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Analysis for Computer Scientists

Foundations, Methods, and Algorithms

Second Edition

Translated in collaboration with Elisabeth Bradley

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Preface to the Second Edition

We are happy that Springer Verlag asked us to prepare the second edition of our textbook *Analysis for Computer Scientists*. We are still convinced that the algorithmic approach developed in the first edition is an appropriate concept for presenting the subject of analysis. Accordingly, there was no need to make larger changes.

However, we took the opportunity to add and update some material. In particular, we added hyperbolic functions and gave some more details on curves and surfaces in space. Two new sections have been added: One on second-order differential equations and one on the pendulum equation. Moreover, the exercise sections have been extended considerably. Statistical data have been updated where appropriate.

Due to the essential importance of the MATLAB programs for our concept, we have decided to provide these programs additionally in Python for the users' convenience.

We thank the editors of Springer, especially Simon Rees and Wayne Wheeler, for their support during the preparation of the second edition.

Innsbruck, Austria
March 2018

Michael Oberguggenberger
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Preface to the First Edition

Mathematics and mathematical modelling are of central importance in computer science. For this reason the teaching concepts of mathematics in computer science have to be constantly reconsidered, and the choice of material and the motivation have to be adapted. This applies in particular to mathematical analysis, whose significance has to be conveyed in an environment where thinking in discrete structures is predominant. On the one hand, an analysis course in computer science has to cover the essential basic knowledge. On the other hand, it has to convey the importance of mathematical analysis in applications, especially those which will be encountered by computer scientists in their professional life.

We see a need to renew the didactic principles of mathematics teaching in computer science, and to restructure the teaching according to contemporary requirements. We try to give an answer with this textbook which we have developed based on the following concepts:

1. algorithmic approach;
2. concise presentation;
3. integrating mathematical software as an important component;
4. emphasis on modelling and applications of analysis.

The book is positioned in the triangle between mathematics, computer science and applications. In this field, algorithmic thinking is of high importance. The algorithmic approach chosen by us encompasses:

- a. development of concepts of analysis from an algorithmic point of view;
- b. illustrations and explanations using MATLAB and maple programs as well as Java applets;
- c. computer experiments and programming exercises as motivation for actively acquiring the subject matter;
- d. mathematical theory combined with basic concepts and methods of *numerical analysis*.

Concise presentation means for us that we have deliberately reduced the subject matter to the essential ideas. For example, we do not discuss the general convergence theory of power series; however, we do outline Taylor expansion with an estimate of the remainder term. (Taylor expansion is included in the book as it is an

indispensable tool for modelling and numerical analysis.) For the sake of readability, proofs are only detailed in the main text if they introduce essential ideas and contribute to the understanding of the concepts. To continue with the example above, the integral representation of the remainder term of the Taylor expansion is derived by integration by parts. In contrast, Lagrange's form of the remainder term, which requires the mean value theorem of integration, is only mentioned. Nevertheless we have put effort into ensuring a self-contained presentation. We assign a high value to *geometric intuition*, which is reflected in a large number of illustrations.

Due to the terse presentation it was possible to cover the whole spectrum from foundations to interesting *applications of analysis* (again selected from the viewpoint of computer science), such as fractals, L-systems, curves and surfaces, linear regression, differential equations and dynamical systems. These topics give sufficient opportunity to enter various *aspects of mathematical modelling*.

The present book is a translation of the original German version that appeared in 2005 (with the second edition in 2009). We have kept the structure of the German text, but took the opportunity to improve the presentation at various places.

The contents of the book are as follows. Chapters 1–8, 10–12 and 14–17 are devoted to the basic concepts of analysis, and Chapters 9, 13 and 18–21 are dedicated to important applications and more advanced topics. The Appendices A and B collect some tools from vector and matrix algebra, and Appendix C supplies further details which were deliberately omitted in the main text. The employed software, which is an integral part of our concept, is summarised in Appendix D. Each chapter is preceded by a brief introduction for orientation. The text is enriched by computer experiments which should encourage the reader to actively acquire the subject matter. Finally, every chapter has exercises, half of which are to be solved with the help of computer programs. The book can be used from the first semester on as the main textbook for a course, as a complementary text or for self-study.

We thank Elisabeth Bradley for her help in the translation of the text. Further, we thank the editors of Springer, especially Simon Rees and Wayne Wheeler, for their support and advice during the preparation of the English text.

Innsbruck, Austria
January 2011

Michael Oberguggenberger
Alexander Ostermann

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