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Pieter Kok

# A First Introduction to Quantum Physics

 Springer

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# Preface

Quantum mechanics is one of the crowning achievements of human thought. There is no theory that is more successful in predicting phenomena over such a wide range of situations—and with such accuracy—than quantum mechanics. From the basic principles of chemistry to the working of the semiconductors in your mobile phone, and from the Big Bang to atomic clocks, quantum mechanics comes up with the goods. At the same time, we still have trouble pinpointing exactly what the theory tells us about nature. Quantum mechanics is hard, but perhaps not as hard as you think. Let us compare it to another great theory of physics: electromagnetism.

When we teach electricity and magnetism in school and university, we start with simple problems involving point charges and line currents. We introduce Coulomb's law, the law of Biot and Savart, the Lorentz force, and so on. After working through some of the most important consequences of these laws, we finally arrive at Maxwell's equations. Advanced courses in electrodynamics then take over and explore the consequences of this unification, treating such topics as waveguides, gauge invariance, relativity. The pedagogical route is going from the simple, tangible problems to the general and abstract theory. You need to know quite a bit of electromagnetism and vector calculus before you can appreciate the beauty of Maxwell's equations.

The situation in teaching quantum mechanics is generally quite different. Instead of simple experimentally motivated problems, a first course in quantum mechanics often takes a historical approach, describing Planck's solution of black-body radiation, Einstein's explanation of the photoelectric effect, and Bohr's model for the atom from 1913. This is then followed by the introduction of the Schrödinger equation. The problem is that appreciating Schrödinger's equation requires a degree of familiarity with the corresponding classical solutions that most students do not yet have at this stage. As a result, many drown in the mathematics of solving the Schrödinger equation and never come to appreciate the subtle and counterintuitive aspects of quantum mechanics as a fundamental theory of nature.

It does not have to be like this. We can develop the core principles of quantum mechanics based on very simple experiments and without requiring much prior mathematical knowledge. By exploring idealised behaviour of photons in

interferometers, electron spins in magnetic fields, and the interaction of simple two-level atoms with light, we can put our finger quite precisely on the strange, puzzling, and wonderful aspects of nature as described by quantum mechanics. We can then illustrate the theory with modern applications such as gravitational wave detection, magnetic resonance imaging, atomic clocks, quantum computing and teleportation, scanning tunnelling microscopy, and precision measurements.

Another reason to write this book was to make use of the wonderful possibilities that are offered by new media. Physics is an experimental science, and seeing how systems behave in interactive figures when you nudge them in the right way hopefully gives the reader an immediate connection between the experiments and the physical principles behind them. That is why I have included many interactive elements to accompany the text, which are available online. I firmly believe that replacing static figures on a page with interactive and animated content can be a great pedagogical tool when used correctly.

This book introduces quantum mechanics from simple experimental considerations, requiring only little mathematics at the outset. The key mathematical techniques such as complex numbers and matrix multiplication are introduced when needed, and are kept to the minimum necessary to understand the physics. However, a full appreciation of the theory requires that you also take a course on linear algebra. Sections labelled with  indicate topics that are not part of the core material in the book, but denote important applications of the theory. They are the reason we care about quantum mechanics. The first half of this book is devoted to the basic description of quantum systems. We introduce the state of a system, evolution operators and observables, and we learn how to calculate probabilities of measurement outcomes. The second half of the book deals with more advanced topics, including entanglement, decoherence, quantum systems moving in space, and a more in-depth treatment of uncertainty in quantum mechanics. We end this book with a chapter on the interpretation of quantum mechanics and what the theory says about reality. This is the most challenging chapter, and it relies heavily on all the material that has been developed in the preceding nine chapters.

I am greatly indebted to my colleague Antje Kohnle from the University of St. Andrews, who helped me navigate the pitfalls of interactive content, and without whom this book would have been much less readable. I also wish to thank Dan Browne, Mark Everitt, and Derek Raine for deep and extended discussions on how to organise a first course in quantum mechanics. Finally, I want to thank Rose, Xander, and Iris for their patience and support during the writing of this book.

Sheffield, UK  
April 2018

Pieter Kok

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