Appendix A

A.1 Review of AC Power Fundamentals

Steady-state AC circuit analysis is conveniently performed in the phasor domain. A sinusoidal voltage in the time domain can be written as

$$v(t) = v_{\max} \cos(\omega t + \theta_{\mathrm{v}}) \tag{A.1}$$

where $v_{\max}$ is the voltage amplitude; ω is the frequency, in radians per second; and θ_{v} is the phase angle in radians. When $v(t)$ is transformed into the phasor domain, it becomes

$$v(t) \leftrightarrow \boldsymbol{V} = |\boldsymbol{V}| \angle \theta_{\mathrm{v}} = \frac{v_{\max}}{\sqrt{2}} \angle \theta_{\mathrm{v}} = V_{\mathrm{RMS}} \angle \theta_{\mathrm{v}} \tag{A.2}$$

where V_{RMS} is the RMS value of the voltage. The phasor transform can also be applied to a sinusoidal current $i(t)$ so that

$$i(t) = i_{\max} \cos(\omega t + \theta_{\mathrm{i}}) \leftrightarrow \boldsymbol{I} = |\boldsymbol{I}| \angle \theta_{\mathrm{i}} = \frac{i_{\max}}{\sqrt{2}} \angle \theta_{\mathrm{i}} = I_{\mathrm{RMS}} \angle \theta_{\mathrm{i}}. \tag{A.3}$$

Note that the frequency of the voltage or current is not explicitly shown in the phasor representation. A circuit can only be analyzed in the phasor domain if the voltages and currents are sinusoidal and have the same frequency. Let the angle difference between the voltage and current be

$$\theta = \theta_{\mathrm{v}} - \theta_{\mathrm{i}}. \tag{A.4}$$

The angle θ is known as the *power factor angle*. The power factor PF is defined as

$$PF = \cos(\theta). \tag{A.5}$$

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When $\theta < 0$, the power factor is "leading" and when $\theta > 0$ the power factor is "lagging." Lagging power factor is associated with inductive loads, such as motors, whereas leading power factor is associated with capacitive loads. The voltage and current are related to the impedance Z as

$$\boldsymbol{V} = \boldsymbol{I}Z \tag{A.6}$$

where $Z = R + \mathrm{j}X\ \Omega$.

The instantaneous power of a circuit element at time t is the product of the time-domain voltage and current at time t.

$$p(t) = v(t)i(t) = v_{\max} i_{\max} \cos(\omega t + \theta_{\mathrm{v}}) \cos(\omega t + \theta_{\mathrm{i}}) \tag{A.7}$$

The average power, also called the "real" power P, is the average of the instantaneous power. It can be shown that

$$P = \frac{v_{\max} i_{\max}}{2} \cos\theta = |\boldsymbol{V}||\boldsymbol{I}| \cos\theta. \tag{A.8}$$

The apparent (complex) power S is defined as

$$S = \boldsymbol{V}\boldsymbol{I}^* = P + \mathrm{j}Q \tag{A.9}$$

where $*$ is the complex conjugate operator, j is $\sqrt{-1}$ and Q is the reactive or imaginary power. The complex power and reactive power are commonly expressed with units of volt-amperes (VA) and volt-amperes reactive (VAR), respectively. The relationship between S, P, Q, and θ are

$$P = \mathrm{Re}\{\boldsymbol{V}\boldsymbol{I}^*\} = \mathrm{Re}\{S\} = |S| \cos(\theta) = |\boldsymbol{V}||\boldsymbol{I}| \cos(\theta) = |S| \times PF \tag{A.10}$$

$$Q = \mathrm{Im}\{\boldsymbol{V}\boldsymbol{I}^*\} = \mathrm{Im}\{S\} = |S| \sin(\theta) = |\boldsymbol{V}||\boldsymbol{I}| \sin(\theta) \tag{A.11}$$

A.2 Three-Phase Analysis

A balanced three-phase source consists of three voltage sources whose RMS values V_ϕ are all the same and have the following relationships:

$$v_{\mathrm{a}}(t) = \sqrt{2} V_\phi \cos(\omega t) \leftrightarrow \boldsymbol{V}_{\mathrm{a}} = V_\phi \angle 0^\circ \tag{A.12}$$

$$v_{\mathrm{b}}(t) = \sqrt{2} V_\phi \cos(\omega t - 120^\circ) \leftrightarrow \boldsymbol{V}_{\mathrm{b}} = V_\phi \angle -120^\circ \tag{A.13}$$

$$v_{\mathrm{c}}(t) = \sqrt{2} V_\phi \cos(\omega t + 120^\circ) \leftrightarrow \boldsymbol{V}_{\mathrm{c}} = V_\phi \angle 120^\circ \tag{A.14}$$

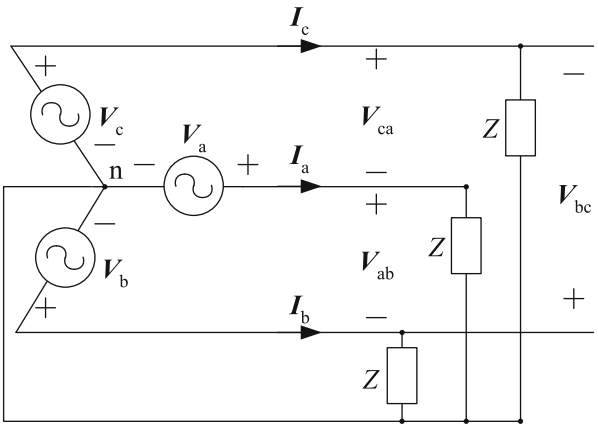

Fig. A.1 A balanced three-phase wye-circuit

That is, the sources are of the same magnitude and are displaced in phase by 120°. The voltage sources can be arranged as a wye or a delta. We will consider a wye-connected source; the fundamental results also apply to delta-connected circuits. A wye-connected circuit is shown in Fig. A.1. Note that common point of connection is known as the "neutral" point and is marked with an "n." The voltages $\boldsymbol{V}_{\mathrm{a}}$, $\boldsymbol{V}_{\mathrm{b}}$, and $\boldsymbol{V}_{\mathrm{c}}$ are known as the "line-to-neutral" voltages. The voltages $\boldsymbol{V}_{\mathrm{ab}}$, $\boldsymbol{V}_{\mathrm{bc}}$, and $\boldsymbol{V}_{\mathrm{ca}}$ are known as the "line-to-line" voltages.

In this book, we are mostly concerned with converting between the magnitude of the line-to-line voltage $V_{\ell\ell}$ and the magnitude of the line-to-neutral voltage V_ϕ. The line-to-line voltage is the voltage between any two lines. For a wye-connected source

$$\boldsymbol{V}_{\mathrm{ab}} = \boldsymbol{V}_{\mathrm{an}} - \boldsymbol{V}_{\mathrm{bn}} = \sqrt{3}V_\phi \angle 30^\circ = V_{\ell\ell}\angle 30^\circ \tag{A.15}$$

$$\boldsymbol{V}_{\mathrm{bc}} = \boldsymbol{V}_{\mathrm{bn}} - \boldsymbol{V}_{\mathrm{cn}} = \sqrt{3}V_\phi \angle -90^\circ = V_{\ell\ell}\angle -90^\circ \tag{A.16}$$

$$\boldsymbol{V}_{\mathrm{ca}} = \boldsymbol{V}_{\mathrm{cn}} - \boldsymbol{V}_{\mathrm{an}} = \sqrt{3}V_\phi \angle 150^\circ = V_{\ell\ell}\angle 150^\circ. \tag{A.17}$$

From this we see that the line-to-line voltage magnitude $V_{\ell\ell}$ is equal to the line-to-neutral voltage V_ϕ multiplied by a factor of $\sqrt{3}$. That is

$$V_{\ell\ell} = \sqrt{3}V_\phi. \tag{A.18}$$

A three-phase load is said to be balanced if the line-to-neutral impedances are equal. In this case, the phase currents have the same magnitude I_ϕ and are displaced from one another by 120°:

$$\boldsymbol{I}_{\mathrm{a}}\angle\theta_{\mathrm{i,a}} = \boldsymbol{I}_{\mathrm{b}}\angle\theta_{\mathrm{i,a}} - 120^\circ = \boldsymbol{I}_{\mathrm{c}}\angle\theta_{\mathrm{i,a}} + 120^\circ \tag{A.19}$$

$$I_\phi = |\boldsymbol{I}_{\mathrm{a}}| = |\boldsymbol{I}_{\mathrm{b}}| = |\boldsymbol{I}_{\mathrm{c}}|. \tag{A.20}$$

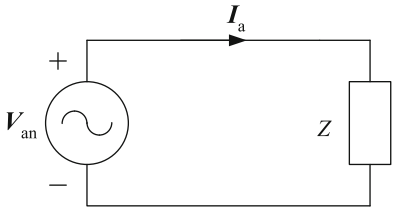

Fig. A.2 Equivalent per-phase circuit of a three-phase wye-connected circuit

By symmetry, each phase supplies the same power:

$$S_\phi = \boldsymbol{V}_{an}\boldsymbol{I}_a^* = \boldsymbol{V}_{bn}\boldsymbol{I}_b^* = \boldsymbol{V}_{cn}\boldsymbol{I}_c^* \tag{A.21}$$

where S_ϕ is the power supplied or consumed by any single phase. The total apparent power is the sum of the power delivered by each phase:

$$S_{total} = 3\boldsymbol{V}_{an}\boldsymbol{I}_a^* = \boldsymbol{V}_\phi \boldsymbol{I}_\phi^* \tag{A.22}$$

and

$$P_{total} = 3\text{Re}\{\boldsymbol{V}_{an}\boldsymbol{I}_a^*\} \tag{A.23}$$

$$Q_{total} = 3\text{Im}\{\boldsymbol{V}_{an}\boldsymbol{I}_a^*\}. \tag{A.24}$$

We can therefore model any balanced three-phase circuit as a single-phase or "per-phase" equivalent circuit as shown in Fig. A.2. We use the line-to-neutral voltage in this circuit, and we must remember to apply a factor of three if we are interested in the total power. It is customary to use a-phase quantities, but this is arbitrary.

Solutions

Problems of Chap. 1

1.1 1 TOE; 1 MWh; 100 kWh; 10,000 calories; 5000 J; 1 Calorie; 1 BTU

1.3 3.4 days; US$0.294

1.7 18 panels (rounding up)

1.8 SSA: 454,666 GWh; USA: 4,069,400 GWh.

Problems of Chap. 2

2.1 851.67 l/yr

2.2 1481.4 Wh/day; 540.7 kWh/year

2.11 Tier 3

Problems of Chap. 3

3.1 69.47 A

3.4 Ababju: infrastructure cost US$600,500; cost per connection US$1201.0

3.5 Ababju: annual cost per connection US$63.97

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Problems of Chap. 4

4.3 $101.34\angle -76.5°$A

Problems of Chap. 5

5.2 $112.68\angle -23.44°$V; 11.8 kVA

5.4 One 50 kW gen set: US$148,263

5.7 9900 MJ/day

Problems of Chap. 6

6.1 3.90 m

6.5 45 m: for 1 kW 2.67 l/s, for 5 kW 13.3 l/s, for 10 kW 26.7 l/s

6.6 Location A: specific speed 4.37, Kaplan Turbine

6.7 PCD: 0.185 m; jet diameter: 0.011 m

Problems of Chap. 7

7.1 Cell Temperature: 56°C. short-circuit current: 9.61 A; open-circuit voltage: 43.17 V, maximum power: 308.77 W. short-circuit current at 500 W/m^2: 4.745 A. power produced: 217.77 W

7.4 Array short-circuit current: 30.28 A; array open-circuit voltage: 171.98 V; array power: 3685.3 W

Problems of Chap. 8

8.1 5 moles: 49.952 V

8.4 Current and C-rate at 72 hour rate: 5.47 A, 0.0139C

8.6 47.24 h

8.9 11 h, 63.36 kWh

Problems of Chap. 9

9.1 0.414 (41.4%)

9.3 Duty cycle: 0.6055; PV current: 9.081 A; MPPT current: 5.916 A

9.6 Power to battery: 0.487 kW; average DC voltage: 58.07 V; average DC current 47.80 A

9.10 $\boldsymbol{I} = 4.06\angle -2.93°$A; real power 1.266 kW; reactive power: 67.79 VAR

Problems of Chap. 10

10.3 Charge controller current: 40.83 A; PV power: 610.4 W

10.7 Battery voltage: 58.40 V; diversion load current: 2 A; charge controller current: 0 A

10.8 Gen set 1 power:57.2 kW; gen set 2 power: 52.8 kW; frequency: 59.91 Hz (both gen sets)

Problems of Chap. 11

11.3 User A: 140 Wh/day, load factor: 0.194

11.8 Capacity factor at 20 l/s: 1.00 (summer), 1.00 (winter); capacity factor at 30 l/s: 0.70 (summer), 1.00 (winter)

Problems of Chap. 12

12.9 Voltage drop segment 1: 12.375 V; segment 2: 8.500 V; segment 3: 4.125 V. power loss segment 1: 371.25 W; segment 2: 170 W; segment 3: 41.25 W

12.12 US$0.350/kWh

Problems of Chap. 13

13.5 14.06 Ah

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