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Quantum Mechanics for Pedestrians 2

Applications and Extensions

Second Edition

Springer

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ISSN 2192-4791 ISSN 2192-4805 (electronic)
Undergraduate Lecture Notes in Physics
ISBN 978-3-030-00466-8 ISBN 978-3-030-00467-5 (eBook)
https://doi.org/10.1007/978-3-030-00467-5

Library of Congress Control Number: 2018954852

Originally published with the title: *Quantum Mechanics for Pedestrians 2: Applications and Extensions*

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface to the Second Edition, Volume 2

In this second edition of Volume 2, a short introduction to the basics of quantum field theory has been added. The material is placed in the Appendix. It is not a comprehensive and complete presentation of the topic, but, in the sense of a primer, a concise account of some of the essential ideas.

Fundamentals from other areas can be found in Volume 1, i.e., outlines of special relativity, classical field theory, electrodynamics and relativistic quantum mechanics.

Oldenburg, Germany
February 2018

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Preface to the First Edition, Volume 2

In the first volume of *Quantum Mechanics for Pedestrians,* we worked out the basic structure of quantum mechanics (QM) and summarized it in the form of postulates that provided its framework.

In this second volume, we want to fill that framework with life. To this end, in eight of the 14 chapters we will discuss some key applications, what might be called the 'traditional' subjects of quantum mechanics: simple potentials, angular momentum, perturbation theory, symmetries, identical particles, and scattering.

At the same time, we want to prudently broaden the scope of our treatment, in order to be able to discuss modern developments such as entanglement and decoherence. We begin this theme in Chap. 20 with the question of whether quantum mechanics is a local-realistic theory. In Chap. 22, we introduce the density operator in order to discuss the phenomenon of decoherence and its importance for the measurement process in Chap. 24. In Chap. 27, we address again the realism debate and examine the question as to what extent quantum mechanics can be considered to be a complete theory. Modern applications in the field of quantum information can be found in Chap. 26.

Finally, we outline in Chap. 28 the most common current interpretations of quantum mechanics. Apart from one chapter, what was said in Volume I applies generally: An introduction to quantum mechanics has to take a definite stand on the interpretation question, although (or perhaps because) the question as to which one of the current interpretations (if any) is the 'correct' one it is still quite controversial. We have taken as our basis what is often called the 'standard interpretation.'

In order to formulate the postulates, we worked in the first volume with very simple models, essentially toy models. This is of course not possible for some of the 'real' systems presented in the present volume, and accordingly, these chapters are formally more complex. However, here also, we have kept the mathematical level as simple as possible. Moreover, we always choose that particular presentation which is best adapted to the question at hand, and we maintain the relaxed approach to mathematics which is usual in physics.

This volume is also accompanied by an extensive appendix. It contains some information on mathematical issues, but its principal focus is on physical topics whose consideration or detailed discussion would be beyond the scope of the main text in Chaps. 15–28.

In addition, there is for nearly every chapter a variety of exercises; solutions to most of them are given in the appendix.

Contents

Contents of Volume 1

Introduction

Quantum mechanics is probably the most accurately verified physical theory existing today. To date, there has been no contradiction from any experiments; the applications of quantum mechanics have changed our world right up to aspects of our everyday life. There is no doubt that quantum mechanics 'functions'—it is indeed extremely successful. On a formal level, it is clearly unambiguous and consistent and (certainly not unimportant)—as a theory—it is both aesthetically satisfying and convincing.

The question in dispute is the 'real' meaning of quantum mechanics. What does the wavefunction stand for, and what is the role of chance? Do we actually have to throw overboard our classical and familiar conceptions of reality? Despite the nearly century-long history of quantum mechanics, fundamental questions of this kind are still unresolved and are currently being discussed in a lively and controversial manner. There are two contrasting positions (along with many intermediate views): Some see quantum mechanics simply as the precursor stage of the 'true' theory (although eminently functional); others see it as a valid, fundamental theory itself.

This book aims to introduce its readers to both sides of quantum mechanics, the established side and the side that is still under discussion. We develop here both the conceptual and formal foundations of quantum mechanics, and we discuss some of its 'problem areas.' In addition, this book includes applications—oriented fundamental topics, some 'modern' ones—for example, issues in quantum information—and 'traditional' ones such as the hydrogen and the helium atoms. We restrict ourselves to the field of nonrelativistic physics, although many of the ideas can be extended to the relativistic case.[1] Moreover, we consider only time-independent interactions.

In introductory courses on quantum mechanics, the practice of formal skills often takes priority (this is subsumed under the slogan 'shut up and calculate'). In accordance with our objectives here, we will also give appropriate space to the

[1] In the second edition, some essentials of relativistic quantum mechanics are added; see the Appendix.

discussion of fundamental questions. This special blend of basic discussion and modern practice is in itself very well suited to evoke interest and motivation in students. This is, in addition, enhanced by the fact that some important fundamental ideas can be discussed using very simple model systems as examples. It is not coincidental that some of the topics and phenomena addressed here are treated in various simplified forms in high-school textbooks.

In mathematical terms, there are two main approaches used in introductions to quantum mechanics. The first one relies on differential equations (i.e., analysis) and the other one on vector spaces (i.e., linear algebra); of course, the 'finished' quantum mechanics is independent of the route of access chosen. Each approach (they also may be called the Schrodinger and the Heisenberg routes) has its own advantages and disadvantages; the two are used in this book on an equal footing.

The roadmap of the book is as follows:

The foundations and structure of quantum mechanics are worked out step by step in the first part (Volume 1, Chaps. 1–14), alternatively from an analytical approach (odd chapters) and from an algebraic approach (even chapters). In this way, we avoid limiting ourselves to only one of the two formulations. In addition, the two approaches reinforce each other in the development of important concepts. The merging of the two threads starts in Chap. 12. In Chap. 14, the conclusions thus far reached are summarized in the form of quite general postulates for quantum mechanics.

Especially in the algebraic chapters, we take up current problems early on (interaction-free quantum measurements, the neutrino problem, quantum cryptography). This is possible since these topics can be treated using very simple mathematics. Thus, this type of access is also of great interest for high-school level courses. In the analytical approach, we use as elementary physical model systems the infinite potential well and free particle motion.

In the second part (Volume 2, Chaps. 15–28), applications and extensions of the formalism are considered. The discussion of the conceptual difficulties (measurement problem, locality and reality, etc.) again constitutes a central theme, as in the first volume. In addition to some more traditionally oriented topics (angular momentum, simple potentials, perturbation theory, symmetries, identical particles, scattering), we begin in Chap. 20 with the consideration of whether quantum mechanics is a local realistic theory. In Chap. 22, we introduce the density operator in order to consider in Chap. 24 the phenomenon of decoherence and its relevance to the measurement process. In Chap. 27, we continue the realism debate and explore the question as to what extent quantum mechanics can be regarded as a complete theory. Modern applications in the field of quantum information can be found in Chap. 26.

Finally, we outline in Chap. 28 the most common interpretations of quantum mechanics. Apart from this chapter, a general statement applies: While it is still a controversial issue as to which (if indeed any) of the current interpretations is the 'correct' one, an introduction to quantum mechanics must take a concrete position and has to present the material in a coherent form. In this book, we choose the version commonly known as the 'standard interpretation.'

A few words about the role of mathematics:

In describing objects that—due to their small size—are beyond our everyday experience, quantum mechanics cannot be formulated completely in terms of everyday life and must therefore remain to some extent abstract. A deeper understanding of quantum mechanics cannot be achieved on a purely linguistic level; we definitely need mathematical descriptions.[2] Of course, one can use analogies and simplified models, but that works only to a certain degree and also makes sense only if one is aware of the underlying mathematical apparatus, at least in broad terms.[3]

It is due to this interaction of the need for mathematical formulations and the lack of intuitive access that quantum mechanics is often regarded as 'difficult.' But that is only part of the truth; to be sure, there are highly formalized and demanding aspects. Many wider and interesting issues, however, are characterized by very simple principles that can be described using only a basic formalism.

Nevertheless, beginners in particular perceive the role of mathematics in quantum mechanics as discouraging. Three steps serve to counter this impression or, in the optimum case, to avoid it altogether:

First, we keep the mathematical level as simple as possible and share the usual quite nonchalant attitude of physicists toward mathematics. In particular, the first chapters go step by step, so that the initially diverse mathematics skills of the readers are gradually brought up to a common level.

In addition, we use very simple models, toy models so to speak, especially in the first part of the book, in order to treat the main physical ideas without becoming involved in complicated mathematical questions. Of course, these models are only rough descriptions of actual physical situations. But they manage with relatively simple mathematics, do not require approximation methods or numerics, and yet still permit essential insights into the fundamentals of quantum mechanics.[4] Only in Volume 2, more realistic models are applied, and this is reflected occasionally in a somewhat more demanding formal effort.

The third measure involves exercises and some support from the Appendix. At the end of almost every chapter, there is a variety of exercises, some of them dealing with advanced topics. They invite the reader to work with the material in

[2]This applies at least to physicists; for as Einstein remarked: 'But there is another reason for the high repute of mathematics: it is mathematics that offers the exact natural sciences a certain measure of security which, without mathematics, they could not attain.' To give a layman without mathematical training an understanding of quantum mechanics, one will (or must) rely instead on math-free approaches.

[3]Without appropriate formal considerations, it is impossible to understand, for example, how to motivate the replacement of a physical measurement variable by a Hermitian operator.

[4]We could instead also make use of the large reservoir of historically important experiments. But their mathematical formulation is in general more complex, and since in the frame of our considerations they do not lead to further-reaching conclusions than our 'toy models,' we restrict ourselves to the latter for clarity and brevity.

order to better assimilate and more clearly grasp it, as well as of course to train the necessary formal skills.[5]

The learning aids in the Appendix include chapters with some basic mathematical and physical background information; this allows the reader to refresh 'passive' knowledge without the need to refer to other sources or to become involved with new notations.

Moreover, the no doubt unusually extensive Appendix contains the solutions to many of the exercises and, in addition, some chapters in which further-reaching questions and issues are discussed; although these are very interesting in themselves, their treatment would far exceed the framework of a lecture course.

The footnotes with a more associative character can be skipped on a first reading.

A note on the term 'particle': Its meaning is rather vague in physics. On the one hand, it denotes 'something solid, not wavelike'; on the other hand 'something small', ranging from the elementary particles as structureless building blocks of matter, to objects which themselves are composed of constituent 'particles' like the α particle and other atomic nuclei or even macroscopic particles like sand grains. In quantum mechanics, where indeed it is often not even clear whether a particular object has mainly particle or mainly wave character, the careless use of the term may cause confusion and communication problems.

Accordingly, several terms which go beyond 'wave' or 'particle' have been suggested, such as quantal particle, wavical, wavicle, quantum object, quanton. Throughout this book, we will use the term 'quantum object,' unless there are traditionally established terms such as 'identical particles' or 'elementary particles.' The consistent use of 'quantum object' instead of 'particle' may perhaps seem somewhat pedantic, but we hope that it will help to ensure that fewer false images stick in the minds of readers; it is for this reason that this term is also found in many high-school textbooks.

Quantum mechanics is a fundamental theory of physics, which has given rise to countless applications. But it also extends deep into areas such as philosophy and epistemology and leads to thinking about 'what holds the world together at its core'; in short, it is also an intellectual adventure. The fascinating thing is that the more one becomes acquainted with quantum mechanics, the more one realizes how simple many of its central ideas really are.[6] It would be pleasing if *Quantum Mechanics for Pedestrians* could help to reveal this truth.

[5]'It is a great support to studying, at least for me, to grasp everything that one reads so clearly that one can apply it oneself, or even make additions to it. One is then inclined to believe in the end that one could have invented everything himself, and that is encouraging.' Georg Christoph Lichtenberg, *Scrap Books,* Vol. J (1855).

[6]'The less we know about something, the more complicated it is, and the more we know about it, the easier it is. This is the simple truth about all the complexities.' Egon Friedell, in *Kulturgeschichte der Neuzeit; Kulturgeschichte Agyptens und des alten Orients (Cultural history of modern times; the cultural history of Egypt and the ancient Near East).*

Let us close with a remark by Richard Feynman which holds true not only for physics in general, but even more for quantum mechanics: ‘Physics is like sex: Sure, it may give some practical results, but that’s not why we do it.’

Overview of Volume 2

Now we have established the basic framework of quantum mechanics in Volume 1 in the form of postulates, we turn to two other major topics in Volume 2.

First, we want to fill the framework of quantum mechanics with life, i.e., to discuss some applications (solutions for simple potentials, angular momentum, symmetries, identical particles, scattering, quantum information). These chapters are to some extent more technical, since we cannot use toy models, as in the first volume, but instead have to consider 'real' systems.

Second, we will extend the framework of quantum mechanics prudently to address modern developments such as entanglement and decoherence. Finally, we turn again to the realism debate; the final chapter presents some of the current interpretations of quantum mechanics.