Astronautics

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The Physics of Space Flight

Third Edition

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Cover: The Space Shuttle *Atlantis* launched on February 7, 2008, to ferry on its 29th flight the European science laboratory *Columbus* to the International Space Station. (Used with permission of NASA)

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This book is dedicated to the astronauts and cosmonauts, who lost their lives in the pursuit of space exploration

Preface to the Third Edition

This textbook is about all basic physical aspects of spaceflight. Not all have been covered in the past editions. So, what is new in this third edition? First, there are new sections covering new topics, such as

- Sections 1.2 and 1.3 dealing with the physics of a jet engine and general rocket performance have been widely extended to more sophisticated effects.
- Sections 7.4.5 and 7.4.6 describe two general solutions to Newton's gravitational equation of motion.
- Section 7.7 studies stellar orbits, which are not subject to the standard but more general types of gravitational potentials.
- Hypersonic flow theory for reentry vehicles is expounded in Sect. 6.2 as a basis to understand how lift and drag come about and in particular how both depend on the angle of attack, the most important control parameter to guide a winged body through the flight corridor (see Fig. 10.22).
- Accordingly, the reentry of a Space Shuttle, which in this book even more serves as a case study, is explained in Sect. 10.7 in greater detail and in terms of NASA terminology.
- In Sect. 8.1, the different basic types of orbit maneuvers are discussed and exemplified.
- A new form of solution of Lambert's problem is derived in Sect. 8.2.3, which is visualized in Fig. 8.8.
- Section 8.4.3 discusses modern super-synchronous transfer orbits to GEO.
- Relative motion in near-circular orbits is examined in Sect. 8.5.4.
- The virial theorem for bounded and unbounded n-body systems is derived in Sect. 11.1.2 and used to discuss the stability of an n-body system.
- Section 12.3 (*Gravitational Perturbation Effects*) has been revised and greatly extended including other and higher order perturbation terms.
- Chapter 14 has been radically revised: There is a new Sect. 14.1 on orbit geometric issues (eclipse duration and access area) and a fully revised Sect. 14.2 on orbit determination.

– There is a whole new Chap. 16 dedicated to thermal radiation physics and modeling. It serves the same purpose as Chap. 15 *Spacecraft Attitude Dynamics*, namely to provide insight into some basic and important physics of a spacecraft in space.

Some sections have been substantially revised and there a hundreds more or less significant extensions of established topics of space fight as already covered in the 2nd version of this textbook.

I put a lot of effort into introducing and using a proper terminology, or establishing one if not existent. An example of the former is the distinction between orbital velocity v, angular velocity ω, angular frequency ω_i, and orbital frequency n, which are sometimes confused. Orbital velocity v is the speed of motion of a body on an orbit. Angular velocity ω is the instantaneous speed of angular motion, while angular frequency ω_i is the number of revolutions in a given time. Finally, orbital frequency n (a.k.a. mean motion) is the time average of the angular velocity over one orbital period T (see Eq. (7.4.10)). Thus, $n = 2\pi/T$; it therefore can be considered both as a mean angular velocity (i.e., mean angular motion) and as a frequency, the orbital frequency. Because proper terminology is essential, the conventional "symbols used" table on the following pages also serves the purpose of enabling one to look up the proper terminology for a physical quantity.

Because physics is independent of the choice of the reference system, the third version consequently uses a reference system-free vector notation (except auxiliary corotating reference systems in Sects. 6.3 and 7.3). All reference systems, the transformations between them, and the vector representations in the different common reference systems are summed up in Sect. 13.1.

Finally, I feel the need to a very personal comment on textbooks in general. When I was a student, I bought some expensive but basic physics textbooks, which are still in my office shelf and serve as my reference books, because true physics is eternal. Compare buying a textbook with a marriage. You do not just buy it. It must have a kind of visual—a tactile sensuality: You open it with joyful anticipation. Your fingers glide over the pages, and they slowly turn one page after the other. You like the layout, the way the book talks to you, and how it explains the world from a point of view you have never considered before. You just love it, and thus it will become part of your daily scientific work. You may forget little physical details, but you will always remember that the one you are looking for is on top of the left-hand page somewhere in the middle of the book. You will never forget that visual detail, and therefore you will always find the answer to your question quite swiftly. I have about a handful of such key textbooks, which I would not sell in my lifetime. I sense that these books were written for guiding me through my scientific life. For me, writing this book was for giving back to other people what many scientists before had given to me. We all are standing on the shoulders of giants. May this textbook keep and pass the body of basic knowledge to you and future generations.

Garching, Germany Ulrich Walter

Preface to the Second Edition

Textbooks are subject to continuous and critical scrutiny of students. So is this one. Having received many questions to the book in my lectures and by e-mail, I constantly improved and updated the content such that already after three years it was time to have also the reader benefit from this. You will therefore find the textbook quite revised as for instance rocket staging (Chapter 3), engine design (Section 4.4), radial orbits (Section 7.5), or the circular restricted thee-body problem (Section 11.4). But there are also new topics, namely Lambert transfer (Section 8.2), relative orbits (Section 8.5), and orbital rendezvous (Section 8.6), higher orbit perturbations including frozen orbits (Sections 12.3.6 and 12.3.7), resonant perturbations and resonant orbits (Section 12.4), and relativistic perturbations (Section 12.6.2). Along with this also the structure of the content has changed slightly. Therefore the section and equation numbers are not always identical to the first edition.

Nevertheless the overall structure still serves the same intention: It is set up for a two semester course on astronautics. Chapter 1–7 (except Sect. 1.4), Section 8.1, and Chapters 9–10 is the basic subject matter an aerospace student should know or have been exposed to at least once. The sequence of the chapters is first rocket basics (Chapter 1–5), thereafter a flight into space "once around", starting with ascent flight (Chapter 6), then space orbits (Chapter 7) and basic orbital maneuvers (Section 8.1), interplanetary flight (Chapter 9), and finally reentry (Chapter 10). The second part of the textbook is more advanced material, which I lecture together with satellite technology in an advanced course for true rocket scientists and space engineers.

The careful reader might have noticed that the book now comes with a subtitle: *The Physics of Space Flight*. This was decided to provide a quick comprehension of the nature of this textbook. In addition, because the Space Shuttle and the ISS are running examples in this textbook, a picture of the launching Space Shuttle Atlantis was chosen as a new frontispiece. Unfortunately, I couldn't find an equally attractive picture of my Space Shuttle Columbia.

Preface to the First Edition

There is no substitute for true understanding

Kai Lai Chung

If you want to cope with science, you have to understand it – truly understand it. This holds in particular for astronautics. "To understand" means that you have a network of relationships in your mind, which permits you to deduce an unknown fact from well-known facts. The evolution of a human being from birth to adulthood and beyond consists of building up a comprehensive knowledge network of the world, which makes it possible to cope with it. That you are intelligent just means that you are able to do that – sometimes you can do it better, and sometimes worse.

True understanding is the basis of everything. There is nothing that would be able to substitute true understanding. Computers do not understand – they merely carry out programmed deterministic orders. They do not have any understanding of the world. This is why even a large language computer will always render a false translation of the phrase: "He fed her cat food." Our world experience intuitively tells us that "He fed a woman's cat some food." But a computer does not have world experience, and thus does not generally know that cat food is nasty for people. Most probably, and according to the syntax, it would translate it as: "He fed a woman some food that was intended for cats.", what the Google translator actually does when translating this phase into other languages. No computer program in the world is able to substitute understanding. You have to understand yourself. Only when you understand are you able to solve problems by designing excellent computer programs. Nowadays, real problems are only solved on computers – written by bright engineers and scientists.

The goal of this book is to build up a network of astronautic relationships in the mind of the reader. If you don't understand something while reading this book, I made a mistake. The problem of a relational network, though, is that the underlying logic can be very complex, and sometimes it seems that our brains are not suitable for even the simplest logic. If I asked you, "You are not stupid, are you?", you would normally answer, "No!" From a logical point of view, a double negation of

an attribute is the attribute itself. So your "No!" means that you consider yourself stupid. You, and also we scientists and engineers, do not want this embarrassing mistake to happen time and time again, and so we use mathematics. Mathematical logic is the guardrail of human thinking. Physics, on the other hand, is the art of applying this logic consistently to nature in order to be able to understand how it works. So it comes as no surprise to find a huge amount of formulas and a lot of physics in this book.

Some might think this is sheer horror. But now comes the good news. Most of the formulas are just intermediate steps of our elaborations. To understand astronautics, you only need to engage in the formulas shaded gray and to remember those bordered black. There you should pause and try to understand their meaning because they will tell you the essential story and lift the secrets of nature. Though you don't need to remember all the other formulas, as a student you should be able to derive these stepping stones for yourself. Thereby you will always be able to link nodes in your relational network whenever you deem it necessary. To treat formulas requires knowing a lot of tricks. You will learn them only by watching others doing such "manipulation" and, most importantly, by doing it yourself. Sometimes you will see the word "exercise" in brackets. This indicates that the said calculation would be a good exercise for you to prove to yourself that you know the tricks. Sometimes it might denote that there is not the space to fully lay out the needed calculation because it is too lengthy or quite tricky. So, you have to guess for yourself whether or not you should do the exercise. Nonetheless, only very few of you will have to derive formulas professionally later. For the rest of you: just try to follow the story and understand how consistent and wonderful nature is. Those who succeed will understand the words of Richard Feynman, the great physicist, who once expressed his joy about this by saying: "The pleasure of finding things out."

Take the pleasure to find out about astronautics.

The original version of the book was revised: The correction to the book is available at https://doi.org/10.1007/978-3-319-74373-8_17

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Many figures in this book were drawn by the interactive plotting program *gnuplot v4.0*. My sincere thanks to its authors Thomas Williams, Colin Kelley, Hans-Bernhard Bröker, and many others for establishing and maintaining this versatile and very useful tool for free public use. The author is grateful to the GeoForschungsZentrum Potsdam, Germany's National Research Centre for Geosciences for providing the geoid views and the visualization of the spherical harmonics in the color tables on pages 566, 568, and 569.

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Abbreviations

AOA	Angle of attack
AU	Astronomical unit
CFPAR	Constant flight path angle rate
CM	Center of mass
CPR	Constant pitch rate
CR3BP	Circular restricted three-body problem
EGM96	Earth gravitational model 1996
EL_1	Sun-Earth libration point L_1
EoM	Equation of motion
ET	External tank
EQW	Equinoctial coordinate system EQW (see Sect. 7.3.5)
FPA	Flight path angle
GEO	Geostationary orbit
GEODSS	Ground-based electro-optical deep space surveillance
GG	Gravity gradient
GMT	Greenwich mean time
GMST	Greenwich mean sidereal time
GSO	Geosynchronous orbit
GTO	Geostationary transfer orbit
GVE	Gaussian variational equation
IAU	International Astronomical Union
ICRF	International Celestial Reference Frame
IJK	Cartesian equatorial coordinate system (see Sect. 13.1.4)
ITRF	International Terrestrial Reference Frame
ISS	International Space Station
JD	Julian date
LEO	Low earth orbit, 100 km $< h <$ 2000 km
LL_1	Earth-Moon (Lunar) libration point L_1
LPE	Lagrange's planetary equations
LVLH	Local vertical, local horizontal (reference frame)

MECO	Main engines cut-off
MEO	Medium earth orbit, 2000 km $< h <$ *GEO*
MJD	Modified julian date
NTW	Corotating Cartesian topocentric satellite coordinate system *NTW* (see Sect. 13.1.4)
OMS	Orbital maneuvering system
PQW	Cartesian geocentric perifocal coordinate system *PQW* (see Sect. 13.1.4)
R&D	Rendezvous and docking
R3BP	Restricted three-body problem
RAAN	Right ascension of ascending node
RSW	Corotating Cartesian topocentric satellite coordinate system *RSW* (see Sect. 13.1.4)
RTG	Radioisotope thermoelectric generator
S/C	Spacecraft
SOI	Sphere of influence
SRB	Solid rocket booster
SSME	Space shuttle main engine
SSO	Sun-synchronous orbit
SSTO	Super-synchronous transfer orbit
TAEM	Terminal area energy management
TDRS	Tracking and data relay satellite
TDRSS	Tracking and data relay satellite system
TTPR	Thrust-to-power ratio
UT	Universal time
VDF	Velocity distribution function
WSB	Weak stability boundary

Symbols Used and Terminology

$x,\ a,\ b(.),\ f(\cdot)$	Scalars/scalar-valued functions
$\boldsymbol{x}, \boldsymbol{a}, \boldsymbol{b}(.), \boldsymbol{f}(\cdot)$	Vectors/vector-valued functions
$\boldsymbol{X},\ \boldsymbol{\Omega},\ \boldsymbol{J}(\cdot), \boldsymbol{M}(\cdot)$	Matrices/matrix-valued functions

Superscripts

T	Transpose of a vector or matrix

Subscripts

0	At the beginning (zero); or osculating (momentary)
a	With respect to the atmosphere
air	Atmosphere
apo	Apoapsis
B	Body system
c	Combustion; or commensurate
CM	Center of mass (a.k.a. barycenter)
col	Collision
crit	Critical (maximal deceleration)
D	Aerodynamic drag
div	(Jet) divergence
e	At exit, *or* ejection; or at entry interface
eff	Effective
esc	Escape (velocity)
ex	Exhaust
ext	External
EQW	Equinoctial coordinate system *EQW* (see Sect. 7.3.5)

f	Final (mass); or frozen orbit
F	Force
G	Gravitation
GEO	Geostationary
GG	Gravity gradient
h	Horizontal
H	Hohmann
i	Initial (mass)
id	Ideal engine
I	Inertial reference frame
IJK	Cartesian equatorial coordinate system (see Sect. 13.1.4)
IR	Infrared
ion	Ionic
in	Initial, at entry, incoming
int	Internal
jet	Propellant exhaust jet
kin	Kinetic (energy)
L	Aerodynamic lift; or payload; or libration point
LVLH	Local vertical, local horizontal (reference frame)
max	Maximum
min	Minimum
micro	Microscopic
n	Nozzle; or normal (vertically to …)
NTW	Corotating Cartesian topocentric satellite coordinate system *NTW* (see Sect. 13.1.4)
opt	Optimal (value)
out	Final, at exit, outgoing
p	Propellant; or planet; or perturbation; or periapsis (only in the case of epoch t_p)
P	Principal axes system; or orbital period
per	Periapsis; or periodic
PQW	Cartesian geocentric perifocal coordinate system *PQW* (see Sect. 13.1.4)
pot	Potential (energy)

r	Radial; or reflection; or radiation
rms	Root-mean-square (a.k.a. quadratic mean)
RSW	Corotating Cartesian topocentric satellite coordinate system *RSW* (see Sect. 13.1.4)
s	Structural
S/C	Spacecraft
sec	Secular
sk	Station keeping
sol	Solar
SOI	Sphere of influence
syn	Synodic
t	Tangential; or throat (of thruster)
T	Transfer orbit
tot	Total
trans	Translation; or transition
TTPR	Thrust-to-power ratio
v	Vertical
VDF	Velocity distribution function
vib	Vibration
θ	Vertically to radial
ω	Rotation (or centrifugal); or argument of periapsis (apsidal line)
Ω	Relating to the ascending node (draconitic)
∞	External, at infinity
$*$	Effective (thrust), total
$\oplus$	Earth
$\odot$	Sun
∇	Spacecraft
$\bullet$	Inner (orbit); or black body
$\bigcirc$	Outer (orbit); or in orbit plane
$\varnothing$	Diameter; or cross section
$\parallel$	Parallel to …
$\perp$	Vertical to … ($A_\perp$ = effectively wetted surface area)
$\times$	At orbit crossing

Latin Symbols

a	Semi-major axis (of a Keplerian orbit); or speed of sound; or acceleration
A	Area
A_p	Daily global index of geomagnetic activity, $0 \leq A_p \leq 400$
b	Semi-minor axis (of a Keplerian orbit); or $b := L \cdot \tan \gamma_e / (2D)$
B	Ballistic coefficient (without index: for drag), (see Eq. (6.2.19))
c	$c := L \cdot \cot \gamma_e / (2D)$; or speed of light
c^*	Characteristic velocity, $c^* := p_0 A_t / m_p$
c_p	Specific heat capacity at constant pressure
c_V	Specific heat capacity at constant volume
C	Jacobi constant
C_3	Characteristic energy, $C_3 := v_\infty^2$
C_∞	Infinite-expansion coefficient
C_D	Drag coefficient
C_f	Thrust coefficient; or skin friction drag coefficient
C_L	Lift coefficient
C_n	Nozzle coefficient (a.k.a. nozzle efficiency)
C_n^m	Multipole coefficient of the cosine term
δx	Variation (small changes) of x
$\delta v_{\|\|}$	Differential increase in orbital velocity due to kick-burn in flight direction
$\delta v_{\perp \mathrm{O}}$	Differential increase in orbital velocity due to kick-burn vertical to flight direction, within orbital plane, outbound
$\delta v_{\perp\perp}$	Differential increase in orbital velocity due to kick-burn vertical to flight direction and vertical to orbital plane, parallel to angular momentum
Δv	Delta-v budget
$\boldsymbol{D}$	Drag force
D	Aerodynamic drag, $D = \|\boldsymbol{D}\|$; or diameter
$diag(\ldots)$	Diagonal matrix with elements $(\ldots)$
e	Eccentricity; or electrical charge unit; or Eulerian number, e = 2.718281828 …
E	Energy; or (elliptic) eccentric anomaly
$Ei(x)$	Exponential integral (see Eq. (10.4.5))
f^x	f function (see definition Eq. (10.4.6))
F	Force (without index: gravitational force); or hyperbolic anomaly

$\boldsymbol{F}_*$	Thrust force (total), (a.k.a. propellant force)
F_*	Thrust (total)
F_e	Ejection thrust
F_{ex}	Momentum thrust
F_p	Pressure thrust
$F_{10.7}$	Daily solar flux index at wavelength of 10.7 cm in units 10^{-22} W m^2 Hz^{-1} (= 1 *Jansky* = 1 *solar flux unit*)
$\boldsymbol{g}(\boldsymbol{r})$	(Earth's) gravitational field
g	(Earth's) mean gravitational acceleration, $g = GM_\oplus/r^2$ (see Sect. 7.1.2)
g_o	(Earth's) mean gravitational acceleration at its surface, $g_0 := g(R_\oplus) = GM_\oplus/R_\oplus^2 = \mu_\oplus/R_\oplus^2 = 9.7982876\ \mathrm{m\,s^{-2}}$ (see Sect. 7.1.2)
G	Gravitational constant, $G = 6.67259 \times 10^{-11}$ m^3 kg^{-1} s^{-2}; or generic anomaly
h	(Mass-specific) angular momentum (i.e., per mass unit); or molar enthalpy; or height (above sea level); altitude
H	Enthalpy; or scale height
i	Inclination
$\boldsymbol{I}$	Inertia tensor
$I_x,\ I_y,\ I_z$	Principal moments of inertia
I_{sp}	(Weight-)specific impulse
j	Charge flow density (a.k.a. charge flux)
j_n	Reduced harmonic coefficient of order n
J_{nm}	Harmonic coefficient
k_B	Boltzmann constant, $k_B = 1.380650 \times 10^{23}$ J K^{-1}
$\boldsymbol{L}$	Lift force
L	Aerodynamic lift, $L = \lvert\boldsymbol{L}\rvert$; or angular momentum
L^*	Characteristic length of a combustion chamber
m	Body mass (without index: of a spacecraft)
m_i	In the collinear configuration: masses ordered according to their index, $m_3 < m_2 < m_1$
m_i'	In the collinear configuration: masses with m_2' located between m_1' and m_3', and $m_1' \geq m_3'$
$\dot{m}$	Mass flow rate (without index: of a spacecraft)
M	Central mass (central body); or total mass of a system of bodies; or mean anomaly; or molar mass
Ma	Mach number $Ma := v/a$

n	Rocket stage number; or mean motion (a.k.a orbital frequency); or mean number of excited degrees of freedom of gas molecules; or particle density
N	Particle number
p	Pressure; or propellant; or linear momentum $p = mv$; or semi-latus rectum, $p := h^2/\mu = a(1 - e^2)$; or $p := H/(\varepsilon_e R)$
P	Power
$P_n(x)$	Legendre polynomials of degree n
$P_n^m(x)$	Unnormalized associated Legendre polynomials of degree n and order m
q	Electrical charge density; or $q := H \cot^2 \gamma_e / (\varepsilon_e R)$
$\dot{q}$	Heat flux (*a.k.a.* heat flow density), $\dot{q} := \dot{Q}/A$
$\dot{Q}$	Heat flow rate
r	Orbit radius; or ratio
$\boldsymbol{r}$	Radial vector, a.k.a. position vector
R	Radius (of a celestial body, in particular Earth's radius); or residual perturbational potential; or universal gas constant, $R = 8.314\ \mathrm{J\,K^{-1}\ mol^{-1}}$
$\boldsymbol{R}$	Rotation (matrix)
$R_\oplus$	Equatorial scale factor of the Earth gravitational model EGM96, $R_\oplus = 6378.1363$ km, equaling roughly Earth's mean equatorial radius
R_s	Specific gas constant of standard atmosphere, $R_s = 286.91\ \mathrm{J\,K^{-1}\ kg^{-1}}$
Re	Reynolds number
S_n^m	Multipole coefficient of the sine term
$\mathrm{sgn}(x)$	Sign function (sign of x): $\mathrm{sgn}(x) = x/\lvert x\rvert$
St	Stanton number, $St \approx 0.1\%$
t	Time
t_p	Time at passage through periapsis, a.k.a. epoch (see end of Sect. 7.3.1)
T	Temperature; or orbital period; or torque
u	Argument of latitude
$\boldsymbol{u}$	Unit vector as a basis of a reference system (example: $\boldsymbol{u}_r \equiv \hat{\boldsymbol{r}} = \boldsymbol{r}/r$)

U	Internal energy of a gas; or total electrical voltage; or potential (for which holds $\boldsymbol{F} = -m \cdot dU/d\boldsymbol{r}$) (without index: gravitational potential)
v	Velocity (orbital v of the spacecraft, or drift v of propellant gas)
$\boldsymbol{v}$	Velocity vector
$v_{\triangleright}$	First cosmic velocity, $v_{\triangleright} = \sqrt{g_0 R_{\oplus}} = 7.905\ \mathrm{km\,s^{-1}}$ (see Eq. (7.4.4))
$v_{\triangleright\triangleright}$	Second cosmic velocity, $v_{\triangleright\triangleright} = \sqrt{2 g_0 R_{\oplus}} = 11.180\ \mathrm{km\,s^{-1}}$ (see Sect. 7.4.4)
v_*	Effective exhaust velocity
v_e	Ejection velocity
$\bar{v}_e$	Mean ejection velocity, $\bar{v}_e := \langle v_e \rangle_\mu$
v_{ex}	Exhaust velocity, $v_{ex} = \eta_{div} \bar{v}_e$
v_h	$v_h := \mu/h$
V	Volume; or electric potential
yr	Year(s)

Greek Symbols

α	Thrust angle; or angle of attack (AOA); or proper acceleration; or mass-specific power output of an electrical plant; or absorptivity (*a.k.a.* absorption coefficient)
β	$\beta := v/c$; or geocentric latitude; or (orbit) beta angle
δ	Deflection angle
$\delta(x)$	Dirac delta function
δ_{nm}	Kronecker delta
δx	A finite (not differential) but small variation of x; or small variation of an orbital element over one orbital revolution; or small error of x
δv	An impulsive maneuver (kick-burn) varying the orbital velocity by δv
Δ	Impact parameter
Δ_i	Distance of the libration point i normalized to the distance between the two primaries in the R3BP (see Sect. 11.3.1)

Δx	The amount of change of parameter x
ε	Structural ratio; or specific orbital energy (*a.k.a.* specific mechanical energy); or expansion ration; or $\varepsilon := v^2/v_c^2$ (see Eq. (10.2.11)); or emissivity (*a.k.a.* emission coefficient, emittance)
ε_0	Vacuum permittivity
γ	Flight path angle; or $\gamma := 1\Big/\sqrt{1-\beta^2} = 1\Big/\sqrt{1-v^2/c^2}$
κ	Heat capacity ratio; or reduced drag; κ_D, or lift, κ_L, coefficient
λ	Payload ratio; or dimensionless altitude variable (see definition Eq. (10.2.12)); or geographic longitude
λ_{nm}	Equilibrium longitude
ζ_d	Discharge correction factor
ζ_v	Velocity correction factor
η	Efficiency (a.k.a. loss factor), *in particular* thermal efficiency
η_{div}	Nozzle-divergence loss factor
η_{ec}	Energy conversion efficiency
ρ	(Atmospheric) mass density; or normalized position vector, $\rho = (\xi, \eta, \zeta) = \boldsymbol{r}/d$; or surface reflectivity; or inverse radius, $\rho := 1/r$
μ	Standard gravitational parameter: $\mu := GM$. For Earth: $\mu_\oplus = GM_\oplus = 3.9860044105 \times 10^5$ km^3 s^{-2} (as of EGM96); or reduced mass, $\mu := m_2/(m_1 + m_2)$ with $m_2 < m_1$; or mass ratio (see Eq. (1.3.1)); or mass flux (*a.k.a.* mass flow density), $\mu := \dot{m}_p/A$ (see Sect. 4.1.3)
$\mu(\theta)$	Angular mass flow distribution function; or bank angle (roll angle)
μ_i	Mass ratio of the ith partial rocket (see Eq. (3.1.7))
σ	Stefan–Boltzmann constant, $\sigma = 5.6704 \times 10^{-8}\, W\, m^{-2}\, K^{-4}$; or proper speed, $\sigma = c \cdot \operatorname{arctanh}\beta$
τ	Proper time (*a.k.a.* eigentime); or dimensionless time
θ	True anomaly, (*a.k.a.* orbit angle); or pitch angle, $\theta := \alpha + \gamma$
χ	Collinear configuration parameter (see Sect. 11.2.2)
$\chi(\lambda, \varepsilon_e)$	See definition Eq. (10.4.7)

$\boldsymbol{\omega}$	Angular velocity vector
ω	Angular velocity, $\omega = d\theta/dt = \lvert\boldsymbol{\omega}\rvert$; or argument of periapsis
ω_i	Angular frequency (*a.k.a.* circular frequency)
Ω	Right ascension of ascending node (RAAN); or effective potential in the CR3BP; or solid angle

Diacritics

$\dot{x}$	First-order time derivative of a quantity x, $\dot{x} = dx/dt$
$\ddot{x}$	Second-order time derivative of a quantity x, $\ddot{x} = d^2x/dt^2$
x'	First-order derivative with respect to a specified variable
x''	Second-order derivative with respect to a specified variable
$\bar{x}$	Geometric mean
$\underline{\text{ex}}$pression	The underlined letters of an arbitrary expression will be used as subscript for an upcoming variable to indicate its special meaning. Example: $\underline{\text{ex}}$haust velocity $\boldsymbol{v}_{ex}$
$\hat{\boldsymbol{r}}$	Unit vector along direction $\boldsymbol{r}$, $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$

Others

:= or =:	Definition equation. The symbol on the colon's side is defined by the expression on the other side of the equation
$= const$	The expression preceding the equation sign is constant (invariant) with respect to a given variable
$\Rightarrow$	From this follows …
@	The condition following this symbol applies to the equation preceding it
$O(\varepsilon^n)$	Landau notation (a.k.a. Big O notation): $O(\varepsilon^n)$ is the magnitude ($\underline{\text{o}}$rder) of the residual power (here ε^n) of a power series expansion. $O(\varepsilon^n)$ means: The residual is of order ε^n
$n!$	Factorial of the nonnegative integer n, $n! = \prod_{k=1}^{n} k$, $0! = 1$
$(2n-1)!!$	Double factorial of the odd positive integer $i = 2k-1$, $(2n-1)!! = \prod_{k=1}^{n} (2k-1)$
[…]	Square brackets denote the units of a given physical quantity
[a, b]	Closed interval between numbers a and b
$\angle(\boldsymbol{a}, \boldsymbol{b})$	Angle between vector **a** and vector **b**
$\langle \boldsymbol{y} \rangle_x$	Average of y with respect to x over interval [a, b], $\langle y \rangle_x := \frac{1}{b-a} \int_a^b y \cdot dx$
$\langle y \rangle$	Time average, $\langle y \rangle \equiv \langle y \rangle_t$

$\bullet$	Inner orbit (relative to another given orbit)
$\bigcirc$	Outer orbit (relative to another given orbit)
$\blacksquare$	What was to be shown (quod erat demonstrandum)
Υ	First point of Aries, a.k.a. vernal point (see Sect. 13.1)
$\uparrow$	Increasing
$\uparrow\uparrow$	Strongly increasing
$\downarrow$	Decreasing
$\downarrow\downarrow$	Strongly decreasing