
References

- Abraham, R., & Marsden, J. E. (1978). *Foundations of mechanics*. San Francisco: Benjamin Cummings.
- Abraham, R., & Shaw, C. D. (1985). *Dynamics-the geometry of behavior*. Santa Cruz, CA: Aerial Press.
- Aihara, K., Matsumoto, G., & Ichikawa, M. (1985). An alternating periodic-chaotic sequence observed in neural oscillators. *Physics Letters A*, *111*(5), 251–255.
- Allesina, S., & Pascual, M. (2009). Googling food webs: Can an eigenvector measure species' importance for coextinctions? *PLoS Computational Biology*, *5*(9), e1000494.
- Anderson, R. M., May, R. M., & Anderson, B. (1992). *Infectious diseases of humans: Dynamics and control*. Oxford, UK: Oxford University Press.
- Beuter, A., Titcombe, M., Richer, F., Gross, C., & Guehl, D. (2001). Effect of deep brain stimulation on amplitude and frequency characteristics of rest tremor in Parkinson's disease. *Thalamus and Related Systems*, *1*(3), 203–211.
- Bodine, E. N., Lenhart, S., & Gross, L. J. (2014). *Mathematics for the life sciences*. Princeton, NJ: Princeton University Press.
- Boiteux, A., Goldbeter, A., & Hess, B. (1975). Control of oscillating glycolysis of yeast by stochastic, periodic, and steady source of substrate: a model and experimental study. *Proceedings of the National Academy of Sciences*, *72*(10), 3829–3833.
- Brin, S., & Page, L. (1998). The anatomy of a large-scale hypertextual web search engine. *Computer Networks and ISDN systems*, *30*(1), 107–117.
- Chou, H., Berman, N., & Ipp, E. (1992). Oscillations of lactate released from islets of Langerhans: evidence for oscillatory glycolysis in beta-cells. *American Journal of Physiology-Endocrinology and Metabolism*, *262*(6), E800–E805.
- Crouse, D. T., Crowder, L. B., & Caswell, H. (1987). A stage-based population model for loggerhead sea turtles and implications for conservation. *Ecology*, *68*(5), 1412–1423.

- Crutchfield, J., Farmer, J. D., Packard, N., & Shaw, R. (1986). Chaos. *Scientific American*, 254(12), 46–57.
- Farkas, I., Helbing, D., & Vicsek, T. (2002). Social behaviour: Mexican waves in an excitable medium. *Nature*, 419(6903), 131–132.
- Fath, B. D., & Patten, B. C. (1999). Review of the foundations of network environ analysis. *Ecosystems*, 2(2), 167–179.
- Gardner, T. S., Cantor, C. R., & Collins, J. J. (2000). Construction of a genetic toggle switch in *Escherichia coli*. *Nature*, 403(6767), 339–342.
- Garfinkel, A. (1981). *Forms of explanation: Rethinking the questions in social theory*. New Haven, CT: Yale University Press.
- Garfinkel, A. (1983). A mathematics for physiology. *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology*, 245(4), R455–R466.
- Garfinkel, A., Spano, M., Ditto, W., & Weiss, J. N. (1992). Controlling cardiac chaos. *Science*, 257, 1230–1235.
- Ghosh, A., & Chance, B. (1964). Oscillations of glycolytic intermediates in yeast cells. *Biochemical and biophysical research communications*, 16(2), 174–181.
- Gragani, A., Rinaldi, S., & Feichtinger, G. (1997). Cyclic dynamics in romantic relationships. *International Journal of Bifurcation and Chaos*, 7(11), 2611–2619.
- Grier, D. A. (2013). *When computers were human*. Princeton, NJ: Princeton University Press.
- Hannon, B. (1973). The structure of ecosystems. *Journal of Theoretical Biology*, 41(3), 535–546.
- Hastings, A., Hom, C. L., Ellner, S., Turchin, P., & Godfray, H. C. J. (1993). Chaos in ecology: Is mother nature a strange attractor? *Annual Review of Ecology and Systematics*, 24(1), 1–33.
- Hastings, A., & Powell, T. (1991). Chaos in a three-species food chain. *Ecology*, 72(3), 896–903.
- Hayashi, H., Ishizuka, S., Ohta, M., & Hirakawa, K. (1982). Chaotic behavior in the onchidium giant neuron under sinusoidal stimulation. *Physics Letters A*, 88(8), 435–438.
- Hilborn, R. C. (2000). *Chaos and nonlinear dynamics: An introduction for scientists and engineers*. Oxford, UK: Oxford University Press.
- Hirata, H., Yoshiura, S., Ohtsuka, T., Bessho, Y., Harada, T., Yoshikawa, K., et al. (2002). Oscillatory expression of the bHLH factor Hes1 regulated by a negative feedback loop. *Science*, 298(5594), 840–843.
- Hirsch, M. W., Smale, S., & Devaney, R. L. (2012). *Differential equations, dynamical systems, and an introduction to chaos*. Cambridge, MA: Academic Press.

- Hodgkin, A. L., & Huxley, A. F. (1939). Action potentials recorded from inside a nerve fibre. *Nature*, *144*(3651), 710–711.
- Hodgkin, A. L., & Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, *117*(4), 500.
- Holdaway, R. N., & Jacomb, C. (2000). Rapid extinction of the moas (aves: Dinornithiformes): model, test, and implications. *Science*, *287*(5461), 2250–2254.
- Ingalls, B. (2013). *Mathematical modeling in systems biology: An introduction*. Cambridge, MA: MIT Press.
- Izhikevich, E. M. (2007). *Dynamical systems in neuroscience*. Cambridge, MA: MIT Press.
- Keener, J. P., & Sneyd, J. (2009). *Mathematical physiology*. New York: Springer.
- Lahav, G., Rosenfeld, N., Sigal, A., Geva-Zatorsky, N., Levine, A. J., Elowitz, M. B., et al. (2004). Dynamics of the p53-Mdm2 feedback loop in individual cells. *Nature Genetics*, *36*(2), 147–150.
- Laugesen, J., & Mosekilde, E. (2006). Border-collision bifurcations in a dynamic management game. *Computers and Operations Research*, *33*(2), 464–478.
- Licinio, J., Negrão, A. B., Mantzoros, C., Kaklamani, V., Wong, M.-L., Bongiorno, P. B., et al. (1998). Synchronicity of frequently sampled, 24-h concentrations of circulating leptin, luteinizing hormone, and estradiol in healthy women. *Proceedings of the National Academy of Sciences*, *95*(5), 2541–2546.
- Luciani, D. S., Misler, S., & Polonsky, K. S. (2006). Ca²⁺ controls slow nad (p) h oscillations in glucose-stimulated mouse pancreatic islets. *The Journal of physiology*, *572*(2), 379–392.
- Luo, C.-H., & Rudy, Y. (1991). A model of the ventricular cardiac action potential. depolarization, repolarization, and their interaction. *Circulation Research*, *68*(6), 1501–1526.
- Lux, T. (1995). Herd behaviour, bubbles and crashes. *The Economic Journal*, *105*(431), 881–896.
- Mackey, M. C., & Glass, L. (1977). Oscillation and chaos in physiological control systems. *Science*, *197*(4300), 287–289.
- Martien, P., Pope, S., Scott, P., & Shaw, R. (1985). The chaotic behavior of the leaky faucet. *Physics Letters A*, *110*(7), 399–404.
- Martin, C. H., & Wainwright, P. C. (2013). Multiple fitness peaks on the adaptive landscape drive adaptive radiation in the wild. *Science*, *339*(6116), 208–211.
- May, R. M. (1976). Simple mathematical models with very complicated dynamics. *Nature*, *261*(5560), 459–467.
- McLean, A. R., Emery, V. C., Webster, A., & Griffiths, P. D. (1991). Population dynamics of HIV within an individual after treatment with zidovudine. *AIDS*, *5*(5), 485–490.

- Meltzer, M. I., Atkins, C. Y., Santibanez, S., Knust, B., Petersen, B. W., Ervin, E. D., et al. (2014). Estimating the future number of cases in the Ebola epidemic—Liberia and Sierra Leone, 2014–2015. *MMWR Surveill Summ*, 63(Suppl 3), 1–14.
- Miller, R. E., & Blair, P. D. (2009). *Input-output analysis: Foundations and extensions*. Cambridge, UK: Cambridge University Press.
- Monod, J., & Jacob, F. (1961). General conclusions: Teleonomic mechanisms in cellular metabolism, growth, and differentiation. In *Cold Spring Harbor Symposia on Quantitative Biology* (vol. 26, pp. 389–401). Cold Spring Harbor Laboratory Press.
- Mosekilde, E., & Laugesen, J. L. (2007). Nonlinear dynamic phenomena in the beer model. *System Dynamics Review*, 23(2–3), 229–252.
- Patten, B. (1985). Energy cycling, length of food chains, and direct versus indirect effects in ecosystems. *Canadian Bulletin of Fisheries and Aquatic Science*, 213, 119–138.
- Phillips, A. N. (1996). Reduction of HIV concentration during acute infection: Independence from a specific immune response. *Science*, 271(5248), 497.
- Putnam, H. (1975). Philosophy and our mental life. In Language Mind (Ed.), *Mind, language, and reality*. Cambridge, UK: Cambridge University Press.
- Riemann, B. (1873). On the hypotheses which lie at the bases of geometry (translated by William Kingdom Clifford). *Nature*, 8(183, 184), 14–17, 36, 37.
- Scheffer, M., Carpenter, S., Foley, J. A., Folke, C., & Walker, B. (2001). Catastrophic shifts in ecosystems. *Nature*, 413(6856), 591–596.
- Shetterly, M. L. (2016). *Hidden figures: The untold story of the African-American women who helped win the space race*. New York: HarperCollins.
- Smith, W. R. (1983). Qualitative mathematical models of endocrine systems. *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology*, 245(4), R473–R477.
- Sprott, J. (2004). Dynamical models of love. *Nonlinear Dynamics, Psychology, and Life Sciences*, 8(3), 303–314.
- Sterman, J. D. (1989). Modeling managerial behavior: Misperceptions of feedback in a dynamic decision making experiment. *Management Science*, 35(3), 321–339.
- Stewart, I. (1997). *Does God play dice?: The new mathematics of chaos*. Malden, MA: Blackwell Publishers.
- Strogatz, S. H. (1988). Love affairs and differential equations. *Mathematics Magazine*, 61(1), 35.
- Strogatz, S. H. (2014). *Nonlinear dynamics and chaos: With applications to physics, biology, chemistry, and engineering*. Boulder, CO: Westview Press.
- Sturis, J., Polonsky, K. S., Blackman, J. D., Knudsen, C., Mosekilde, E., & Van Cauter E. (1991a). Aspects of oscillatory insulin secretion. In E. Mosekilde & L. Mosekilde (Eds.), *Complexity, Chaos, and Biological Evolution* (vol. 270, pp. 75–93). New York: Plenum Press.

- Sturis, J., Polonsky, K. S., Mosekilde, E., & Van Cauter, E. (1991b). Computer model for mechanisms underlying ultradian oscillations of insulin and glucose. *American Journal of Physiology-Endocrinology and Metabolism*, 260(5), E801–E809.
- Tanner, J. T. (1975). The stability and the intrinsic growth rates of prey and predator populations. *Ecology*, 56(4), 855–867.
- Watt, J., & Young, A. (1962). An attempt to simulate the liver on a computer. *The Computer Journal*, 5(3), 221–227.
- Weinstein, M. S. (1977). Hares, lynx, and trappers. *The American Naturalist*, 111(980), 806–808.
- Wright, S (1932). The roles of mutation, inbreeding, crossbreeding, and selection in evolution. In *Proceedings of the Sixth International Congress on Genetics* (pp. 356–366).
- Yodzis, P. (1998). Local trophodynamics and the interaction of marine mammals and fisheries in the benguela ecosystem. *Journal of Animal Ecology*, 67(4), 635–658.

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