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# Appendices

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## Appendix I: Answers to Problems in Selected Chapters

### Chapter 1

$$1.1 \quad T = 2\pi L \sqrt{\frac{W}{g} \cdot \frac{L}{3EI + 2kL^3}}$$

$$1.2 \quad u(t=1) = -0.89 \text{ in} \\ \dot{u}(t=1) = 22.66 \text{ in/sec}$$

$$1.3 \quad f = 2.24 \text{ cps}$$

$$1.4 \quad (a) f = 2.87 \text{ cps} \\ (b) f = 2.74 \text{ cps}$$

$$1.5 \quad f = \frac{4}{\pi} \sqrt{\frac{3EIg}{L^3W}}$$

$$1.6 \quad u(t=2) = -0.474 \text{ in} \\ \dot{u}(t=2) = -21.05 \text{ in/sec} \\ \ddot{u}(t=2) = 4065 \text{ in/sec}^2$$

$$1.7 \quad \theta = \theta_0 \cos \sqrt{\frac{g}{L}t} + \frac{\dot{\theta}_0}{\omega} \sin \sqrt{\frac{2}{L}t}$$

$$1.8 \quad k = 73.64 \text{ lb/in} \\ EI = 3.39 \times 10^5 (\text{lb} \cdot \text{in}^2)$$

$$1.9 \quad u(t) = 0.0995 \sin 48.03t \text{ (in)} \\ \dot{u}(t) = 4.78 \cos 48.03t \text{ (in/sec)}$$

$$1.10 \quad u(t) = 0.5 \cos 27.78t$$

$$1.11 \quad f = \frac{1}{2\pi} \sqrt{\frac{ka^2 - mgL}{mL^2}}$$

$$1.13 \quad f = \frac{1}{2\pi} \sqrt{\frac{3EI}{mL^3} - \frac{3g}{2L}}$$

$$1.14 \quad f - f_0 \sqrt{1 - \frac{w}{w_{er}}}$$

$$1.15 \quad (a) f = \frac{1}{2\pi} \sqrt{\frac{3EIk g}{(3EI + kL^3)W}}$$

$$(b) f = \frac{1}{2\pi} \sqrt{\frac{48EIk g}{(48EI + kL^3)W}}$$

$$(c) f = \frac{1}{2\pi} \sqrt{\frac{3EILg}{a^2 b^2 W}}$$

$$(d) f = \frac{1}{2\pi} \sqrt{\frac{3EILk g}{(3EIL + a^2 b^2 k)W}}$$

$$1.16 \quad f = \frac{1}{2\pi} \sqrt{k \left( \frac{1}{m_1} + \frac{1}{m_2} \right)}$$

$$1.17 \quad \omega = \sqrt{\frac{k}{m}}$$

$$1.18 \quad \omega = 35.018 \text{ (rad/sec)}$$

$$1.19 \quad \omega = \sqrt{\frac{k - Wh}{I_0}}$$

unstable for  $k \leq Wh$

## Chapter 2

$$2.1 \quad u(t=1) = -0.037 \text{ in}$$

$$\dot{u}(t=1) = -0.570 \text{ in/sec}$$

$$2.2 \quad u(t=2) = -4.65 \times 10^{-9} \text{ in}$$

$$\dot{u}(t=2) = -4.083 \times 10^{-8} \text{ in/sec}$$

$$\ddot{u}(t=2) = 4.18 \times 10^{-5} \text{ in/sec}^2$$

$$2.3.1 \quad c = 0.73 \text{ lb} \cdot \text{sec/in}$$

$$2.3.2 \quad \xi = 1.5\%$$

$$2.5 \quad (a) \text{ for } \xi = 1, u = [u_0(1 + \omega t) + v_0 t] e^{-\omega t}$$

$$(b) \text{ for } \xi > 1, u = e^{-\xi \omega t} \left[ u_0 \cosh \omega'_D t + \frac{v_0 + u_0 \xi \omega}{\omega'_D} \sinh \omega'_D t \right]$$

$$\text{where } \omega'_D = \omega \sqrt{\xi^2 - 1}$$

$$2.6 \quad (a) \xi = 0.4167$$

$$(b) T_D = 0.2765 \text{ sec}$$

$$(c) \delta = 2.8801$$

$$(d) \frac{u_1}{u_2} = 17.8161$$

$$2.9 \quad f_D = 0.5 \sqrt{\frac{ka^2 - mgL}{mL^2}}$$

$$2.10 \quad (a) \xi = 0.076$$

$$(b) f_D = 8.69 \text{ cps}$$

$$(c) \delta = 0.48$$

$$(d) \frac{u_1}{u_2} = 1.61$$

- 2.11 (a)  $\xi = 0.018$   
 (b)  $\omega_D = 57.76 \text{ rad/sec}$   
 (c)  $\delta = 0.113$   
 (d)  $\frac{u_1}{u_2} = 1.12$

2.12 (a)  $K_e = \frac{3EI k}{3EI + kL^3}$

$$C_E = 0.2 \sqrt{\frac{3EI k w}{g(3EI + kL^3)}}$$

(b)  $K_E = \frac{48EI k}{48EI + kL^3}$

$$C_E = 0.2 \sqrt{\frac{48EI k m}{48EI + kL^3}}$$

(c)  $K_E = \frac{3EIL}{a^2 b^2}$

$$C_E = 0.2 \sqrt{\frac{3EIL w}{a^2 b^2 g}}$$

(d)  $K_E = \frac{3EIKL}{3EIL + ka^2 b^2}$

$$C_E = 0.2 \sqrt{\frac{3EIKL w}{(3EIL + ka^2 b^2) g}}$$

2.13  $\xi = 6.7\%$

2.14  $m_1 m_2 \ddot{u} + (m_1 + m_2) \dot{c} u + (m_1 + m_2) k u = 0$

2.15  $\ddot{u} + 2\xi \omega \dot{u} + \omega^2 u = 0$

where:  $\omega = \sqrt{\frac{k}{M}}$ ,  $M = \frac{m_1 m_2}{m_1 + m_2}$ ,  $\omega_D = \omega \sqrt{1 - \xi^2}$ ,  $\xi = \frac{c}{c_{cr}}$ ,  $c_{cr} = 2\sqrt{kM}$

### Chapter 3

3.1  $Y = 0.0037 \text{ in}$

3.2  $A_T = 51.2 \text{ lb}$

3.3  $Y = 0.823 \text{ in}$

3.4  $Y = 0.746 \text{ in}$

3.5 (a)  $A_T = 15,803 \text{ lb}$

(b)  $T_R = 3.16$

3.6  $k = 93 \text{ lb/in}$

3.7  $u = 0.013 \text{ in}$

3.8  $T_R = 0.34$

3.9 (a)  $Y_1 = 0.064 \text{ in for } f_1 = 800 \text{ RPM}$

$Y_2 = 0.0446 \text{ in for } f_2 = 1000 \text{ RPM}$

$Y_3 = 0.0302 \text{ in for } f_3 = 1200 \text{ RPM}$

(b)  $Y(r = 1) = 0.076 \text{ in}$

3.10  $\xi = 3.3\%$

3.11  $\xi = 6.3\%$

$$\omega = 49.2 \text{ (rad/sec)}$$

3.12  $Y = 0.387 \text{ (cm)}$

$$\sigma_{\max} = 110.7 \text{ (Mpa)}$$

3.13  $f = f_r \sqrt{1 - \frac{m_s}{m}}$

3.14  $\omega_p = \omega \sqrt{1 - 2\xi^2}$  for  $\xi < \frac{1}{\sqrt{2}}$

$$U_p = \frac{u_{st}}{2\xi\sqrt{1 - \xi^2}}$$

3.15 (a)  $f = 18.58 \text{ cps}$

(b)  $\xi = 0.0735$

(c)  $F_0 = 4825 \text{ lb}$

(d)  $F_0 = 4840 \text{ lb}$

3.16 
$$\xi = \frac{U_1(1 - r_1^2)}{2r_1\sqrt{U_r^2 r_1^2 - U_1^2}}$$

$$F_r = \frac{U_r U_1 (1 - r_1^2) k}{r_1 \sqrt{U_r^2 r_1^2 - U_1^2}}$$

3.17 (a)  $M\ddot{u} + c\dot{u} + ku = \frac{m_1 F_0}{m_1 + m_2} \sin t$

where  $M = \frac{m_1 m_2}{m_1 + m_2}$

(b) 
$$u = \frac{m_1 F_0 \sin(t - \theta)}{k(m_1 + m_2) \sqrt{(1 - r^2)^2 + (2r\xi)^2}}$$

## Chapter 4

4.1 (a)  $u(t = 0.5) = -0.407 \text{ in}$

(b)  $u_{\max} = 1.37 \text{ in}$

4.2 (a)  $u(t = 0.5) = -0.102 \text{ in}$

(b)  $u_{\max} = 1.17 \text{ in}$

4.3  $DLF = \frac{t}{t_d} - \frac{\sin \omega t}{\omega t_d}$  for  $t \leq t_d$

$$DLF = 1 + \frac{1}{\omega t_d} (\sin \omega t - \sin \omega(t + t_d))$$
 for  $t \geq t_d$

4.4  $V_{\max} = 18,093 \text{ lb}$  for left column

$V_{\max} = 1908 \text{ lb}$  for right column

4.5  $V_{\max} = 15,640 \text{ lb}$  for left column

$V_{\max} = 1649 \text{ lb}$  for right column

4.8  $u_1(t) = -\frac{v}{\omega} \cos \omega t + vt$

$$4.9 \quad u(t) = \frac{u_{st}}{t_d} \left[ t - \frac{\sin \omega t}{\omega} \right] \text{ for } t \leq t_d$$

$$= \frac{u_{st}}{t_d} \left\{ t_d - \frac{\sin \omega t}{\omega} \right\} \cos \omega(1 - t_d) + \frac{1}{\omega} \sin \omega(t - t_d) \text{ for } t \geq t_d$$

- 4.10  $u_{\max} = 1.348$  (in)  
 $\sigma_{\max} = 11,906$  (psi)
- 4.11  $u_{\max} = 0.72$  (in)  
 $\sigma_{\max} = 16,282$  (psi)
- 4.12  $u(t = 0.5) = -1.903$  in
- 4.13  $u(t = 0.5) = -0.060$  in
- 4.14  $u(t = 1) = -2.809$  in
- 4.15  $u(t = 1) = -2.397$  in
- 4.16  $u_{\max} = 6.03$  in (undamped system)  
 $u_{\max} = 4.59$  in (with 20% damping)
- 4.17  $u_{\max} = 1.51$  in
- 4.18  $u_{\max} = 1.42$  in
- 4.19  $u_{\max} = 0.58$  in
- 4.20  $u_{\max} = 1.00$  in
- 4.21  $u_{\max} = 0.79$  in
- 4.22  $u_{\max} = 0.66$  in
- 4.23  $u_{\max} = 0.34$  in
- 4.24  $\sigma_{1\max} = 3842$  psi  
 $\sigma_{2\max} = 6831$  psi
- 4.25  $u_{\max} = 0.71$  in
- 4.26 (a)  $\sigma = \pm 6193$  psi  
 (b)  $F_{\max} = 11,376$  lb

## Chapter 5

- 5.1  $u_{\max} = 0.374$  in
- 5.2  $\sigma_{\max} = 7.246$  ksi
- 5.3  $u_{\max} = 0.418$  in
- 5.4  $\sigma_{\max} = 12.788$  ksi
- 5.5  $S_D = 1.9$  in  
 $S_V = 22.4$  in/sec  
 $S_a = 0.68$  g
- 5.6  $S_D = 1.28$  in  
 $S_V = 15.36$  in/sec  
 $S_a = 0.48$  g
- 5.7  $S_D = 11.0$  in
- 5.8  $(F_s)_{\max} = 88.0$  Kip
- 5.9  $(F_s)_{\max} = 36.0$  Kip  
 $(F_T)_{\max} = 44.0$  Kip

5.10  $S_D = 8.0$  in

5.11  $\mu = 1.8$

5.12 (a)  $S_D = 40$  in

$S_v = 50.3$  in/sec

$S_a = 1.63$  g

(b)  $S_D = 4.8$  in

$S_v = 60.0$  in/sec

$S_a = 1.96$  g

5.13 (a)  $S_D = 0.46$  in

$S_v = 5.8$  in/sec

$S_a = 0.19$  g

(b)  $S_D = 6.0$  in

$S_v = 18$  in/sec

$S_a = 0.6$  g

5.14  $S_D = 0.78$  in (at  $f = 0.5$  cps)

$S_v = 2.46$  in/sec

$S_a = 7.72$  in/sec<sup>2</sup>

5.15  $S_D = 8.03$  in (at  $f = 1.00$  cps)

$S_v = 50.45$  in/sec

$S_a = 317.00$  in/sec<sup>2</sup>

5.16  $S_D = 3.26$  in (at  $f = 1.00$  cps)

$S_v = 20.38$  in/sec

$S_a = 127.40$  in/sec<sup>2</sup>

## Chapter 6

6.1  $u_{\max} = -10.27$  in

6.2  $u_{\max} = 2.56$  in

6.3  $u_{\max} = 6.47$  in

6.4  $u_{\max} = 1.03$  in

6.5  $u_{\max} = 5.19$  in

6.6  $u_{\max} = 2.38$  in

6.7  $\mu = 1.7$

6.8  $u(t = 0.5) = 0.4477$  in

6.9  $u(t = 0.5) = 0.2654$  in

6.10  $u(t = 0.5) = 0.1423$  in

6.11  $u(t = 0.5) = 0.1340$  in

6.12  $a_0 = 2.78$

## Chapter 19

19.1  $F(t) = \frac{120}{\pi} \left[ \sin 2\pi t + \frac{1}{3} \sin 6\pi t + \frac{1}{5} \sin 10\pi t \dots \right]$

19.2  $F(t) = 10^{-6} [357 \sin 2\pi t - 26 \cos 2\pi t + 36 \sin 6\pi t - 532 \cos 6\pi t - 35 \sin 10\pi t - 7 \cos 10\pi t + \dots]$

19.3  $u(t = 0.5) = 0.3518$  in

- 19.4 (a)  $a_n = \frac{720}{\pi(1-n^2)}, n = 2, 4, 6, \dots$   
 $a_n = 0, n = 1, 3, 5, \dots$   
 $b_n = 0, n = 1, 2, 3, \dots$
- 19.5  $u(t = 0.05) = -0.2065$  in
- 19.6  $u(t = 0.05) = -0.2064$  in
- 19.7  $u(t = 0.05) = 0.1295$  in
- 19.8 (a)  $a_0 = 0.0350$   
 $a_1 = 0.0069 \quad b_1 = -0.0361$   
 $a_2 = -0.0724 \quad b_2 = -0.0402$   
 (b)  $u(t = 0.35) = 0.2570$  in
- 19.9  $u(t = 0.35) = 0.2327$  in
- 19.10  $u(i = 0.5) = 0.027$  in
- 19.11  $u(t = 0.5) = (0.02842 - 0.00011i)$  in  
 (a)  $a_0 = \frac{P_0}{\pi}$
- 19.12  $a_n = 0, n = 1, 3, 5, \dots; b_1 = \frac{P_0}{2}$   
 $a_n = \frac{P_0}{\pi} \cdot \frac{2}{1-n^2}, n = 2, 4, 6, \dots; b_n = 0, n > 1$
- 19.13  $u(t = 0.5) = 0.0731$  in
- 19.14  $u(t = 0.5) = 0.0543$  in
- 19.15  $u_1(t) = 0.229(\sin \pi t - 0.75 \sin 4.19t)$   
 $u_2(t) = 0.229[\sin \pi(t-1) - 0.75 \sin 4.19(t-1)]$   
 $u(t) = u_1(t)$  for  $0 \leq t \leq 1.0$  sec  
 $u(t) = u_1(t) + u_2(t)$  for  $t \leq 1.0$  sec
- 19.16  $u(t = 1 \text{ sec}) = 0.2419(\text{in})$
- 19.17  $u(t = 1 \text{ sec}) = 0.0532(\text{in})$
- 19.18  $u(t = 1 \text{ sec}) = 0.0035(\text{in})$
- 19.19  $u(t = 1 \text{ sec}) = 0.0034(\text{in})$
- 19.20  $u(t = 1 \text{ sec}) = 0.0028(\text{in})$

## Chapter 21

- 21.1  $M^* = 4.48 \text{ lb} \cdot \text{sec}^2/\text{in}$   
 $C^* = 2250 \text{ lb} \cdot \text{sec}/\text{in}$   
 $K^* = 45,000 \text{ lb}/\text{in}$   
 $F^*(t) = 625 \cdot f(t) \text{ lb}$
- 21.2  $M^* = \frac{5}{6}m$   
 $C^* = c$   
 $K^* = k$   
 $F^*(t) = \frac{M(t)}{L}$

- 21.3  $M^* = \frac{2}{3}\bar{m}L^2$   
 $C^* = cL$   
 $K^* = kL$   
 $F^* = \frac{P_0L}{6}f(t)$
- 21.4  $M^* = \frac{m}{2\pi}(5\pi - 8)$   
 $K^* = \frac{EI\pi^4}{32L^3}$   
 $F^*(t) = 0.2929F_0f(t)$
- 21.5  $K_G^* = -\frac{N\pi^4}{8L}$
- 21.6  $M^* = 0.1237\frac{\gamma d}{g}$   
 $K^* = -\frac{E_c\pi d^4}{128L^3}$   
 $F^*(t) = -0.1807P_0(t)Ld$
- 21.7  $\omega = \sqrt{\frac{48EI}{L^3(m + \frac{17}{35}m_b)}} \text{ rad/sec}$
- 21.8  $\omega = 7.825\sqrt{\frac{EI}{m_bL^3}} \text{ rad/sec}$
- 21.9  $f = 3.51 \text{ cps}$
- 21.10  $\omega = 2.15\sqrt{\frac{gEI}{WL^3}} \text{ rad/sec}$
- 21.11  $\omega = 0.62\sqrt{\frac{gAG}{WL}} \text{ rad/sec}$
- 21.12  $f = 0.496 \text{ cps}$
- 21.13  $f = 0.492 \text{ cps}$

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## Appendix II: Glossary

Accelerometer	An instrument for measuring ground acceleration as a function of time.
Aliasing	The phenomenon in which higher harmonics introduce spurious low frequency components. This occurs when the number of sampled points of a function is insufficient to describe the function. (See Nyquist Frequency.)
Amplitude	Maximum value of a function as it varies with time. If the variation with time can be described by either a sine or cosine function, it is said to vary harmonically.
Angular Frequency/Circular Frequency	The frequency of periodic function in cycles per second (Hertz) multiplied by $2\pi$ ; expressed in rad/sec.

Autocorrelation of a Random Function $x(t)$	Correlation between the function $x(t)$ and the out-of-phase function $x(t + \tau)$ as defined by Eq. (22.19).
Base Shear Force	The total lateral force on the structure equivalent to the earthquake excitation at the base of the structure.
Basic Design Spectra	Smooth or average plots of maximum response of single degree-of-freedom systems used in seismic design of structures.
Boundary Condition	A constraint applied to the structure independent of time.
Braced Frame	An essentially vertical truss system of the concentric or eccentric type which is provided to resist lateral forces.
Building Frame System	An essentially complete space frame which provides support for gravity loads.
Characteristic Equation	An equation whose roots are the natural frequencies.
Circular Frequency	See Angular Frequency.
Complementary Solution	The solution of a homogeneous differential equation (no external excitation).
Complete Quadratic Combination (CQC)	A method of combining maximum values of modal contributions which is based on random vibration theory and includes cross correlation terms.
Concentric Braced Frame	A brace frame in which the members are subjected primarily to axial forces.
Consistent Mass	Mass influence coefficients determined by assuming that the dynamic displacement functions are equal to the static displacement functions.
Correlation between Random Variables $x_1(t)$ and $x_2(t)$	The time average of the product of the functions $x_1(t)$ and $x_2(t)$ .
Coupled Equations	A system of differential equations in which the equations are not independent from each other.
Critical Damping	Minimum amount of viscous damping for which the system will not vibrate.
D' Alembert Principle	This principle states that a dynamic system may be assumed to be in equilibrium provided that the inertial forces are considered as external forces.
Damped Frequency	The frequency at which a viscously damped system oscillates in free vibration.
Damping	The property of the structure to absorb vibrating energy.
Damping Ratio	The ratio of the viscous damping coefficient to the critical damping.
Degrees of Freedom	The number of independent coordinates required to completely define the position of the system at any time.
Deterministic Vibration	A process which can be predicted by an exact mathematical expression.
Dirac's Delta Function	A generalized function having the properties described in Eq. (22.52).
Direct Stiffness Method	The method of assembling the system stiffness matrix by proper summation of the stiffness coefficients of the elements in the system.

Discrete Fourier Transform	A summation of harmonic terms to express the Fourier transform for a function defined by a finite number of points.
Dual System	A combination of a special or intermediate moment-resisting space frame and shear walls or braced frames.
Ductility Ratio	The ratio between the maximum displacement for elastoplastic behavior and the displacement corresponding to yield point.
Dynamic Condensation	A method of reducing the dimension of the eigenproblem by establishing the dynamic relationship between primary and secondary coordinates.
Dynamic Magnification Factor	The ratio of the maximum displacement of a single degree of freedom excited by a harmonic force to the deflection that would result if a force of that magnitude were applied statically.
Earthquake	The vibrations of the Earth caused by the passage of seismic waves radiating from some source of elastic energy.
Eigenproblem	The problem of solving a homogeneous system of equation: containing a parameter which should be determined to provide nontrivial solutions.
Elastic Rebound Theory	The theory of earthquake generation proposing that faults remain locked while strain energy slowly accumulates in the surrounding rock, and then suddenly slip, releasing this energy.
Elastoplastic	A system which behaves elastically for a force that does not exceed a maximum value and plastically above this maximum.
Ensemble	A set of samples or records of a random process.
Epicenter	The point on the Earth's surface directly above the focus.
Ergodic Process	A stationary random process for which the time average of any record is equal to the average across the ensemble.
Fast Fourier Transform (FFT)	A very efficient algorithm implemented in a computer program for the calculation of the response in the frequency domain.
Flexibility Coefficient	$f_{ij}$ is the displacement at coordinate $i$ due to a unit force (or unit moment) applied at coordinate $j$ .
Forced Vibration	Vibration in which the response is due to external excitation of the system.
Fourier Analysis	Method of determining the response by superposition of the responses to the harmonic components of the excitation.
Fourier Transform	The Fourier transform $C(\omega)$ of a function $F(t)$ is defined by Eq. (22.23).
Fourier Transform Pair	In reference to the function $F(t)$ , the Fourier transform pair is given by Eqs. (22.23) and (22.24).
Free Body Diagram	A sketch of the system, isolated from all other bodies, in which all the forces external to the body are shown.
Free Vibration	The vibration of a system in absence of external excitation.
Frequency Analyzer or Spectral Density Analyzer	An instrument that measures electronically the spectral density function of a signal.
Frequency Ratio	The ratio between the forcing frequency to the natural frequency for a system excited by a harmonic load.
Fundamental Frequency	The lowest natural frequency of a multidegree-of freedom vibrating system.

Gauss-Jordan Reduction or Elimination	A computational technique in which elementary row operations are applied systematically to solve a linear system of equations.
Generalized Coordinates	A set of independent quantities which describe the dynamic system at any time. These quantities are generally functions of the geometric Coordinates.
Geometric Stiffness Coefficient	$k_{Gij}$ is the force at coordinate $i$ due to a unit displacement at coordinate $j$ and resulting from the axial forces in the structure.
Harmonic	A sinusoidal function having a frequency that is an integral multiple of the fundamental frequency.
Harmonic Force	A force expressed by a sine, cosine (or equivalent exponential) function.
Hypocenter or Focus	The Point in the interior of the earth at which rupture is initiated during an earthquake.
Impulsive Load	A load that is applied during a relatively short time interval producing an instantaneous change in velocity.
Initial Conditions	The initial values of specific functions such as displacement, velocity, or acceleration evaluated at time $t = 0$ .
Intensity (of Earthquakes)	A measure of ground shaking obtained from the damage done to man-made structures, changes in the Earth's surface, and witness reports.
Intermediate Moment-Resisting Space Frame (IMRSF)	A concrete space frame designed in conformance with Section 1921.8 (k) of UBC-97.
Isolation	The reduction of severity of the response, usually attained by proper use of a resilient support.
Lateral Force Method	A method of analysis in which lateral horizontal forces at various levels of the structure are considered equivalent to seismic excitation at the base of the structure.
Lateral Force-Resisting System	That part of the structural system assigned to resist lateral forces.
Linear Acceleration Method	A step-by-step method for the integration of the differential equations of motion in which the acceleration is assumed to be a linear function during each time step.
Linear System	A system of differential equations in which no term contains products (or exponents) of the dependent variables or their derivatives.
Logarithmic Decrement	The natural logarithm of the ratio of any two successive amplitudes of the same sign obtained in the decay curve in a free vibration test.
Lumped Mass	A method of discretization in which the distributed mass of the elements is lumped at the nodes or joints.
Mathematical Model	The idealization of a system including all the assumptions imposed on the physical problem.
Mean-Square Value	The time average of the square of a random function as defined by Eq. (22.2).
Mean Value	The time average of a random function defined by Eq. (23.1).
Modal Shapes (also Normal Modes)	The relative amplitude of the displacements at the coordinates of a multidegree-of-freedom system vibrating at one of the natural frequencies.

Modal Superposition Method	A method of solution of multidegree-of-freedom systems in which the response is determined from the solution of independent modal (or normal) equations.
Modified Mercalli Intensity (MMI)	A measure of the effect of an earthquake at a particular location.
Moment-Resisting Space Frame	A space frame in which the members and joints are capable of resisting forces primarily by flexure.
Narrow-Band Process	A random process whose spectral density function has nonzero values only in a narrow frequency range.
Natural Frequency	The number of cycles per second at which a single degree-of-freedom system vibrates freely or a multidegree-of-freedom system vibrates in one of the normal modes.
Natural Period	The time interval for a vibrating system in free vibration to do one oscillation.
Newmark Beta Method	A numerical method to calculate the response of a structure subjected to external excitation (force or motion).
Node or Joint	A point joining elements of the structure and at which displacements are known or to be determined.
Normal Distribution or Gaussian Distribution	A function whose probability density function is given by Eq. (22.12).
Normal Modes	See modal Shapes.
Nyquist Frequency	The maximum frequency component that can be detected from a function sampled at time spacing $\Delta t$ [ $N_y = 1 / 2\Delta t$ (Hz)].
Occupancy Factor	A numerical factor in the calculation of the base shear force that depends on the intended use of the structure.
Ordinary Moment-Resisting Space Frame (OMRSF)	A moment-resisting not meeting special detailing requirements for ductile behavior.
P-Delta Effect	The secondary effect on shears, axial forces, and moments of frame members induced by the vertical loads on the laterally displaced building frame.
Periodic Function	A function that repeats itself at a fixed time interval known as the period of the function.
Power Spectral Density	A term used to describe the intensity of random vibration at a given frequency, measured in $g^2/H_z$ .
Principle of Virtual Work	The work done by all the forces acting on a system in static or dynamic equilibrium, which occurs during a virtual displacement compatible with the constraints of the system equal to zero, for a rigid system and for an elastic frame, equal to the work of the internal forces during the virtual displacement.
Probability Density Function	A function $p(x)$ such that the probability of $x(t)$ of being in the range $(x, x + dx)$ is $p(x) dx$ .
Pseudo-Velocity	The velocity calculated by analogy with the apparent harmonic motion for a system seismically excited.
Random Function	A function (as opposed to a deterministic function) whose value at any time cannot be determined exactly, but can only be predicted in probabilistic terms by statistical methods.

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Random Vibration or Random Process	A process which cannot be predicted in a deterministic sense, but only probabilistically using the theory of statistics.
Rayleigh Distribution	A function whose probability density function is given by Eq. (22.14).
Rayleigh's Formula	A formula to estimate the fundamental period of the structure.
Resonance	The condition in which the frequency of the excitation equals the natural frequency of the vibrating system.
Response	The force or motion that results from external excitation on the structure.
Response Spectrum	A plot of maximum response (displacement, velocity, or acceleration) for a single degree-of-freedom system defined by its natural frequency (or period) subjected to a specific excitation.
Richter Magnitude (M)	A measure related to total energy released during an earthquake.
Root Mean Square (RMS)	The square root of the mean-square value of a random function [Eq. (22.5)].
Sample	A record of random process.
Seismic Zone Factor	A numerical factor in the calculation of the base shear force at a given geographic location.
Seismograph	An instrument for recording, as a function of time, the motions of the Earth's surface that are caused by seismic waves.
Shear Wall	A wall designed to resist lateral forces parallel to the plane of the wall (sometimes referred to as a vertical diaphragm or structural wall).
Shock Spectrum	See Response Spectrum.
Simple Harmonic Motion	The motion of a system which may be expressed by a sine or cosine function of time.
Site-Structure Resonance Coefficient	A numerical factor in the calculation of the base shear force that depends on the condition of the soil.
Space Frame	A three-dimensional structural system, without bearing walls, composed of members interconnected so as to function as a complete self-contained unit with or without the aid of horizontal diaphragms or floor-bracing systems.
Special Moment-Resisting Space Frame (SMRSF)	A moment-resisting space frame specially detailed to provide ductile behavior and comply with the requirements given in Chap. 19 or 22 of UBC-97 (International, Conference of Building Officials 1997).
Spectral Analysis or Spectrum	A description of contributions of the frequency components to the mean-square value of a random function.
Spectral Density Function	A function that describes the intensity of random vibration in terms of the mean-square value per unit of frequency.
Square Root Sum of Squares (SRSS)	A method of combining maximum values of modal contributions by taking the square root of the sum of the squared modal contributions.
Standard Deviation	The square root of the variance. It may be calculated by Eq. (22.6).

Static Condensation	A method of reducing the dimensions of the stiffness and of the mass matrices by establishing the static relation between primary and secondary coordinates.
Stationary Process	A random process for which the average across the ensemble has the same value at any selected time.
Steady-State Vibration	The motion of the system that remains after the transient motion existing at the initiation of the motion has vanished.
Stiffness Coefficient	$k_{ij}$ is the force at coordinate $i$ due to a unit displacement at coordinate $j$ .
Story Drift	The relative lateral displacements of consecutive levels of a building.
Spring Constant	The change in load on a linear elastic structure required to produce a unit increment of deflection.
Strong Motion Accelerograph Structure	An instrument to register seismic motions higher than a specified amplitude. An assemblage of framing members designed to support gravity loads and resist lateral forces. Structures may be categorized as building structures or non-building structures.
Structural Factor	A numerical factor in the calculation of the base shear force that depends on the type of structural system.
Tectonic Earthquakes	Earthquakes resulting from the sudden release of energy stored by a major deformation of the earth.
Time History Response	The response (motion or force) of the structure evaluated as a function of time.
Transient Vibration	The initial portion of the motion which vanishes due to the presence of damping forces in the system.
Transmissibility	The non-dimensional ratio, in the steady-state condition, of the response motion to the input motion. Or the non-dimension the amplitude of the force transmitted to the foundation to the amplitude of the force exciting the system.
Variance of $x(t)$	The average of the squares of the deviations of $x(t)$ values from the mean value $x$ .
Viscous Damping	Dissipation of energy such that the motion is resisted by a force proportional to the velocity but in the opposite direction.
White Noise	A wide-band random process for which the spectral density function is constant over the whole frequency range.
Wide-band Process	A random process whose spectral function has nonzero value over a large range of frequencies.
Wiener-Kinchin Equations	These are equations that relate the autocorrelation function and the spectral density function [Eqs. (22.49) and (22.50)].
Wilson's $\theta$ Method	A modification of the step-by-step linear acceleration method in which the time step is multiplied by a factor necessary to render the method unconditionally stable.
Wood-Anderson Seismograph	An instrument used to register seismic motions.

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