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Jochen Pade

Quantum Mechanics for Pedestrians 1

Fundamentals

Second Edition

 Springer

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Preface to the Second Edition, Volume 1

The first edition of ‘Physics for Pedestrians’ was very well received. Repeatedly, I was asked to extend the considerations to relativistic phenomena. This has now been done in this second edition. Volume 1 contains elements of relativistic quantum mechanics, and Volume 2 contains elements of quantum field theory.

These extensions are placed in the Appendix. They are not comprehensive and complete presentations of the topics, but rather concise accounts of some essential ideas of relativistic quantum physics.

Furthermore, for the sake of completeness and to guarantee a consistent notation, there are outlines of relevant topics such as special relativity, classical field theory, and electrodynamics.

In addition, a few minor bugs have been fixed and some information has been updated.

I gratefully thank Svend-Age Biehs, Heinz Helmers, Stefanie Hoppe, Friedhelm Kuypers and Lutz Polley who have helped me in one way or another to prepare this second edition.

Oldenburg, Germany
February 2018

Jochen Pade

Preface to the First Edition, Volume 1

There are so many textbooks on quantum mechanics—do we really need another one?

Certainly, there may be different answers to this question. After all, quantum mechanics is such a broad field that a single textbook cannot cover all the relevant topics. A selection or prioritization of subjects is necessary *per se*, and moreover, the physical and mathematical foreknowledge of the readers has to be taken into account in an adequate manner. Hence, there is undoubtedly not only a certain leeway, but also a definite need for a wide variety of presentations.

Quantum Mechanics for Pedestrians has a thematic blend that distinguishes it from other introductions to quantum mechanics (at least those of which I am aware). It is not just about the conceptual and formal foundations of quantum mechanics, but from the beginning and in some detail it also discusses both current topics as well as advanced applications and basic problems as well as epistemological questions. Thus, this book is aimed especially at those who want to learn not only the appropriate formalism in a suitable manner, but also those other aspects of quantum mechanics addressed here. This is particularly interesting for students who want to teach quantum mechanics themselves, whether at the school level or elsewhere. The current topics and epistemological issues are especially suited to generate interest and motivation among students.

Like many introductions to quantum mechanics, this book consists of lecture notes which have been extended and complemented. The course which I have given for several years is aimed at teacher candidates and graduate students in the master's program, but is also attended by students from other degree programs. The course includes lectures (two sessions/four hours per week) and problem sessions (two hours per week). It runs for 14 weeks, which is reflected in the 28 chapters of the lecture notes.

Due to the usual interruptions such as public holidays, it will not always be possible to treat all 28 chapters in 14 weeks. On the other hand, the later chapters in particular are essentially independent of each other. Therefore, one can make a selection based on personal taste without losing coherence. Since the book consists of extended lecture notes, most of the chapters naturally offer more material than

will fit into a two-hour lecture. But the ‘main material’ can readily be presented within this time; in addition, some further topics may be treated using the exercises.

Before attending the quantum mechanics course, the students have had among others an introduction to atomic physics: Relevant phenomena, experiments, and simple calculations should therefore be familiar to them. Nevertheless, experience has shown that at the start of the lectures, some students do not have enough substantial and available knowledge at their disposal. This applies less to physical and more to the necessary mathematical knowledge, and there are certainly several reasons for this. One of them may be that for teacher training; not only the quasi-traditional combination physics/mathematics is allowed, but also others such as physics/sports, where it is obviously more difficult to acquire the necessary mathematical background and, especially, to actively practice its use.

To allow for this, I have included some chapters with basic mathematical knowledge in the Appendix, so that students can use them to overcome any remaining individual knowledge gaps. Moreover, the mathematical level is quite simple, especially in the early chapters; this course is not just about practicing specifically elaborated formal methods, but rather we aim at a compact and easily accessible introduction to key aspects of quantum mechanics.

As remarked above, there are a number of excellent textbooks on quantum mechanics, not to mention many useful Internet sites. It goes without saying that in writing the lecture notes, I have consulted some of these, have been inspired by them and have adopted appropriate ideas, exercises, etc., without citing them in detail. These books and Internet sites are all listed in the bibliography and some are referred to directly in the text.

A note on the title *Quantum Mechanics for Pedestrians*: It does not mean ‘quantum mechanics light’ in the sense of a painless transmission of knowledge à la Nuremberg funnel. Instead, ‘for pedestrians’ is meant here in the sense of autonomous and active movement—step by step, not necessarily fast, from time to time (i.e., along the more difficult stretches) somewhat strenuous, depending on the level of understanding of each walker—which will, by the way, become steadily better while walking on.

Speaking metaphorically, it is about discovering on foot the landscape of quantum mechanics; it is about improving one’s knowledge of each locale (if necessary, by taking detours); and it is perhaps even about finding your own way.

By the way, it is always amazing not only how far one can walk with some perseverance, but also how fast it goes—and how sustainable it is. ‘Only where you have visited on foot, have you really been.’ (Johann Wolfgang von Goethe).

Klaus Schlupmann, Heinz Helmers, Edith Bakenhus, Regina Richter, and my sons, Jan Philipp and Jonas have critically read several chapters. Sabrina Milke assisted me in making the index. I enjoyed enlightening discussions with Lutz Polley, while Martin Holthaus provided helpful support and William Brewer made useful suggestions. I gratefully thank them and all the others who have helped me in some way or other in the realization of this book.

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Introduction

Quantum mechanics is probably the most accurately verified physical theory existing today. To date, there has been no contradiction from any experiments; the applications of quantum mechanics have changed our world right up to aspects of our everyday life. There is no doubt that quantum mechanics ‘functions’—it is indeed extremely successful. On a formal level, it is clearly unambiguous and consistent and (certainly not unimportant)—as a theory—it is both aesthetically satisfying and convincing.

The question in dispute is the ‘real’ meaning of quantum mechanics. What does the wavefunction stand for, and what is the role of chance? Do we actually have to throw overboard our classical and familiar conceptions of reality? Despite the nearly century-long history of quantum mechanics, fundamental questions of this kind are still unresolved and are currently being discussed in a lively and controversial manner. There are two contrasting positions (along with many intermediate views): Some see quantum mechanics simply as the precursor stage of the ‘true’ theory (although eminently functional); others see it as a valid, fundamental theory itself.

This book aims to introduce its readers to both sides of quantum mechanics, the established side and the side that is still under discussion. We develop here both the conceptual and formal foundations of quantum mechanics, and we discuss some of its ‘problem areas.’ In addition, this book includes applications—oriented fundamental topics, some ‘modern’ ones—for example, issues in quantum information—and ‘traditional’ ones such as the hydrogen and the helium atoms. We restrict ourselves to the field of nonrelativistic physics, although many of the ideas can be extended to the relativistic case.¹ Moreover, we consider only time-independent interactions.

In introductory courses on quantum mechanics, the practice of formal skills often takes priority (this is subsumed under the slogan ‘shut up and calculate’). In accordance with our objectives here, we will also give appropriate space to the discussion of fundamental questions. This special blend of basic discussion and

¹In the second edition, some essentials of relativistic quantum mechanics are added; see the Appendix.

modern practice is in itself very well suited to evoke interest and motivation in students. This is, in addition, enhanced by the fact that some important fundamental ideas can be discussed using very simple model systems as examples. It is not coincidental that some of the topics and phenomena addressed here are treated in various simplified forms in high-school textbooks.

In mathematical terms, there are two main approaches used in introductions to quantum mechanics. The first one relies on differential equations (i.e., analysis) and the other one on vector spaces (i.e., linear algebra); of course, the ‘finished’ quantum mechanics is independent of the route of access chosen. Each approach (they also may be called the Schrodinger and the Heisenberg routes) has its own advantages and disadvantages; the two are used in this book on an equal footing.

The roadmap of the book is as follows:

The foundations and structure of quantum mechanics are worked out step by step in the first part (Volume 1, Chaps. 1–14), alternatively from an analytical approach (odd chapters) and from an algebraic approach (even chapters). In this way, we avoid limiting ourselves to only one of the two formulations. In addition, the two approaches reinforce each other in the development of important concepts. The merging of the two threads starts in Chap. 12. In Chap. 14, the conclusions thus far reached are summarized in the form of quite general postulates for quantum mechanics.

Especially in the algebraic chapters, we take up current problems early on (interaction-free quantum measurements, the neutrino problem, quantum cryptography). This is possible since these topics can be treated using very simple mathematics. Thus, this type of access is also of great interest for high-school level courses. In the analytical approach, we use as elementary physical model systems the infinite potential well and free particle motion.

In the second part (Volume 2, Chaps. 15–28), applications and extensions of the formalism are considered. The discussion of the conceptual difficulties (measurement problem, locality and reality, etc.) again constitutes a central theme, as in the first volume. In addition to some more traditionally oriented topics (angular momentum, simple potentials, perturbation theory, symmetries, identical particles, scattering), we begin in Chap. 20 with the consideration of whether quantum mechanics is a local realistic theory. In Chap. 22, we introduce the density operator in order to consider in Chap. 24 the phenomenon of decoherence and its relevance to the measurement process. In Chap. 27, we continue the realism debate and explore the question as to what extent quantum mechanics can be regarded as a complete theory. Modern applications in the field of quantum information can be found in Chap. 26.

Finally, we outline in Chap. 28 the most common interpretations of quantum mechanics. Apart from this chapter, a general statement applies: While it is still a controversial issue as to which (if indeed any) of the current interpretations is the ‘correct’ one, an introduction to quantum mechanics must take a concrete position and has to present the material in a coherent form. In this book, we choose the version commonly known as the ‘standard interpretation.’

A few words about the role of mathematics:

In describing objects that—due to their small size—are beyond our everyday experience, quantum mechanics cannot be formulated completely in terms of everyday life and must therefore remain to some extent abstract. A deeper understanding of quantum mechanics cannot be achieved on a purely linguistic level; we definitely need mathematical descriptions.² Of course, one can use analogies and simplified models, but that works only to a certain degree and also makes sense only if one is aware of the underlying mathematical apparatus, at least in broad terms.³

It is due to this interaction of the need for mathematical formulations and the lack of intuitive access that quantum mechanics is often regarded as ‘difficult.’ But that is only part of the truth; to be sure, there are highly formalized and demanding aspects. Many wider and interesting issues, however, are characterized by very simple principles that can be described using only a basic formalism.

Nevertheless, beginners in particular perceive the role of mathematics in quantum mechanics as discouraging. Three steps serve to counter this impression or, in the optimum case, to avoid it altogether:

First, we keep the mathematical level as simple as possible and share the usual quite nonchalant attitude of physicists toward mathematics. In particular, the first chapters go step by step, so that the initially diverse mathematics skills of the readers are gradually brought up to a common level.

In addition, we use very simple models, toy models so to speak, especially in the first part of the book, in order to treat the main physical ideas without becoming involved in complicated mathematical questions. Of course, these models are only rough descriptions of actual physical situations. But they manage with relatively simple mathematics, do not require approximation methods or numerics, and yet still permit essential insights into the fundamentals of quantum mechanics.⁴ Only in Volume 2, more realistic models are applied, and this is reflected occasionally in a somewhat more demanding formal effort.

The third measure involves exercises and some support from the Appendix. At the end of almost every chapter, there is a variety of exercises, some of them dealing with advanced topics. They invite the reader to work with the material in

²This applies at least to physicists; for as Einstein remarked: ‘But there is another reason for the high repute of mathematics: it is mathematics that offers the exact natural sciences a certain measure of security which, without mathematics, they could not attain.’ To give a layman without mathematical training an understanding of quantum mechanics, one will (or must) rely instead on math-free approaches.

³Without appropriate formal considerations, it is impossible to understand, for example, how to motivate the replacement of a physical measurement variable by a Hermitian operator.

⁴We could instead also make use of the large reservoir of historically important experiments. But their mathematical formulation is in general more complex, and since in the frame of our considerations they do not lead to further-reaching conclusions than our ‘toy models,’ we restrict ourselves to the latter for clarity and brevity.

order to better assimilate and more clearly grasp it, as well as of course to train the necessary formal skills.⁵

The learning aids in the Appendix include chapters with some basic mathematical and physical background information; this allows the reader to refresh ‘passive’ knowledge without the need to refer to other sources or to become involved with new notations.

Moreover, the no doubt unusually extensive Appendix contains the solutions to many of the exercises and, in addition, some chapters in which further-reaching questions and issues are discussed; although these are very interesting in themselves, their treatment would far exceed the framework of a lecture course.

The footnotes with a more associative character can be skipped on a first reading.

A note on the term ‘particle’: Its meaning is rather vague in physics. On the one hand, it denotes ‘something solid, not wavelike’; on the other hand ‘something small’, ranging from the elementary particles as structureless building blocks of matter, to objects which themselves are composed of constituent ‘particles’ like the α particle and other atomic nuclei or even macroscopic particles like sand grains. In quantum mechanics, where indeed it is often not even clear whether a particular object has mainly particle or mainly wave character, the careless use of the term may cause confusion and communication problems.

Accordingly, several terms which go beyond ‘wave’ or ‘particle’ have been suggested, such as quantal particle, wavical, wavicle, quantum object, quanton. Throughout this book, we will use the term ‘quantum object,’ unless there are traditionally established terms such as ‘identical particles’ or ‘elementary particles.’ The consistent use of ‘quantum object’ instead of ‘particle’ may perhaps seem somewhat pedantic, but we hope that it will help to ensure that fewer false images stick in the minds of readers; it is for this reason that this term is also found in many high-school textbooks.

Quantum mechanics is a fundamental theory of physics, which has given rise to countless applications. But it also extends deep into areas such as philosophy and epistemology and leads to thinking about ‘what holds the world together at its core’; in short, it is also an intellectual adventure. The fascinating thing is that the more one becomes acquainted with quantum mechanics, the more one realizes how simple many of its central ideas really are.⁶ It would be pleasing if *Quantum Mechanics for Pedestrians* could help to reveal this truth.

⁵‘It is a great support to studying, at least for me, to grasp everything that one reads so clearly that one can apply it oneself, or even make additions to it. One is then inclined to believe in the end that one could have invented everything himself, and that is encouraging.’ Georg Christoph Lichtenberg, *Scrap Books*, Vol. J (1855).

⁶‘The less we know about something, the more complicated it is, and the more we know about it, the easier it is. This is the simple truth about all the complexities.’ Egon Friedell, in *Kulturgeschichte der Neuzeit; Kulturgeschichte Agyptens und des alten Orients* (*Cultural history of modern times; the cultural history of Egypt and the ancient Near East*).

Let us close with a remark by Richard Feynman which holds true not only for physics in general, but even more for quantum mechanics: ‘Physics is like sex: Sure, it may give some practical results, but that’s not why we do it.’

Overview of Volume 1

In the following 14 chapters, we want to work out the fundamental structure of quantum mechanics, *videlicet* on the basis of a few simple models. The use of these ‘toy systems’ has two advantages.

First, their simplicity allows us to identify the essential mechanisms of quantum mechanics without getting lost in complex mathematical considerations. These mechanisms, which we summarize in Chap. 14 in the form of postulates, can nevertheless be formulated in a rather general manner.

Second, we can emphasize the essential ideas very quickly in this manner, so that we can treat and understand current topics quite soon along the trail.