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Introduction to General Relativity

A Course for Undergraduate Students of Physics

Springer

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Fatti non foste a viver come bruti,
ma per seguir virtute e canoscenza.
(Dante Alighieri, Inferno, Canto XXVI)

Preface

The formulations of the theories of special and general relativity and of the theory of quantum mechanics in the first decades of the twentieth century are a fundamental milestone in science, not only for their profound implications in physics but also for the research methodology. In the same way, the courses of special and general relativity and of quantum mechanics represent an important milestone for every student of physics. These courses introduce a different approach to investigate physical phenomena, and students need some time to digest such a radical change.

In Newtonian mechanics and in Maxwell's theory of electrodynamics, the approach is quite empirical and natural. First, we infer a few fundamental laws from observations (e.g., Newton's Laws) and then we construct the whole theory (e.g., Newtonian mechanics). In modern physics, starting from special and general relativity and quantum mechanics, this approach may not be always possible. Observations and formulation of the theory may change order. This is because we may not have direct access to the basic laws governing a certain physical phenomenon. In such a case, we can formulate a number of theories, or we can introduce a number of ansatzes to explain a specific physical phenomenon within a certain theory if we already have the theory, and then we compare the predictions of the different solutions to check which one, if any, is consistent with observations.

For example, Newton's First, Second, and Third Laws can be directly inferred from experiments. Einstein's equations are instead obtained by imposing some "reasonable" requirements and they are then confirmed by comparing their predictions with the results of experiments. In modern physics, it is common that theorists develop theoretical models on the basis of "guesses" (motivated by theoretical arguments but without any experimental support), with the hope that it is possible to find predictions that can later be tested by experiments.

At the beginning, a student may be disappointed by this new approach and may not understand the introduction of *ad hoc* assumptions. In part, this is because we are condensing in a course the efforts of many physicists and many experiments, without discussing all the unsuccessful—but nevertheless necessary and important—attempts that eventually led to a theory in its final form. Moreover, different students may have different backgrounds, not only because they are students of different disciplines (e.g., theoretical physics, experimental physics, astrophysics, and mathematical physics) but also because undergraduate programs in different countries can be very different. Additionally, some textbooks may follow approaches appreciated by some students and not by others, who may instead prefer different textbooks. This point is quite important when we study for the first time the theories of special and general relativity and the theory of quantum mechanics, because there are some concepts that at the beginning are difficult to understand, and a different approach may make it easier or harder.

In the present textbook, the theories of special and general relativity are introduced with the help of the Lagrangian formalism. This is the approach employed in the famous textbook by Landau and Lifshitz. Here, we have tried to have a book more accessible to a larger number of students, starting from a short review of Newtonian mechanics, reducing the mathematics, presenting all the steps of most calculations, and considering some (hopefully illuminating) examples. The present textbook dedicates quite a lot of space to the astrophysical applications, discussing Solar System tests, black holes, cosmological models, and gravitational waves at a level adequate for an introductory course of general relativity. These lines of research have become very active in the past couple of decades and have attracted an increasing number of students. In the last chapter, students can get a quick overview of the problems of Einstein's gravity and current lines of research in theoretical physics.

The textbook has 13 chapters, and in a course of one semester (usually 13–15 weeks) every week may be devoted to the study of one chapter. Note, however, that Chaps. 1–9 are almost "mandatory" in any course of special and general relativity, while Chaps. 10–13 cover topics that are often omitted in an introductory course for undergraduate students. Exercises are proposed at the end of most chapters and are partially solved in Appendix I.

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Shanghai, China Cosimo Bambi
April 2018

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Conventions

There are several conventions in the literature and this, unfortunately, can sometimes generate confusion.

In this textbook, the spacetime metric has signature $(-+++)$ (convention of the gravity community). The Minkowski metric thus reads

$$||\eta_{\mu\nu}|| = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where here and in the rest of the book the notation $||A_{\mu\nu}||$ is used to indicate the matrix of the tensor $A_{\mu\nu}$.

Greek letters (μ, ν, ρ,…) are used for spacetime indices and can assume the values 0, 1, 2,…, n, where n is the number of spatial dimensions. Latin letters (i, j, k,…) are used for space indices and can assume the values 1, 2,…, n. The time coordinate can be indicated either as t or as x^0. The index associated with the time coordinate can be indicated either as t or as 0, for example V^t or V^0.

The Riemann tensor is defined as

$$R^{\mu}_{\nu\rho\sigma} = \frac{\partial \Gamma^{\mu}_{\nu\sigma}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\mu}_{\nu\rho}}{\partial x^{\sigma}} + \Gamma^{\mu}_{\lambda\rho}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\lambda}_{\nu\rho},$$

where $\Gamma^{\mu}_{\nu\rho}$s are the Christoffel symbols

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\frac{\partial g_{\lambda\rho}}{\partial x^{\nu}} + \frac{\partial g_{\nu\lambda}}{\partial x^{\rho}} - \frac{\partial g_{\nu\rho}}{\partial x^{\lambda}} \right).$$

The Ricci tensor is defined as $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$. The Einstein equations read

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_{\rm N}}{c^4} T_{\mu\nu} \, .$$

Since the present textbook is intended to be an introductory course on special and general relativity, unless stated otherwise we will explicitly show the speed of light c, Newton's gravitational constant $G_{\rm N}$, and Dirac's constant $\hbar$. In some parts (Chaps. 10 and 13 and Sects. 8.2 and 8.6), we will employ units in which $G_{\rm N} = c = 1$ to simplify the formulas.

Note that ρ will be sometimes used to indicate the energy density, and sometimes it will indicate the mass density (so the associated energy density will be ρc^2).