

# Index

## Symbols

3-J symbol, 493  
properties, 494

## A

action, classical, 318  
adiabatic approximation, *see* time-dependent problems  
angular momentum  
addition, 492  
classical, 186  
angular momentum operator, 187  
commutation relations, 187  
with other operators, 188  
eigenfunctions, 192  
in spherical coordinates, 190  
matrix elements in Dirac notation, 263, 284  
total, 300

## $\hat{L}^2$

eigenfunctions, 194  
eigenvalues, 193  
in spherical coordinates, 191

## B

Baker-Campbell-Hausdorff theorem, 93  
Bell state, 488  
Bell's theorem, 322  
Bell inequality, 21, 324  
CHSH inequality, 324  
electron spin measurements, 324  
hidden variables, 322  
optical polarization measurements, 326  
proof, 323

Bessel's equation, 218  
spherical Bessel functions, 218  
spherical Hankel functions, 224  
spherical Neumann functions, 218  
blackbody spectrum, 7, 25  
Bloch equations  
magnetic, 579, 580  
optical, 580, 581  
Bloch sphere, 582  
Bloch vector, 581  
pseudofield vector, 581  
Bloch's theorem, 150  
Bohr magneton, 292  
Bohr radius, 12  
Bohr theory, 11  
Boltzmann constant, 7  
Born approximation, *see* scattering  
boson, 291

## C

classically allowed region, 120  
classically forbidden region, 120  
Clebsch-Gordan coefficients, 300, 492  
properties, 493  
commutator, 90  
fundamental relationships, 90  
simultaneous eigenfunctions, 91  
Compton wavelength, 12  
continuous eigenvalues, 86, 109  
completeness, 87  
orthogonalization, 87  
units, 87  
Coulomb potential, *see* spherically symmetric potentials

creation operator, *see* raising operator  
 cyclotron frequency, 579  
 cylindrically symmetric potentials, 244  
   2-D isotropic oscillator, 246  
   Landau levels, 248  
   radial momentum, 245

**D**

de Broglie wavelength, 14  
 density matrix, 575  
   time evolution, 576  
 density of states  
   energy, 608  
   matter wave, 110  
   radiation field, 21  
     box normalization, 22  
     periodic boundary conditions, 23  
 destruction operator, *see* lowering operator  
 dipole-dipole interaction, 510  
 Dirac delta function, 46  
 Dirac notation  
   basis kets, 253  
   bra, 254  
   connection with linear algebra, 276  
   eigenvalue equation, 256  
   ket, 253  
   matrix element, 255, 261  
   state vector, 253  
   time-dependent Schrödinger equation, 256,  
     280  
   unit matrices, 255  
   wave function, 260

**E**

effective potential, 187  
 Ehrenfest's theorem, 314  
 eigenfunctions, 78  
 eigenvalues, 78  
 electron  $g$  factor, 294  
 entangled states, 19, 107  
 EPR paradox, 19  
 equation of continuity, 102  
 Euler angles, 466  
 exponential decay, 610

**F**

Fermi energy, 304  
 Fermi's Golden Rule, 610  
   photoionization, 610  
     angular distribution, 613  
     total decay rate, 613

fermion, 291  
 field quantization, *see* photon states  
 fine structure constant, 12  
 Fourier transforms, 43

**G**

Galilean transformation, 490  
 Gauss theorems, 38  
 Gaussian distribution, 41  
 generating function  
   associated Laguerre polynomials, 241  
   Hermite polynomials, 171  
   Legendre polynomials, 195

**H**

Heaviside function, 419  
 Heisenberg representation, 317  
 Heisenberg uncertainty relation, 94  
 Hermite polynomials, 169  
   recursion relations, 170  
 Hilbert space, 253  
 hydrogen atom  
   Dirac equation, 511  
   hyperfine interaction  
     contact terms, 540  
     ground state splitting, 542  
     Hamiltonian, 540  
     level shifts, 522  
     non-contact terms, 546  
     Zeeman splitting, 543  
   in a magnetic field, 523  
     Paschen-Back region, 526, 527  
     Zeeman splitting, 525  
   in an electric field, 527  
   Lamb shift, 521  
   radial integrals, 535  
   relativistic corrections  
     Darwin term, 520  
     fine structure splitting, 518  
     mass correction, 513  
     spin-orbit, 518  
     total fine structure, 520  
 hydrogen spectrum, 13

**I**

infinite square well potential, 120  
   classical probability distributions, 121  
   eigenenergies, 123  
   eigenfunctions, 123, 124  
    $k$ -space distribution, 156  
   momentum probability distribution, 126

quantum revivals, 130, 159  
 spatial probability distribution, 125  
 three dimensions, 181  
 two dimensions, 180  
 wave packet dynamics, 127  
 interaction representation, 557  
 irreducible tensor operators, *see* operators

**K**

Kronig-Penney model, 151  
 Bloch states, 155  
 eigenfunctions, 151  
 transmission coefficient, 153

**L**

ladder operators  
 angular momentum, 270  
   action on kets, 272  
   raising and lowering operators, 271  
 simple harmonic oscillator  
   lowering operator, 264, 265  
   raising operator, 264, 265  
 Laguerre's equation, 228  
   associated Laguerre function, 229  
   properties, 229, 242  
   differential equations, 241  
   generalized Laguerre polynomials, 228  
   properties, 229, 241  
 Landé  $g$  factor, 534  
 Legendre's equation, 192  
   associated Legendre polynomials, 192  
   Legendre polynomials, 195  
   properties, 195

**M**

magnetic moment  
 classical, 291  
 quantum, 292  
 Markov process, 610  
 matrix  
   Hermitian, 284  
   inverse, 283  
   orthogonal, 284  
   transpose, 283  
   unitary, 284  
 measurement, 106  
   collapse of the wave function, 107  
 momentum operator  
   in coordinate space, 89

**N**

normal distribution, *see* Gaussian distribution  
 nuclear  $g$  factor, 539

**O**

one-dimensional potentials, 115  
 barrier, 116, 146  
   eigenfunctions, 146  
   tunneling, 147  
 bound state, 118  
 delta function, 147  
   bound state energy, 149  
   derivative discontinuity, 148  
   transmission, 149  
 double barrier, 159  
 general considerations, 115  
 reflecting, 117  
 square well, 118, 137  
   eigenenergies, 139  
   eigenfunctions for  $E < V_0$ , 138  
   eigenfunctions for  $E > V_0$ , 143  
   electromagnetic scattering analogue,  
     145  
   strength parameter, 140  
 step, 118  
   eigenfunctions for  $E < V_0$ , 133  
   eigenfunctions for  $E > V_0$ , 135  
 operators, 69  
   commutator, 90  
   coupled tensor, 510  
   expectation value, 79  
     time dependence, 104, 576  
     using density matrix, 575  
   function of coordinates, 72  
   function of momenta, 89  
   Hamiltonian, 71  
   Hermitian  
     completeness, 80, 85  
     definition, 80  
     orthogonality, 81  
   inner product, 81  
   irreducible tensor, 499  
     commutators with  $J$ , 500  
   kinetic energy, 78  
   linear, 252  
   momentum, 78  
     acting on functions of coordinates, 93  
     eigenfunctions, 97  
     in coordinate space, 97  
     matrix elements in coordinate  
       representation, 258

operators (*cont.*)  
 position, 78  
   acting on functions of momentum, 93  
   eigenfunctions, 97  
   matrix elements in momentum representation, 259  
 potential energy, 78  
 scalar, 496  
 tensor, 498  
 transformation  
   under rotation, 479  
   under translation, 478  
 vector, 496  
   commutators with  $J$ , 498  
 optical theorem, *see* scattering

## P

parity operator, 98, 472  
   eigenvalues and eigenfunctions, 98  
 Parseval's theorem, 49  
 path integral approach, 318  
 Pauli matrices, 296  
   properties, 296  
 permutation operator, 301  
   eigenfunctions, 301  
 perturbation theory, 339  
   degenerate, 350  
     linear Stark effect, 354  
     perturbed matrix, 350  
   non-degenerate, 340  
     eigenenergies, 342  
     eigenfunctions, 342  
     eigenkets, 342  
     perturbed matrix, 343  
     perturbed oscillator, 346  
     van der Waals interaction, 347  
   time-dependent, 593  
     atom + radiation pulse, 594  
     oscillator in electric field, 595  
 photoelectric effect, 9  
 photon states, 614  
   field quantization, 614  
   vacuum state, 614  
 Planck constant, 8  
 plane wave expansion  
   in Legendre polynomials, 426, 454  
   in spherical harmonics, 290, 453  
 Poisson bracket, 257  
 postulates, 77  
 potentials  
   separable, 179  
 power broadening, 579  
 probability current density, 103

propagator  
   Feynman, 319, 321  
   free-particle, 62, 322  
   gravitational potential, 334  
   simple harmonic oscillator, 335  
 pseudofield vector, *see* Bloch equations  
 pulse area, *see* two-level atom

## Q

quantum nondemolition, 107  
 quantum revivals, 607  
 quantum teleportation, 329  
   Bell states, 330  
   key distribution, 330  
   quantum gate operation, 330

## R

Rabi frequency, 565  
   generalized, 570  
 Rabi oscillations, 561, 571  
 radial momentum  
   classical, 210  
   cylindrical coordinates, 212  
   quantum operator, 211  
 Rayleigh-Jeans Law, 24  
 reduced mass, 226  
 reflection coefficient  
   amplitude  
     square well, 144  
     step potential, 133, 135  
   intensity  
     square well, 144  
     step potential, 133, 136  
 rotating-wave approximation, 566, 567, 572, 576, 580  
   for circular polarization, 592  
 rotation  
   active, 465  
   matrices, 474  
     for  $j = 1/2$ , 477, 482  
     for  $\ell = 1$ , 482  
     for spin, 476  
     properties, 475  
   operator, 473  
     in terms of Euler angles, 473  
   passive, 463  
   transformation of kets, 474  
 rotation matrix  
   active, 465, 467  
   passive, 464, 466  
 RWA, *see* rotating-wave approximation  
 Rydberg, 12

## S

- scattering
  - Born approximation, 446
  - Gaussian potential, 449
  - Green function, 447
  - scattering amplitude, 448
  - spherical well potential, 450
  - validity conditions, 449
- classical, 409
  - attractive potentials, 416
  - deflection angle, 411
  - differential cross section, 410
  - glory, 418
  - hard sphere, 413, 436
  - orbiting, 418
  - rainbow, 416
  - repulsive potentials, 415
  - scattering angle, 411
  - spherical well, 418
  - total cross section, 411
- differential cross section, 423
- electromagnetic
  - by dielectric sphere, 419
  - deflection angle, 420, 421
  - glory, 421
  - rainbow, 421
- hard sphere, 436
  - high energy limit, 455
- one-dimensional
  - by delta function, 407
  - by double barrier, 407
  - by square well, 406
  - by various potentials, 404
  - steady-state approach, 391
  - time delay, 403
  - wave packet approach, 395
- optical theorem, 429
- partial waves, 424
  - differential cross section, 428
  - hard sphere, 434
  - partial wave phase, 426
  - scattering amplitude, 427
  - semi-classical approximation, 431
  - spherical barrier, 444
  - spherical well, 436
  - total cross section, 428
- scattering amplitude, 422
- scattering length, 441
- total cross section, 423
  - uncertainty principle estimate, 424
- Schmidt orthogonalization, 84
- Schrödinger equation
  - general solution, 72, 101
  - in momentum space, 262
  - time-dependent, 56
  - time-independent, 72
- Schrödinger's cat, 18
- selection rules, 583
  - circular polarization
    - without RWA, 592
  - electric dipole, 586
  - magnetic dipole, 587
- Semiclassical approximation, *see* WKB
  - approximation
- simple harmonic oscillator
  - 3-D isotropic, 235
    - Bohr theory, 235
    - classical radial probability distribution, 239
    - effective potential, 236
    - eigenenergies, 235
    - eigenfunctions, 238
    - radial probability distribution, 239
    - radial wave functions, 239
  - one dimension, 163
    - classical probability distributions, 165
    - coherent state, 174, 267
    - dimensionless variables, 165
    - eigenenergies, 169
    - eigenfunctions, 171
    - number operator, 264
    - potential energy, 164
    - recursion relations, 171
  - three dimensions, 182
  - two dimensions, 181
- simultaneous eigenfunctions, 91
- Slater determinant, 303
- spherical harmonics, 194
  - classical polar angle probability
    - distribution, 197, 201
  - physical interpretation, 196
- spherically symmetric potentials, 207
  - 3-D Isotropic oscillator, *see* simple harmonic oscillator
- boundary conditions, 215
- classical radial probability distribution, 208
- Coulomb, 225
  - classical radial probability distribution, 232
  - degeneracy, 229
  - effective potential, 207
  - eigenenergies, 228
  - eigenfunctions, 229
  - radial equation, 226
  - radial probability distribution, 232, 233
  - radial wave functions, 230
- effective potential, 207
- finite well, 223

spherically symmetric potentials (*cont.*)  
 effective potential, 223  
 eigenenergy equation, 224  
 eigenfunctions, 224  
 infinite well, 215  
   classical radial probability distribution, 220  
   effective potential, 216  
   eigenenergies, 219  
   eigenfunctions, 218  
 radial equation, 213  
 radial equation for  $u_l(r)$ , 214  
 radial probability distribution, 217

spin  
 in a magnetic field, 558

spin angular momentum  
 magnetic moment, 294  
 singlet state, 302  
 spin matrices, 295  
 triplet state, 302

spin-orbit interaction, 298, 299

spontaneous decay, 359, 613  
 decay rate, 619

square well potential, *see* one-dimensional potentials

Stark effect  
 in hydrogen, 528  
 linear, 354  
 nonlinear, 372

Stefan-Boltzmann law, 26

Stern-Gerlach experiment, 293

Stokes theorems, 38

sudden approximation, *see* time-dependent problems

**T**

Taylor series, 35

tensor operator, *see* operators

time-dependent perturbation theory, *see* perturbation theory

time-dependent problems  
 adiabatic approximation, 597  
   adiabatic switching, 605  
   bounded continuum, 620  
   sudden approximation, 600

translation  
 active, 463  
 operator, 471  
 passive, 462

transmission coefficient  
 amplitude  
   square well, 144  
   step potential, 135

intensity  
 barrier, 147  
 delta function potential, 149  
 square well, 144  
 step potential, 136

tunneling, 147

two-level atom, 563  
 in optical field, 563  
 density matrix equations, 577, 578  
 detuning, 567  
 dipole approximation, 563  
 field-interaction representation, 572  
 pulse area, 569  
 resonance approximation, 567  
 rotating-wave approximation, 567  
 steady-state solution, 578

**U**

ultraviolet catastrophe, 8

**V**

variational approach, 361  
 combined with perturbation theory, 362, 365  
   Stark shift, 372  
   van der Waals energy, 368  
 helium atom, 368  
 linear potential, 364  
 oscillator potential, 362  
 upper energy bound, 361

vector operator, *see* operators

**W**

wave function, 15  
 coordinate space, 77  
 minimum uncertainty, 95  
 momentum space, 78

wave packet  
 Gaussian, 58  
 spreading, 60

Wien's Law, 7

Wigner-Eckart theorem, 501  
 proof, 503

WKB approximation, 375  
 barrier transmission, 384  
 connection formulas, 379  
 derivation, 385  
 partial wave phases, 432  
 radial wave function, 431  
 semiclassical quantization condition, 381  
 power law potential, 381

validity condition, 378  
wave function, 377

**Z**

Zeeman effect

anomalous, 292

for multi-electron atoms

strong field region, 531

weak field region, 534

in hydrogen, 525

normal, 292

vector model, 532, 534

Zeno

effect, 623

experiment, 628

paradox, 622