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Linear Algebra and Analytic Geometry for Physical Sciences

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To our families

Contents

Introduction

This book originates from a collection of lecture notes that the first author prepared at the University of Trieste with Michela Brundu, over a span of fifteen years, together with the more recent one written by the second author. The notes were meant for undergraduate classes on linear algebra, geometry and more generally basic mathematical physics delivered to physics and engineering students, as well as mathematics students in Italy, Germany and Luxembourg.

The book is mainly intended to be a self-contained introduction to the theory of finite-dimensional vector spaces and linear transformations (matrices) with their spectral analysis both on Euclidean and Hermitian spaces, to affine Euclidean geometry as well as to quadratic forms and conic sections.

Many topics are introduced and motivated by examples, mostly from physics. They show how a definition is natural and how the main theorems and results are first of all plausible before a proof is given. Following this approach, the book presents a number of examples and exercises, which are meant as a central part in the development of the theory. They are all completely solved and intended both to guide the student to appreciate the relevant formal structures and to give in several cases a proof and a discussion, within a geometric formalism, of results from physics, notably from mechanics (including celestial) and electromagnetism.

Being the book intended mainly for students in physics and engineering, we tasked ourselves not to present the mathematical formalism *per se*. Although we decided, for clarity's sake of our readers, to organise the basics of the theory in the classical terms of *definitions* and the main results as *theorems* or *propositions*, we do often not follow the standard sequential form of *definition—theorem—corollary—example* and provided some two hundred and fifty solved problems given as exercises.

Chapter 1 of the book presents the Euclidean space used in physics in terms of applied vectors with respect to orthonormal coordinate system, together with the operation of scalar, vector and mixed product. They are used both to describe the motion of a point mass and to introduce the notion of vector field with the most relevant differential operators acting upon them.

Chapters 2 and 3 are devoted to a general formulation of the theory of finite-dimensional vector spaces equipped with a scalar product, while the Chaps. 4–6 present, via a host of examples and exercises, the theory of finite rank matrices and their use to solve systems of linear equations.

These are followed by the theory of linear transformations in Chap. 7. Such a theory is described in Chap. 8 in terms of the Dirac's Bra-Ket formalism, providing a link to a geometric–algebraic language used in quantum mechanics.

The notion of the diagonal action of an endomorphism or a matrix (the problem of diagonalisation and of reduction to the Jordan form) is central in this book, and it is introduced in Chap. 9.

Again with many solved exercises and examples, Chap. 10 describes the spectral theory for operators (matrices) on Euclidean spaces, and (in Chap. 11) how it allows one to characterise the rotations in classical mechanics. This is done by introducing the Euler angles which parameterise rotations of the physical three-dimensional space, the notion of angular velocity and by studying the motion of a rigid body with its inertia matrix, and formulating the description of the motion with respect to different inertial observers, also giving a characterisation of polar and axial vectors.

Chapter 12 is devoted to the spectral theory for matrices acting on Hermitian spaces in order to present a geometric setting to study a finite level quantum mechanical system, where the time evolution is given in terms of the unitary group. All these notions are related with the notion of Lie algebra and to the exponential map on the space of finite rank matrices.

In Chap. 13, we present the theory of quadratic forms. Our focus is the description of their transformation properties, so to give the notion of signature, both in the real and in the complex cases. As the most interesting example of a non-Euclidean quadratic form, we present the Minkowski spacetime from special relativity and the Maxwell equations.

In Chaps. 14 and 15, we introduce through many examples the basics of the Euclidean affine linear geometry and develop them in the study of conic sections, in Chap. 16, which are related to the theory of Kepler motions for celestial body in classical mechanics. In particular, we show how to characterise a conic by means of its eccentricity.

A reader of this book is only supposed to know about number sets, more precisely the natural, integer, rational and real numbers and no additional prior knowledge is required. To try to be as much self-contained as possible, an appendix collects a few basic algebraic notions, like that of group, ring and field and maps between them that preserve the structures (homomorphisms), and polynomials in one variable. There are also a few basic properties of the field of complex numbers and of the field of (classes of) integers modulo a prime number.

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