
Structural Dynamics

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Structural Dynamics

Theory and Computation

Sixth Edition

 Springer

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Preface to the Sixth Edition

The basic structure of the five previous editions is still maintained in this Sixth Edition. After the release of the Fifth Edition in 2004, academic and industrial environments have been changed, although the fundamentals have not changed over 15 years. When the author started to teach structural dynamics since 2011, the most challenging part as an instructor has been to present how students can solve and simulate the structural dynamics using the computer program. There is a limited information available to show how we can solve structural dynamics in finite element method-based commercial software. When understanding the background of undergraduate and graduate students who are first exposed to structural dynamics, the fundamentals are mainly considered as core content. The author believes that a line-by-line computer language is a helpful learning and teaching tool for its application of fundamentals. This is the major motivation of the revision of this textbook.

This revised textbook intends to provide enhanced learning materials for students to learn structural dynamics, ranging from basics to advanced topics, including their application. When a line-by-line programming language is included with solved problems, students can learn course materials easily and visualize the solved problems using a program. Among several programming languages, MATLAB[®] has been adopted by many academic institutions across several disciplines. Many educators and students in the USA and many international institutions can readily access MATLAB[®], which has an appropriate programming language to solve and simulate problems in the textbook. It effectively allows matrix manipulations and plotting of data. Therefore, multi-degree-of-freedom problems can be solved in conjunction with the finite element method using MATLAB[®]. As of 2018, SAP2000 presented in the Fifth Edition is currently outdated, at least regarding user interface procedure. The revision author Young Hoon Kim still believes that SAP2000 includes routines for the analysis and design of structures with linear or nonlinear behavior subjected to static or dynamics loads. However, in this edition exclusion of SAP2000 is necessary to minimize the learner's confusion to link between contents and solving with the aid of computer programming language. The author still believes that SAP2000 can be one of the best tools to solve structural analysis and structural dynamics in complex systems. Practical engineers who are eager to use commercial software can learn from many other textbooks available in the market. Generously, authors

offer the alternative option to navigate other textbooks related to finite element methods. In short, the Sixth Edition mainly targets readers such as senior or master students in structural engineering and earthquake engineering in civil engineering.

This revised edition includes 34 solved examples in Chaps. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, and 22 with basics: inputs and outputs. The solved problems enhance learners' understanding, as well as effective teaching resources: line-by-line programming language. Additional figures printed out from MATLAB[®] codes illustrate time-variant structural behavior and dynamic characteristics (e.g., time versus displacement and spectral chart). This textbook updates basics of earthquake design with current design codes (ASCE 7-16 and IBC 2018). Finally, the Sixth Edition uses (1) basic MATLAB[®] codes for structural dynamics: more than 30 examples in most chapters covering basics and advanced topics, (2) contents to educate undergraduate students and Master of Science/Engineering students who are first exposed to structural dynamics. Graduate and undergraduate students can easily use a contemporary computer program (MATLAB) that is widely used in the USA and other countries.

- Printed code language helps students to understand the application of structural dynamics.
- Graduate students are able to apply the fundamentals to real design problems using current version and practices.
- Enhanced illustrations will enhance the readability of expected readers.

I also like to take this opportunity to thank my colleagues in my home department at Speed School of Engineering at the University of Louisville, KY, especially, Dr. J.P. Mohsen, who continuously encouraged me to revise this book in a timely manner. He recommended the Fifth Edition which also originated from the Third Edition, which I first read in my undergraduate structural dynamics study in South Korea. I also wish to recognize and thank my current PhD student, Jice Zeng. He did diligent work redrawing most figures from the previous edition of this textbook. Especially, I want to thank Dr. Yeeseok Kim at California Baptist University. He introduced the application of programming language in my first course of structural dynamics at the University of Louisville which enabled me to proficiently apply structural dynamics using MATLAB[®]. Finally, I was able to transform my course materials into part of the revision of this version. In addition, I wish to thank Paul Drougas and Flanagan Caroline who patiently waited for my final manuscript.

I am very grateful to serve as the coauthor for original author Mario Paz for enabling my contribution in reviewing and editing this volume, especially those sections which used the computer programs. Finally, I thank my wife, Hye-Jin Baek. Without her support, this revision would be incomplete. Also, my two sons, Edward and William, always provide me all the energy.

For from him and through him and to him are all things. To him be glory forever. Amen. (Romans 11:36)

Louisville, KY, USA
May, 2018

Young Hoon Kim

Preface to the First Edition

Natural phenomena and human activities impose forces of time-dependent variability on structures as simple as a concrete beam or a steel pile, or as complex as a multistory building or a nuclear power plant constructed from different materials. Analysis and design of such structures subjected to dynamic loads involve consideration of time-dependent inertial forces. The resistance to displacement exhibited by a structure may include forces which are functions of the displacement and the velocity. As a consequence; the governing equations of motion of the dynamic system are generally nonlinear partial differential equations which are extremely difficult to solve in mathematical terms. Nevertheless, recent developments in the field of structural dynamics enable such analysis and design to be accomplished in a practical and efficient manner. This work is facilitated through the use of simplifying assumptions and mathematical models, and of matrix methods and modern computational techniques.

In the process of teaching courses on the subject of structural dynamics, the author came to the realization that there was a definite need for a text which would be suitable for the advanced undergraduate or the beginning graduate engineering student being introduced to this subject. The author is familiar with the existence of several excellent texts of an advanced nature but generally these texts are, in his view, beyond the expected comprehension of the student. Consequently, it was his principal aim in writing this book to incorporate modern methods of analysis and techniques adaptable to computer programming in a manner as clear and easy as the subject permits. He felt that computer programs should be included in the book in order to assist the student in the application of modern methods associated with computer usage. In addition, the author hopes that this text will serve the practicing engineer for purposes of self-study and as a reference source.

In writing this text, the author also had in mind the use of the book as a possible source for research topics in structural dynamics for students working toward an advanced degree in engineering who are required to write a thesis. At Speed Scientific School, University of Louisville, most engineering students complete a fifth year of study with a thesis requirement leading to a Master in Engineering degree. The author's experience as a thesis advisor leads him to believe that this book may well serve the students in

their search and selection of topics in subjects currently under investigation in structural dynamics.

Should the text fulfill the expectations of the author in some measure, particularly the elucidation of this subject, he will then feel rewarded for his efforts in the preparation and development of the material in this book.

Louisville, KY, USA
December, 1979

Mario Paz

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