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Arnt Inge Vistnes

Physics of Oscillations and Waves

With use of Matlab and Python

Springer

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To
Kirsten
Ingunn, Torunn, Maria
with families

Preface

Origin

The University of Oslo in Norway is one of the first universities to introduce numerical methods as an integral part of almost all mathematically oriented courses for science students (first attempts started in 1997). This created the need for textbooks in physics covering all the topics included in the syllabus. There were many textbooks on oscillations and waves on the market, but none adhered well with the learning objectives we adopted.

The Norwegian version of this book was originally written in 2008 for use in the course *"FYS2130 Svingninger og bølger"* (Oscillations and Waves) and has undergone many revisions and expansions since then. The course is given in the fourth semester to students enrolled in the Department of Physics at the University of Oslo. These students have taken courses in Python programming, classical mechanics and electromagnetism, but have had limited education in oscillations and wave phenomena.

Scope

In the present book, I have mostly adhered to traditional descriptions of the phenomena; however, I have also tried to point towards potential limitations of such descriptions. When appropriate, analogies between different phenomena are drawn.

The formalism and phenomena are treated quite differently from section to section. Some sections provide only qualitative descriptions and thus only a superficial or introductory understanding of the topics while other sections are more mathematical and demanding. Occasionally, the mathematical derivations are not essential to understand the material, but are included to show the connection between basic physical laws and the phenomena discussed in the text.

Principles from numerical methods are employed as they permit us to handle more realistic problems than pure analytical mathematics alone, and they facilitate to obtain a deeper understanding of some phenomena.

Program codes are given, ready to use, and is a tool for further exploration of the phenomena that are covered. Our experience from teaching this topic to students over years is that, numerical methods based on "hands-on computer code development" expand the experimental attitude and facilitate the learning process.

We try in this book to emphasize how so-called algorithmic thinking can improve understanding. As a personal example, the algorithm for calculating how a wave evolves over time has given me a much deeper understanding of the wave phenomena than by working with analytical mathematics over years. Another example is the realization that all variants of classical interference and diffraction can be calculated using a single computer program, demonstrating not only that numerical methods are powerful, but also that the underlying physical mechanism is identical in all these cases.

We have made an effort to ensure a logical and reader-friendly structure of the book. Especially important parts of the core material in the text are marked by coloured background, and various examples show how the core material can be used in different contexts. Supplementary information and comments are given in small print. Learning objectives point to the most important sections of each chapter. Most of the chapters include suggestions to further reading.

There are three types of exercises in the book. The first type of exercise consists of a list of concepts in each chapter that can be used by students in various ways for active learning. Thereafter follow comprehension/discussion questions and more regular problems often including calculations. Best learning outcome is achieved by trying all the three types of tasks, including oral discussions when working with understanding concepts and the comprehension/discussion questions. The problems used in the exercises are taken from daily life experiences, in order to demonstrate how physics is relevant in many aspects of our everyday life.

For the more regular problems, the aim is to encourage the reader to learn how to *devise* a strategy for solving the problem at hand and to select the appropriate laws. A "correct answer" without an adequate justification and reasoning is worthless. In many tasks, not all the relevant quantities are supplied, and in these cases, the reader must search for the necessary information in other books or the Internet. This is a natural part of working with physics today. A list of answers for the problems is not worked out yet. Some problems require particular data files to be analyzed that will be available from a web page advertised by the publisher.

Content

In our daily life, oscillations and waves play an important role. The book covers sound phenomena, our sense of hearing, and the two sets of measurements of sound and units that are in use: one for physical purposes solely and the other related to

the sense of hearing. Similarly, the book treats light phenomena and our sense of vision, as well as the two sets of measurements and units that are in use for these purposes. In addition, we also discuss colour mixing and important differences between our senses of hearing and vision.

By introducing Fourier transform, Fourier series and fast Fourier transform, we introduce important tools for analysis of oscillatory/wave phenomena. Our aim is to give the reader all necessary details so that she/he can utilize this numeric method to its full potential. We also point out a common misconception we often find in connection with Fourier analysis.

We introduce continuous wavelet transform with Morlet wavelets as a kind of time-resolved Fourier transform and explain why we have chosen this method instead of a short-term Fourier transform. Much emphasis is put on optimizing the analysis and how this is closely related to the time-bandwidth product; a classical analogue to Heisenberg's uncertainty principle. A computer program is provided for this topic as well as for many other parts of the book.

One chapter is devoted to numerical method, mainly in how to solve ordinary and partial differential equations of first or second order. Other topics covered in the book are geometric optics, interference, diffraction, dispersion and coherence. We also briefly cover skin effect, waveguides and lasers.

Intended Audience

The reader of the book should have some basic programming experience, preferably in Matlab or Python, and know basic mechanics and electromagnetism. The principal ingredients of the book encompassing physical phenomena and formalism, analytical mathematics, numerical methods, focus on everyday phenomena and state-of-the-art examples are likely to be of interest to a broader group of readers. For instance, we have experienced that established physicists who want to look up details within the themes like colour vision, geometrical optics and polarization also appreciate the book.

Computer Programs

In this book all computer programs are given in Matlab code. However, all the these programs are available as separate files both in Matlab and in Python code at the "additional resources" Web page at https://urldefense.proofpoint.com/v2/url?u=http-3A__www.physics.uio.no_pow_&d=DwIFAg&c=vh6FgFnduejNhPPD0fl_yRaSfZy8CWbWnIf4XJhSqx8&r=9V0dbmmXGCupx1bqsdDysssYnqDmbKz79g1dipIcPn4&m=FJQIEp2YVoX1g_zLnM3m3k9m6Oa6GBqfvvj68AbJtM0&s=cXDHnCeHUxv0te6xsUN3OL9B2L4V3MHfUpayYSP6_gU&e=.

Some introduction is given to programming style, reproducibility and documentation, but not at a level as is expected for a course fully devoted to programming. We do not provide an introduction to "dimensionless variables".

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A generous grant from *The Norwegian Non-fiction Writers and Translators Association* allowed me to be free from teaching obligations for two fall semesters, during which the first version of the book and some of the illustrations were prepared. A warm "thank you" to Anders Malthe-Sørenssen for providing inspiration for teaching in general and for securing financial support for the translation of the book from Norwegian to English.

Most of all, I thank my dear Kirsten and our children for their loving forbearance during the periods when I have been busy working with this book. I now look forward to take more part in family life.

Kurland, Norway Arnt Inge Vistnes
June 2018

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