine / 392
A.5 The Right Triangle / 392
A.6 Pythagorean Theorem / 392
A.7 Sine, Cosine, and Tangent / 393
A.8 Inverse Sine, Cosine, and Tangent / 394
A.9 Exercise Problems / 397

Appendix B: Vector Algebra 401
B.1 Definitions / 403
B.2 Notation / 403
B.3 Multiplication of a Vector by a Scalar / 404
B.4 Negative Vector / 404
B.5 Addition of Vectors: Graphical Methods / 404
B.6 Subtraction of Vectors / 405
B.7 Addition of More Than Two Vectors / 405
B.8 Projection of Vectors / 406
B.9 Resolution of Vectors / 406
B.10 Unit Vectors / 407
B.11 Rectangular Coordinates / 407
B.12 Addition of Vectors: Trigonometric Method / 409
B.13 Three-Dimensional Components of Vectors / 414
B.14 Dot (Scalar) Product of Vectors / 415
B.15 Cross (Vector) Product of Vectors / 416
B.16 Exercise Problems / 419

Appendix C: Calculus 423
C.1 Functions / 425
C.1.1 Constant Functions / 426
C.1.2 Power Functions / 426
C.1.3 Linear Functions / 428
C.1.4 Quadratic Functions / 428
C.1.5 Polynomial Functions / 429
C.1.6 Trigonometric Functions / 429
C.1.7 Exponential and Logarithmic Functions / 431
C.2 The Derivative / 432
C.2.1 Derivatives of Basic Functions / 432
C.2.2 The Constant Multiple Rule / 433
C.2.3 The Sum Rule / 434
C.2.4 The Product Rule / 435
C.2.5 The Quotient Rule / 435
C.2.6 The Chain Rule / 436
C.2.7 Implicit Differentiation / 438
C.2.8 Higher Derivatives / 438

C.3 The Integral / 439
C.3.1 Properties of Indefinite Integrals / 441
C.3.2 Properties of Definite Integrals / 442
C.3.3 Methods of Integration / 444
C.4 Trigonometric Identities / 445
C.5 The Quadratic Formula / 446
C.6 Exercise Problems / 447

Index 449

The original version of this book was revised. An erratum to this book can be found at https://doi.org/10.1007/978-3-319-44738-4_16