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# Control Engineering: MATLAB Exercises

 Springer

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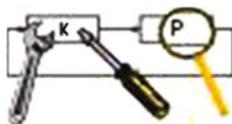
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Frigyes Csáki  
(1921–1977)

*This textbook is devoted to the memory  
of Frigyes Csáki, who was the first professor  
of control in Hungary*



# Preface

This book is intended to aid students in their study of MATLAB™/SIMULINK™ for use in solving control problems. Specifically, 16 labs for an introductory control course have been developed at the Department of Automation and Applied Informatics, Budapest University of Technology and Economics. This book is a collection of these labs. This exercise book is a supplement to the textbook “Control Engineering,” by László Keviczky, Ruth Bars, Jenő Hetthéssy, and Csilla Bányász [1], which is used in the control course. Each chapter of this exercise book is related to the corresponding chapter of the textbook.

The importance of accompanying textbooks by labs using CAD software was recognized decades ago at the department. At that time, a set of FORTRAN libraries supported the instruction both in control systems analysis and design. We still believe that learning control theory is best motivated by applications and simulations rather than by concepts alone. In fact, the use of MATLAB™ allows a lot of theoretical concepts to be easily implemented. If students can immediately show for themselves how certain concepts work in practice, they will go back to the theoretical considerations with greater confidence and an improved ability to move to the next field to study. Well, feedback is around us, anyway.

The problems discussed in this book are limited to linear, time-invariant control systems. Both continuous-time and discrete-time systems are considered, with deterministic inputs.

MATLAB™/SIMULINK™ is useful only for those students, who master the tools offered. Though the application of MATLAB™ commands is simple and straightforward, a systematic introduction together with control-related examples is a must in our opinion. Time should be devoted to practicing fundamental MATLAB™ facilities, alternative command sequences, and visualization capabilities. An introductory lab is devoted to demonstrating the availability and power of MATLAB™ in this respect.

Frequency functions and transfer functions form essential tools in classical control theory. Interestingly enough, the frequency domain considerations gave a remarkable impetus to the postmodern control era, as well. Three labs have been devoted to discussing fundamental analysis of continuous-time systems including

feedback and stability. As far as controller synthesis is concerned, three labs treat YOULA-parameterized control design, as well as *PID* compensation and series compensation for processes with dead time have also been elaborated. The case of controlling unstable processes is also involved. The theoretical discussion of state-space representations is supported by two labs offering a gentle introduction to the subject, as well as demonstrating the efficient algorithms and MATLAB™ commands available for state variable feedback.

These days, controllers are implemented as digital controllers. As most of the processes to be controlled are continuous time in nature, digital control needs additional tools to cover sampled data systems. Just to support the development of a proper view of discrete-time systems, an introductory lab has been added to this topic. Two labs are devoted to discrete controller design. One of them shows controller design using the YOULA parameterization, and the design of a SMITH predictor as well as a deadbeat control as special cases of YOULA parameterization. The second lab discusses discrete-time *PID* controller design. State feedback control for discrete systems is also provided in a lab devoted to this topic.

Two labs deal with the polynomial design method for the compensation of unstable processes, both for the continuous and the discrete case.

In the last lab, the modelling and simulation of a heating process provide a case study.

Each lab is introduced by summarizing the basic concepts and definitions of the topic discussed. The MATLAB™-related functions are discussed in detail. Labs have been designed to be accomplished within a two-hour period, each. Solved examples and reinforcement problems are intended to foster a better understanding. Examples range from simple drills just to demonstrate the MATLAB™ commands to more complex problems, and in most cases a short evaluation completes the lab. It is supposed that the reader writes and runs the codes and evaluates the results. In some cases, the plots are not included in the book, but the evaluation is given, supposed that the reader, after having run the codes, sees the figures.

It is to be emphasized that this set of labs is not a substitute for a textbook in any respect. The textbook of our introductory control course intends to give a deep and comprehensive treatment of control-related subjects. The labs in this book are intended to serve as pedagogical tools offering the student a chance for active learning and experimenting. The present set of labs have been employed in instruction for several semesters.

The authors hope that through active problem solving the students will understand better the control principles and get practice how to apply them in analysis and design of control systems.

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# Notations

Transfer functions of continuous-time systems	$H$ (or $P$ )
Transfer functions of discrete-time systems	$G$
Controller transfer function	$C$
Process transfer function	$P$
Discrete-time process pulse transfer function	$G$ (or $P_d$ )
Sensitivity function	$S$
Complementary sensitivity function	$T$
Transfer function of an open control loop	$L$
Gain of a control loop	$K$
Transfer coefficient of a control loop	$k$
Youla parameter	$Q$
Continuous time	$(t)$
Discrete time	$[k]$
Laplace transformation	$\mathcal{L}\{\dots\}$
Fourier transformation	$\mathcal{F}\{\dots\}$
$z$ -transformation	$\mathcal{Z}\{\dots\}$
Complex variable ( $\mathcal{L}$ transformation)	$s$
Complex variable ( $\mathcal{Z}$ transformation)	$z$
Reference signal	$r$ (or $y_r$ )
Controlled variable	$y$
Error signal	$e$
Actuating signal (or output of the regulator)	$u$
Input noise	$y_{ni}$
Output noise	$y_n$ (or $y_{no}$ )
Measurement noise	$y_z$
Vector	$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$
Row vector	$\mathbf{a}^T, \mathbf{b}^T, \mathbf{c}^T, \dots$
Matrix	$\mathbf{A}, \mathbf{B}, \mathbf{C}$
Transpose of a matrix	$\mathbf{A}^T$
Adjunct of a matrix	$\mathbf{adj}(\mathbf{A})$

Determinant of a matrix	$\det(\mathbf{A})$ (or $ \mathbf{A} $ )
State variable	$\mathbf{x}$
Parameters of the state equation (continuous)	$\mathbf{A}, \mathbf{b}, \mathbf{c}, d$
Parameters of the state equation (discrete)	$\mathbf{F}, \mathbf{g}, \mathbf{h}, d$ (or $\mathbf{F}, \mathbf{g}, \mathbf{c}, d$ )
Diagonal matrix	$\mathbf{diag}[a_{11}, a_{22}, \dots, a_{mm}]$
Unit matrix	$\mathbf{I} = \mathbf{diag}[1, 1, \dots, 1]$
Sampling time	$T_s$
Dead time (continuous)	$T_d$
Time delay (discrete)	$d$
Additional time delay	$T_h$
Step response function	$v(t)$
Weighting function	$w(t)$
Frequency	$\omega$
Crossover (cutoff) frequency	$\omega_c$
Frequency spectrum of a continuous signal	$F(j\omega)$
Frequency spectrum of a sampled signal series	$F^*(j\omega)$
Frequency spectrum of a discrete-time model	$G(j\omega)$ (or $P_d(j\omega)$ )
Polynomials	$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{G}, \mathcal{F}, \mathcal{R}, \mathcal{X}, \mathcal{Y}, \mathcal{V}$
Degree of a polynomial	$\deg\{\mathbf{A}\}$
Characteristic equation	$\mathcal{A}(s) = 0$
Limit of the control output	$\cup$
Gradient vector	$\mathbf{grad}[f(\mathbf{x})]$
For all $\omega$	$\forall \omega$
Angle of a complex number or function	$\angle$ (or $\text{arc}(\dots)$ )
Exponential function	$e^{(\dots)}$ (or $\exp(\dots)$ )
Natural logarithm	$\ln(\dots)$
Base 10 logarithm	$\lg(\dots)$
Expected value	$E\{\dots\}$
Probability limit value	$\text{plim}\{\dots\}$
Matrix exponential	$e^{\mathbf{A}}$
Matrix logarithm	$\ln(\mathbf{A})$
Continuous time	CT
Discrete time	DT
Step response equivalent	SRE
Partial fractional expansion	PFE