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Operators of Differential Dynamic Logic (dL)

dL	Operator	Meaning
$e \geq \tilde{e}$	greater or equals	true if value of e greater-or-equal to $\tilde{e}$
$\neg P$	negation / not	true if P is false
$P \wedge Q$	conjunction / and	true if both P and Q are true
$P \vee Q$	disjunction / or	true if P is true or if Q is true
$P \rightarrow Q$	implication / implies	true if P is false or Q is true
$P \leftrightarrow Q$	bi-implication / equivalent	true if P and Q are both true or both false
$\forall x P$	universal quantifier / for all	true if P is true for all values of variable x
$\exists x P$	existential quantifier / exist	true if P is true for some value of variable x
$[\alpha]P$	$[\cdot]$ modality / box	true if P is true after all runs of HP α
$\langle\alpha\rangle P$	$\langle\cdot\rangle$ modality / diamond	true if P is true after some run of HP α

Statements and effects of Hybrid Programs (HPs)

HP Notation	Operation	Effect
$x := e$	discrete assignment	assigns current value of term e to variable x
$x := *$	nondet. assignment	assigns any real value to variable x
$x' = f(x) \,\&\, Q$	continuous evolution	follow differential equation $x' = f(x)$ within evolution domain Q for any duration
$?Q$	state test / check	test first-order formula Q at current state
$\alpha; \beta$	seq. composition	HP β starts after HP α finishes
$\alpha \cup \beta$	nondet. choice	choice between alternatives HP α or HP β
α^*	nondet. repetition	repeats HP α any $n \in \mathbb{N}$ times

Semantics of dL formula P is the set of states $[\![P]\!] \subseteq \mathscr{S}$ in which it is true

$$
\begin{aligned}
[\![e \geq \tilde{e}]\!] &= \{\omega \in \mathscr{S} : \omega[\![e]\!] \geq \omega[\![\tilde{e}]\!]\} \\
[\![P \wedge Q]\!] &= [\![P]\!] \cap [\![Q]\!] \\
[\![P \vee Q]\!] &= [\![P]\!] \cup [\![Q]\!] \\
[\![\neg P]\!] &= [\![P]\!]^{\complement} = \mathscr{S} \setminus [\![P]\!] \\
[\![\langle\alpha\rangle P]\!] &= [\![\alpha]\!] \circ [\![P]\!] = \{\omega : \nu \in [\![P]\!] \text{ for some state } \nu \text{ such that } (\omega, \nu) \in [\![\alpha]\!]\} \\
[\![[\alpha]P]\!] &= [\![\neg\langle\alpha\rangle\neg P]\!] = \{\omega : \nu \in [\![P]\!] \text{ for all states } \nu \text{ such that } (\omega, \nu) \in [\![\alpha]\!]\} \\
[\![\exists x P]\!] &= \{\omega : \nu \in [\![P]\!] \text{ for some state } \nu \text{ that agrees with } \omega \text{ except on } x\} \\
[\![\forall x P]\!] &= \{\omega : \nu \in [\![P]\!] \text{ for all states } \nu \text{ that agree with } \omega \text{ except on } x\}
\end{aligned}
$$

Semantics of HP α is relation $[\![\alpha]\!] \subseteq \mathscr{S} \times \mathscr{S}$ between initial and final states

$$
\begin{aligned}
[\![x := e]\!] &= \{(\omega, \nu) : \nu = \omega \text{ except that } \nu[\![x]\!] = \omega[\![e]\!]\} \\
[\![?Q]\!] &= \{(\omega, \omega) : \omega \in [\![Q]\!]\} \\
[\![x' = f(x) \,\&\, Q]\!] &= \big\{(\omega, \nu) : \varphi(0) = \omega \text{ except at } x' \text{ and } \varphi(r) = \nu \text{ for a solution} \\
&\qquad \varphi{:}[0, r] \to \mathscr{S} \text{ of any duration } r \text{ satisfying } \varphi \models x' = f(x) \wedge Q\big\} \\
[\![\alpha \cup \beta]\!] &= [\![\alpha]\!] \cup [\![\beta]\!] \\
[\![\alpha; \beta]\!] &= [\![\alpha]\!] \circ [\![\beta]\!] = \{(\omega, \nu) : (\omega, \mu) \in [\![\alpha]\!], (\mu, \nu) \in [\![\beta]\!]\} \\
[\![\alpha^*]\!] &= [\![\alpha]\!]^* = \bigcup_{n \in \mathbb{N}} [\![\alpha^n]\!] \text{ with } \alpha^{n+1} \equiv \alpha^n; \alpha \text{ and } \alpha^0 \equiv ?\mathit{true}
\end{aligned}
$$

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Axiomatization (dL)

$\langle\cdot\rangle$ $\langle\alpha\rangle P \leftrightarrow \neg[\alpha]\neg P$

M[·] $\dfrac{P \to Q}{[\alpha]P \to [\alpha]Q}$

[:=] $[x:=e]p(x) \leftrightarrow p(e)$

G $\dfrac{P}{[\alpha]P}$

[?] $[?Q]P \leftrightarrow (Q \to P)$

[′] $[x' = f(x)]p(x) \leftrightarrow \forall t \geq 0\, [x := y(t)]p(x)$ $\quad (y'(t) = f(y))$

[∪] $[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$

[;] $[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$

[*] $[\alpha^*]P \leftrightarrow P \wedge [\alpha][\alpha^*]P$

K $[\alpha](P \to Q) \to ([\alpha]P \to [\alpha]Q)$

I $[\alpha^*]P \leftrightarrow P \wedge [\alpha^*](P \to [\alpha]P)$

V $p \to [\alpha]p$ $\quad (FV(p) \cap BV(\alpha) = \emptyset)$

Differential equation axioms

DW $[x' = f(x)\,\&\,Q]P \leftrightarrow [x' = f(x)\,\&\,Q](Q \to P)$

DI $\big([x' = f(x)\,\&\,Q]P \leftrightarrow [?Q]P\big) \leftarrow (Q \to [x' = f(x)\,\&\,Q](P)')$

DC $\big([x' = f(x)\,\&\,Q]P \leftrightarrow [x' = f(x)\,\&\,Q \wedge C]P\big) \leftarrow [x' = f(x)\,\&\,Q]C$

DE $[x' = f(x)\,\&\,Q]P \leftrightarrow [x' = f(x)\,\&\,Q][x' := f(x)]P$

DG $[x' = f(x)\,\&\,Q]P \leftrightarrow \exists y\, [x' = f(x), y' = a(x)\cdot y + b(x)\,\&\,Q]P$

$+'$ $(e+k)' = (e)' + (k)'$

$\cdot'$ $(e\cdot k)' = (e)'\cdot k + e\cdot(k)'$

c' $(c())' = 0$ $\quad$ (for numbers or constants $c()$)

x' $(x)' = x'$ $\quad$ (for variable $x \in \mathcal{V}$)

Differential equation proof rules

dW $\dfrac{Q \vdash P}{\Gamma \vdash [x' = f(x)\,\&\,Q]P, \Delta}$ $\qquad$ dI $\dfrac{Q \vdash [x' := f(x)](F)'}{F \vdash [x' = f(x)\,\&\,Q]F}$

dC $\dfrac{\Gamma \vdash [x' = f(x)\,\&\,Q]C, \Delta \quad \Gamma \vdash [x' = f(x)\,\&\,(Q \wedge C)]P, \Delta}{\Gamma \vdash [x' = f(x)\,\&\,Q]P, \Delta}$

Sequent calculus proof rules

$$\neg\text{R}\ \frac{\Gamma, P \vdash \Delta}{\Gamma \vdash \neg P, \Delta} \qquad \wedge\text{R}\ \frac{\Gamma \vdash P, \Delta \quad \Gamma \vdash Q, \Delta}{\Gamma \vdash P \wedge Q, \Delta} \qquad \vee\text{R}\ \frac{\Gamma \vdash P, Q, \Delta}{\Gamma \vdash P \vee Q, \Delta}$$

$$\neg\text{L}\ \frac{\Gamma \vdash P, \Delta}{\Gamma, \neg P \vdash \Delta} \qquad \wedge\text{L}\ \frac{\Gamma, P, Q \vdash \Delta}{\Gamma, P \wedge Q \vdash \Delta} \qquad \vee\text{L}\ \frac{\Gamma, P \vdash \Delta \quad \Gamma, Q \vdash \Delta}{\Gamma, P \vee Q \vdash \Delta}$$

$$\rightarrow\text{R}\ \frac{\Gamma, P \vdash Q, \Delta}{\Gamma \vdash P \rightarrow Q, \Delta} \qquad \text{id}\ \frac{}{\Gamma, P \vdash P, \Delta} \qquad \text{WR}\ \frac{\Gamma \vdash \Delta}{\Gamma \vdash P, \Delta}$$

$$\rightarrow\text{L}\ \frac{\Gamma \vdash P, \Delta \quad \Gamma, Q \vdash \Delta}{\Gamma, P \rightarrow Q \vdash \Delta} \qquad \text{cut}\ \frac{\Gamma \vdash C, \Delta \quad \Gamma, C \vdash \Delta}{\Gamma \vdash \Delta} \qquad \text{WL}\ \frac{\Gamma \vdash \Delta}{\Gamma, P \vdash \Delta}$$

$$\forall\text{R}\ \frac{\Gamma \vdash p(y), \Delta}{\Gamma \vdash \forall x\, p(x), \Delta}\ (y \notin \Gamma, \Delta, \forall x\, p(x)) \qquad \exists\text{R}\ \frac{\Gamma \vdash p(e), \Delta}{\Gamma \vdash \exists x\, p(x), \Delta}\ (\text{arbitrary term } e)$$

$$\forall\text{L}\ \frac{\Gamma, p(e) \vdash \Delta}{\Gamma, \forall x\, p(x) \vdash \Delta}\ (\text{arbitrary term } e) \qquad \exists\text{L}\ \frac{\Gamma, p(y) \vdash \Delta}{\Gamma, \exists x\, p(x) \vdash \Delta}\ (y \notin \Gamma, \Delta, \exists x\, p(x))$$

$$\text{CER}\ \frac{\Gamma \vdash C(Q), \Delta \quad \vdash P \leftrightarrow Q}{\Gamma \vdash C(P), \Delta} \qquad =\text{R}\ \frac{\Gamma, x = e \vdash p(e), \Delta}{\Gamma, x = e \vdash p(x), \Delta}$$

$$\text{CEL}\ \frac{\Gamma, C(Q) \vdash \Delta \quad \vdash P \leftrightarrow Q}{\Gamma, C(P) \vdash \Delta} \qquad =\text{L}\ \frac{\Gamma, x = e, p(e) \vdash \Delta}{\Gamma, x = e, p(x) \vdash \Delta}$$

Derived axioms and derived rules

$$\wedge'\ \ (P \wedge Q)' \leftrightarrow (P)' \wedge (Q)'$$

$$\vee'\ \ (P \vee Q)' \leftrightarrow (P)' \wedge (Q)'$$

$$[]\wedge\ \ [\alpha](P \wedge Q) \leftrightarrow [\alpha]P \wedge [\alpha]Q$$

$$\overleftarrow{[*]}\ \ [\alpha^*]P \leftrightarrow P \wedge [\alpha^*][\alpha]P$$

$$[**]\ \ [\alpha^*;\alpha^*]P \leftrightarrow [\alpha^*]P$$

$$[:=]_=\ \frac{\Gamma, y = e \vdash p(y), \Delta}{\Gamma \vdash [x := e]p(x), \Delta}\ (y \text{ new})$$

$$\text{iG}\ \frac{\Gamma \vdash [y := e]p, \Delta}{\Gamma \vdash p, \Delta}\ (y \text{ new})$$

$$\text{dG}\ \frac{\Gamma \vdash \exists y\, [x' = f(x), y' = a(x) \cdot y + b(x)\,\&\,Q]P, \Delta}{\Gamma \vdash [x' = f(x)\,\&\,Q]P, \Delta}$$

$$\text{dA}\ \frac{\vdash J \leftrightarrow \exists y\, G \quad G \vdash [x' = f(x), y' = a(x) \cdot y + b(x)\,\&\,Q]G}{J \vdash [x' = f(x)\,\&\,Q]J}$$