Chapter 16

Mining Spatial Data

"*Time and space are modes by which we think and not conditions in which we live.*"—Albert Einstein

16.1 Introduction

Spatial data arises commonly in geographical data mining applications. Numerous applications related to meteorological data, earth science, image analysis, and vehicle data are spatial in nature. In many cases, spatial data is integrated with temporal components. Such data is referred to as *spatiotemporal data*. Some examples of applications in which spatial data arise, are as follows:

1. *Meteorological data:* Quantifications of important weather characteristics, such as the temperature and pressure, are typically measured at different geographical locations. These can be analyzed to discover interesting events in the underlying data.

2. *Mobile objects:* Moving objects typically create trajectories. Such trajectories can be analyzed for a wide variety of insights, such as characteristic trends, or anomalous paths of objects.

3. *Earth science data:* The land cover types at different spatial locations may be represented as behavioral attributes. Anomalies in such patterns provide insights about anomalous trends in human activity, such as deforestation or other anomalous vegetation trends.

4. *Disease outbreak data:* Data about disease outbreaks are often aggregated by spatial locations such as ZIP code and county. The analysis of trends in such data can provide information about the causality of the outbreaks.

5. *Medical diagnostics:* Magnetic resonance imaging (MRI) and positron emission tomography (PET) scans are spatial data in 2 or 3 dimensions. The detection of unusual

C. C. Aggarwal, *Data Mining: The Textbook*, DOI 10.1007/978-3-319-14142-8_16

localized regions in such data can help in detecting diseases such as brain tumors, the onset of Alzheimer disease, and multiple sclerosis lesions. In general, any form of image data may be considered spatial data. The analysis of shapes in such data is of considerable importance in a variety of applications.

6. *Demographic data:* Demographic (behavioral) attributes such as age, sex, race, and salary can be combined with spatial (contextual) attributes to provide insights about the demographic patterns in distributions. Such information can be useful for target-marketing applications.

Most forms of spatial data may be classified as a contextual data type, in which the attributes are partitioned into contextual attributes and behavioral attributes. This partitioning is similar to that in time series and discrete sequence data:

- *Contextual attribute(s):* These represent the attributes that provide the *context* in which the measurements are made. In other words, the contextual attributes provide the reference points at which the behavioral values are measured. In most cases, the contextual attributes contain the spatial coordinates of a data point. In some cases, the contextual attribute might be a logical location, such as a building or a state. In the case of spatiotemporal data, the contextual attributes may include time. For example, in an application in which the sea surface temperatures are measured at sensors at specific locations, the contextual attributes may include both the position of the sensor, and the time at which the measurement is made.

- *Behavioral attribute(s):* These represent the behavioral values at the reference points. For example, in the aforementioned sea surface temperature application, these correspond to the temperature attribute values.

In most forms of spatial data, the spatial attributes are contextual, and may optionally include temporal attributes. An exception is the case of trajectory data, in which the spatial attributes are behavioral, and time is the only contextual attribute. In fact, trajectory data can be considered equivalent to multivariate time series data. This equivalence is discussed in greater detail in Sect. 16.3.

This chapter separately studies cases where the spatial attributes are contextual, and those in which the spatial attributes are behavioral. The latter case typically corresponds to trajectory data, in which the contextual attribute corresponds to time. Thus, trajectory data is a form of spatiotemporal data. In other forms of spatiotemporal data, both spatial and temporal attributes are contextual.

This chapter is organized as follows. Section 16.2 addresses data mining scenarios in which the spatial attributes are contextual. In this context, the chapter studies several important problems, such as pattern mining, clustering, outlier detection, and classification. Section 16.3 discusses algorithms for mining trajectory data. Section 16.4 discusses the summary.

16.2 Mining with Contextual Spatial Attributes

In many forms of meteorological data, a variety of (behavioral) attributes, such as temperature, pressure, and humidity, are measured at different spatial locations. In these cases, the spatial attributes are contextual. An example of sea surface temperature contour charts is illustrated in Fig. 16.1. The different shades in the chart represent the different sea surface temperatures. These correspond to the values of the behavioral attributes at different spatial locations.

Another example is the case of image data, where the intensity of an image is measured in pixels. Such data is often used to capture diagnostic images. Examples of PET scans for a cognitively healthy person and an Alzheimer's patient are illustrated in Fig. 16.2. In this case, the values of the pixels represent the behavioral attributes, and the spatial locations of these pixels represent the contextual attributes. The behavioral attributes in spatial data may present themselves in a variety of ways, depending on the application domain:

1. For some types of spatial data, such as images, the analysis may be performed on the contour of a specific shape extracted from the data. For example, in Fig. 16.3, the contour of the insect may be extracted and analyzed with respect to other images in the data.

2. For other types of spatial data, such as meteorological applications, the behavioral attributes may be abstract quantities such as temperature. Therefore the analysis can be performed in terms of the trends on these abstract quantities. In such cases, the spatial data needs to be treated as a contextual data type with multiple reference points corresponding to spatial coordinates. Such an analysis is generally more complex.

The specific choice of data mining methodology often depends on the application at hand. Both these forms of data are often transformed into other data types such as time series or multidimensional data before analysis.

16.2.1 Shape to Time Series Transformation

In many spatial data sets such as images, the data may be dominated by a particular shape. The analysis of such shapes is challenging because of the variations in sizes and orientations. One common technique for analyzing spatial data is to transform it into a different format that is much easier to analyze. In particular, the contours of a shape are often transformed to time series for further analysis. For example, the contours of the insect shapes in Fig. 16.3 are difficult to analyze directly because of their complexity. However, it is possible to create a representation that is friendly to data processing by transforming them into time series.

A common approach is to use the distance from the centroid to the boundary of the object, and compute a sequence of real numbers derived in a clockwise sweep of the boundary. This yields a time series of real numbers, and is referred to as the *centroid distance signature*. This transformation can be used to map the problem of mining shapes to that of mining time series. The latter domain is much easier to analyze. For example, consider the elliptical shape illustrated in Fig. 16.4a. Then, the time series representing the distance from the centroid, using 360 different equally spaced angular samples, is illustrated in Fig. 16.4b. Note that the contextual attribute here is the number of degrees, but one can "pretend" that this represents a timestamp. This facilitates the use of all the powerful data mining techniques available for time series analysis. In this case, the sample points are started at one of the major axes of the ellipse. If the sample point starts at a different position, or if the shape is rotated (with the same angular starting point), then this causes a cyclic translation of the time series. This is quite important because the precise orientation of a shape may not be known in advance. For example, the shapes in Figs. 16.3b and c are rotated from the

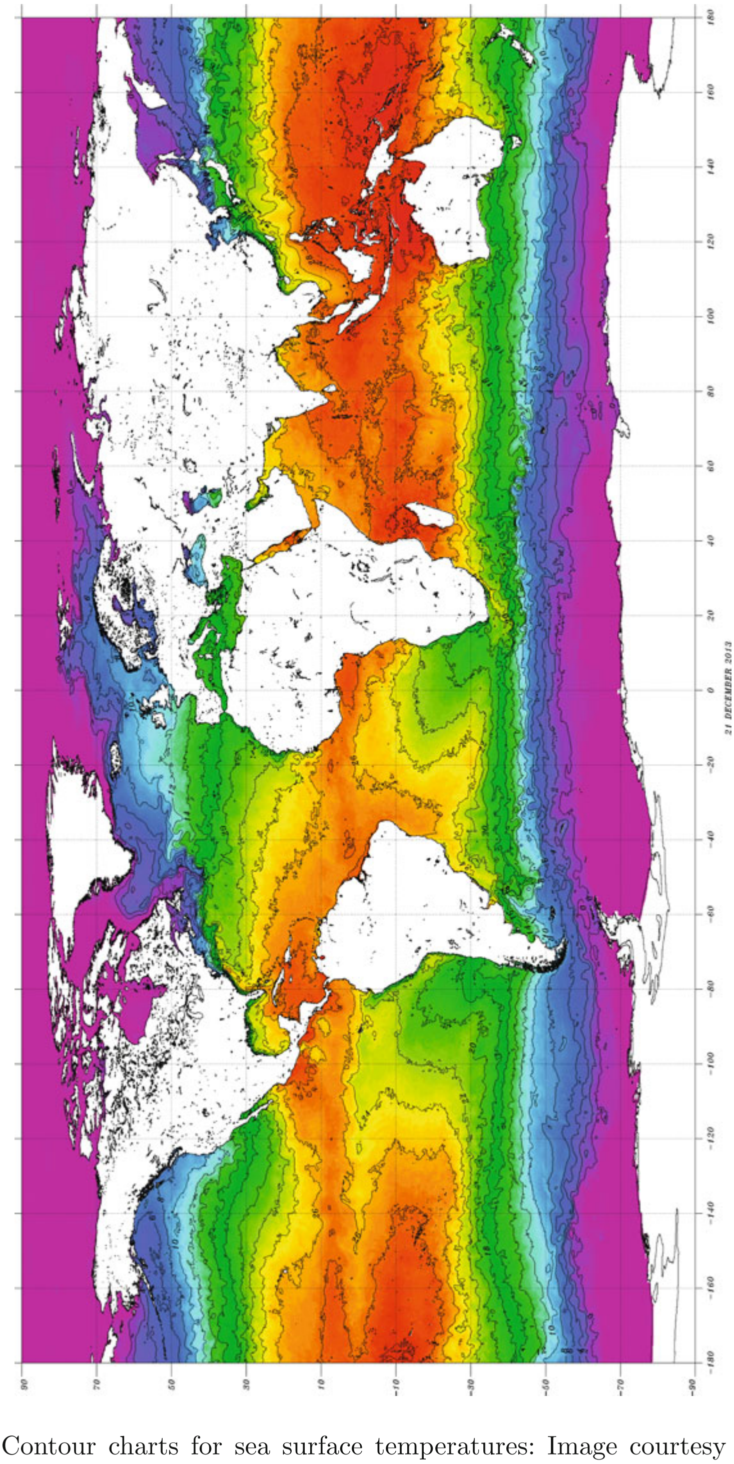


Figure 16.1: Contour charts for sea surface temperatures: Image courtesy of the NOAA Satellite and Information Service

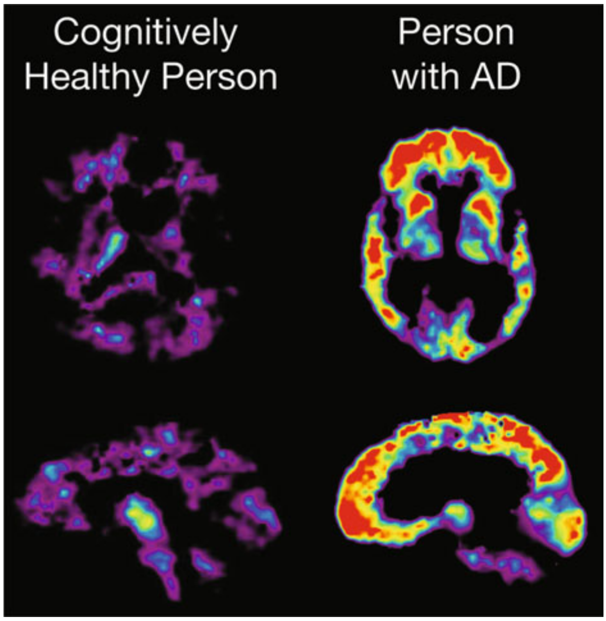


Figure 16.2: PET Scans of the brain of a cognitively healthy person versus an Alzheimer's patient. (Image courtesy of the National Institute on Aging/National Institutes of Health)

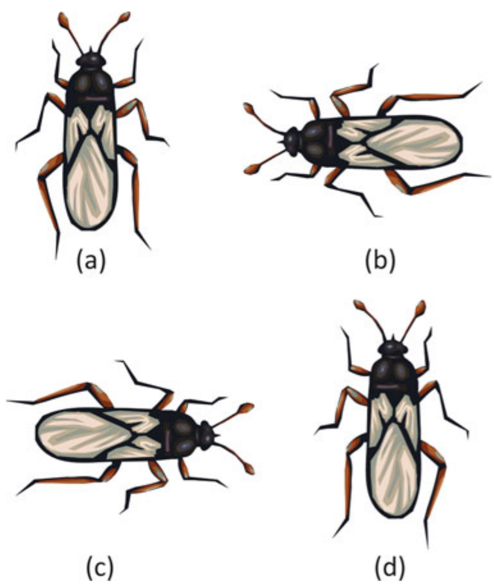
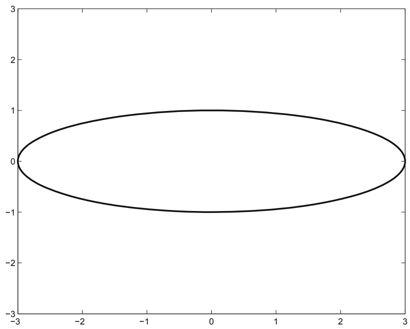
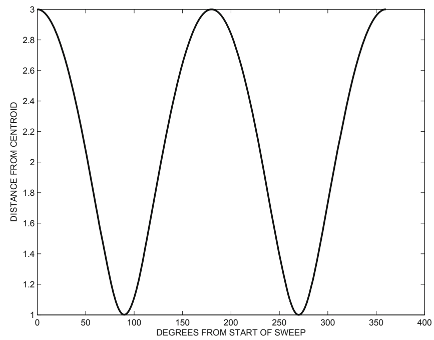


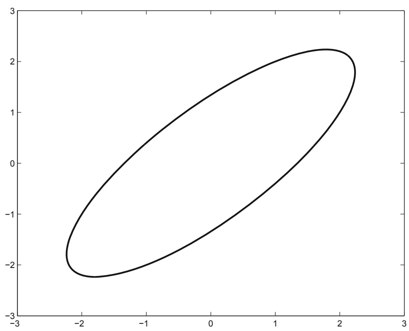
Figure 16.3: Rotation and mirror image effects on shape matching



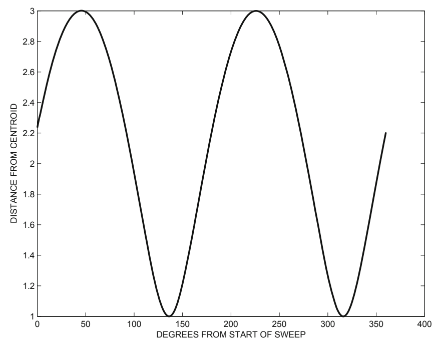
(a) elliptical shape



(b) distance from centroid for (a)



(c) elliptical shape



(d) distance from centroid for (c)

Figure 16.4: Conversion from shapes to time series

shape of Fig. 16.3a. The shape in Fig. 16.3d is a mirror image of the shape of Fig. 16.3a. While rotations result in cyclic translations, mirror images result in a reversal of the series.

Figure 16.4c represents a rotation of the shape of Fig. 16.4a by 45°. Correspondingly, the time series representation in Fig. 16.4d is a (cyclic) translation of time series representation in Fig. 16.4b. Similarly, the mirror image of a shape corresponds to a reversal of the time series. It will be evident later that the impact of rotation or mirror images needs to be explicitly incorporated into the distance or similarity function for the application at hand. After the time series has been extracted, it may be normalized in different ways, depending on the needs of the application:

- If no normalization is performed, then the data mining approach is sensitive to the absolute sizes of the underlying objects. This may be the case in many medical images such as MRI scans, in which all spatial objects are drawn to the same scale.

- If all time series values are multiplicatively scaled down by the same factor to unit mean, such an approach will allow the matching of shapes of different sizes, but discriminate between different levels of relative variations in the shapes. For example, two ellipses with very different ratios of the major and minor axes will be discriminated well.

- If all time series are standardized to zero mean and unit variance, then such an approach will match shapes where *relative* local variations in the shape are similar, but the overall shape may be quite different. For example, such an approach will not discriminate very well between two ellipses with very different ratios of the major and minor axes, but will discriminate between two such shapes with different relative local deviations in the boundaries. The only exception is a circular shape that appears as a straight line. Furthermore, noise effects in the contour will be differentially enhanced in shapes that are less elongated. For example, for two ellipses with similar noisy deviations at the boundaries, but different levels of elongation (major to minor axis ratio), the overall shape of the time series will be similar, but the local noisy deviations in the extracted time series will be *differentially* suppressed in the elongated shape. This can sometimes provide a distorted picture from the perspective of shape analysis. A perfectly circular shape may show unstable and large noisy deviations in the extracted time series because of trivial variations such as image rasterization effects. Thus, the usual mean and variance normalization of time series analysis often leads to unintended results.

In general, it is advisable to select the normalization method in an application-specific way. After the shapes have been converted to time series, they can be used in the context of a wide variety of applications. For example, motifs in the time series correspond to frequent contours in the spatial shapes. Similarly, clusters of similar shapes may be discovered by determining clusters in the time series. Similar observations apply to the problems of outlier detection and classification.

16.2.2 Spatial to Multidimensional Transformation with Wavelets

For data types such as meteorological data in which behavioral attribute values vary across the entire spatial domain, a contour-based shape may not be available for analysis. Therefore, the shape to time series transformation is not appropriate in these cases.

Wavelets are a popular method for the transformation of time series data to multidimensional data. Spatial data shares a number of similarities with time series data. Time series data has a single contextual attribute (time) along which a behaviorial attribute (e.g., temperature) may exhibit a smooth variation. Correspondingly, spatial data has two contextual attributes (spatial coordinates), along which a behavioral attribute (e.g., sea surface temperature) may exhibit a smooth variation. Because of this analogy, it is possible to generalize the wavelet-based approach to the case of multiple contextual attributes with appropriate modifications.

Assume that the spatial data is represented in the form of a 2-dimensional grid of size $q \times q$. Thus, each coordinate of the grid contains an instance of the behavioral attribute, such as the temperature. As discussed for the time series case in Sect. 2.4.4.1 of Chap. 2, differencing operations are applied over contiguous segments of the time series by successive division of the time series in hierarchical fashion. The corresponding basis vectors have $+1$ and -1 at the relevant positions. The 2-dimensional case is completely analogos, where contiguous *areas* of the spatial grid are used for successive divisions. These divisions are alternately performed along the different axes. The corresponding basis vectors are 2-dimensional *matrices* of size $q \times q$ that regulate how the differencing operations are performed. An example of how sea surface temperatures in a spatial data set may be converted to a multidimensional representation is provided in Fig. 16.5. This will result in a total of q^2 wavelet coefficients, though only the large coefficients need to be retained for analysis. A more detailed description of the generation of the spatial wavelet coefficients may be found in Sect. 2.4.4.1 of Chap. 2. The aforementioned description is for the case of a single behavioral attribute and multiple contextual attributes (spatial coordinates). Multiple behavioral attributes can also

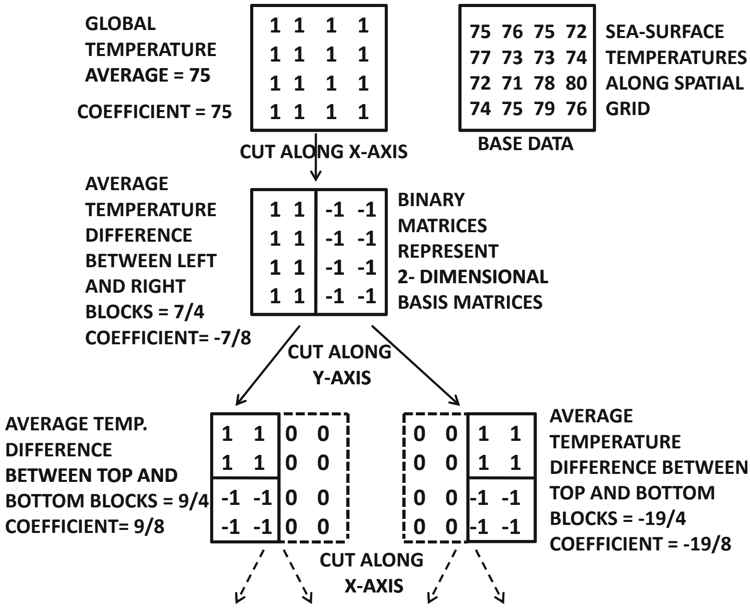


Figure 16.5: Illustration of the top levels of the wavelet decomposition for spatial data in a grid containing sea surface temperatures (Fig. 2.7 of Chap. 2 revisited)

be addressed by performing the decomposition separately for each behavioral attribute, and creating a separate set of dimensions for each behavioral attribute.

Like the time series wavelet, the spatial wavelet is a multiresolution representation. Trends at different levels of spatial granularity are represented in the coefficients. Higher-level coefficients represent trends in larger spatial areas, whereas lower-level coefficients represent trends in smaller spatial areas. Therefore, this approach is very powerful, and has broad usability for many spatial applications. Spatial wavelets can be used effectively for many image clustering and classification applications where (contextual) spatial data can be converted to (noncontextual) multidimensional data. Once the transformation has been performed, virtually all the multidimensional methods discussed in Chaps. 4 to 11 can be used on this representation. Such an approach opens the door to the use of a wide array of multidimensional data mining methods.

16.2.3 Spatial Colocation Patterns

In this problem, the contextual attributes are spatial and the behavioral attributes are typically boolean and nonspatial. Non-boolean behavioral attributes can be addressed with the use of type conversion via discretization or binarization. The goal of spatial colocation pattern mining is to discover combinations of features occurring at the same spatial location. Consider an ecology data set, where one has behavioral attributes such as fire ignition source, needle vegetation type, and a drought indicator. The spatial colocation of these features may often be a risk factor for forest fires. Therefore, the discovery of such patterns is useful in the context of data mining analysis. In many cases, a spatial event indicator of interest (e.g., disease outbreak, vegetation event, or climate event) is added to the other behavioral attributes. The discovery of useful patterns that include this indicator of interest can be used for discovering event causality. This problem is also closely related to rule-based spatial classification, where the likelihood of the event occurring in previously unseen test regions can be estimated from the resulting patterns.

One challenge in the mining process is that the different behavioral attributes may be derived from different data sources, and therefore may not have precisely the same value of the contextual (spatial) attribute in their measurements. Therefore, proper data preprocessing is crucial. The data can be homogenized by partitioning the spatial region into smaller regions. For each of these regions, each behavioral attribute's value is derived heuristically from the values in the original data source. For example, if the boolean attribute has a value of 1 more than predefined fraction of the time in a spatial region, then its value is set to 1. The contextual (spatial) attribute can be set to the centroid of that region. The mining can be performed on this preprocessed data. The overall approach is as follows:

1. Preprocess the data to create the behavioral attribute values at the same set of spatial locations.

2. For each spatial location, create a transaction containing the corresponding combination of boolean values.

3. Use any frequent pattern mining algorithm to discover the relevant patterns in these transactions.

4. For each discovered pattern, map it back to the spatial regions containing the pattern. Cluster the relevant spatial regions for each pattern, if necessary for summarization.

In cases where a particular behavioral attribute is an event of interest (e.g., disease outbreak), the transactions containing values of 0 and 1, respectively, for this attribute can be separately processed to discover two sets of patterns on the other behavioral attributes. The differences between these two sets of patterns can provide insights into discriminative factors for the event of interest at each spatial location. Such patterns are also useful for spatial classification of previously unseen test regions. This approach is identical to that of associative classifiers in Chap. 10.

This model can also address time-changing data in a seamless way. In such cases, the time becomes another contextual attribute in addition to the spatial attributes. Patterns can be discovered at different temporal snapshots using the aforementioned methodology. The key changes in these patterns over time can provide insights into the nature of the spatial evolution.

16.2.4 Clustering Shapes

In many applications, it may be desirable to cluster similar shapes prior to analysis. It is assumed that a database of N shapes is available and that a total of k groups of similar shapes need to be created. This can be a useful preprocessing task in many shape categorization applications. The conversion of a shape to a time series (Sect. 16.2.1) is the appropriate approach in this scenario. Many of the time series clustering algorithms discussed in Sect. 14.5 of Chap. 14 may be used effectively, once the shape has been converted to a time series. The k-medoids, hierarchical, and graph-based methods are particularly suitable because they require only the design of an appropriate similarity function for the corresponding time series. This is an issue that will be discussed in more detail later. The main steps of shape-based clustering are as follows:

1. Use the centroid-based sweep method discussed in Sect. 16.2.1 to convert each shape into a time series. This results in a database of N different time series.

2. Use any time series clustering algorithm, such as hierarchical, k-medoids or graph-based method on time series data as discussed in Sect. 14.5 of Chap. 14. This will cluster the N time series into k groups.

3. Map the k groups of time series clusters to k groups of shape clusters, by mapping each time series into its relevant shape.

The aforementioned clustering algorithm depends only on the choice of the distance function. Any of the time series measures discussed in Sect. 3.4.1 of Chap. 3 may be used, depending on the desired degree of error tolerance or distortion (warping) allowed in the matching. Another important issue is the adjustment of the distance function with the varying rotations of the different shapes. In the following, the Euclidean distance will be used as an example, although the general principle can be applied to any distance function.

It is evident from the example of Fig. 16.4 that *a rotation of the shape leads to a linear cyclic shifting of the time series generated by using the distances of the centroid of the shape to the contours of the shape.* For a time series of length n denoted by $a_1 a_2 \ldots a_n$, a cyclic translation by i units leads to the time series $a_{i+1} a_{i+2} \ldots a_n a_1 a_2 \ldots a_i$. Then, the *rotation invariant Euclidean distance* $RIDist(T_1, T_2)$ between two time series $T_1 = a_1 \ldots a_n$ and $T_2 = b_1 \ldots b_n$ is given by the minimum distance between T_1 and all possible rotational translations of T_2 (or vice versa). Therefore, the rotation-invariant distance is expressed as follows:

$$RIDist(T_1, T_2) = \min_{i=1}^{n} \sum_{j=1}^{n} (a_j - b_{1+(j+i) \bmod n})^2.$$

In general, if a cyclic shift of the time series T_2 by i units is denoted by T_2^i, then the rotation invariant distance, using any distance function $Dist(T_1, T_2)$ may be expressed as follows:

$$RIDist(T_1, T_2) = \min_{i=1}^{n} Dist(T_1, T_2^i). \tag{16.1}$$

Note that the reversal of a time series corresponds to the mirror image of the underlying shape. Therefore, mirror images can also be addressed using this approach, by incorporating the reversals of the series (and its rotations) in the distance function. This will increase the computation by a factor of 2. The precise choice of distance function used is highly application-specific, depending on whether rotations or mirror image conversions are required.

16.2.5 Outlier Detection

In the context of spatial data, outliers can be either point outliers and shape outliers. These two kinds of outliers are also encountered in time series data, and in discrete sequences. In the case of spatial data, these two kinds of outliers are defined as follows:

1. *Point outliers:* These outliers are defined on a single spatial object with a variety of spatial and behavioral attributes. For example, a weather map is a spatial object that contains both spatial locations, and environmental measurements (behavioral values) at these locations. Abrupt changes in the behavioral attributes that violate spatial continuity provide useful information about the underlying contextual anomalies. For example, consider a meteorological application in which sea surface temperatures and

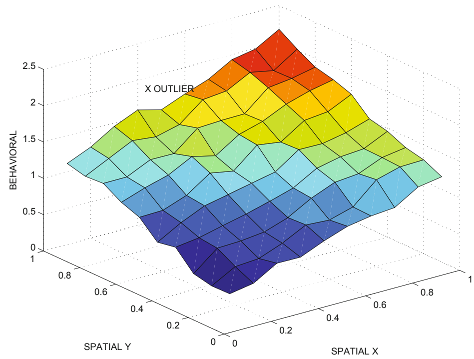


Figure 16.6: Example of point outlier for spatial data

pressure are measured. Unusually high sea surface temperature in a very small localized region is a hot-spot that may be the result of volcanic activity under the surface. Similarly, unusually low or high pressure in a small localized region may suggest the formation of hurricanes or cyclones. In all these cases, spatial continuity is violated by the attribute of interest. Such attributes are often tracked in meteorological applications on a daily basis. An example of a point outlier for spatial data is illustrated in Fig. 16.6.

2. *Shape outliers:* The application settings for these kinds of outliers are quite different. These outliers are defined in a database of multiple shapes. For example, the shapes may be extracted from the different images. In such cases, the unusual shapes in the different objects need to be reported as outliers.

This chapter studies both the aforementioned formulations.

16.2.5.1 Point Outliers

Neighborhood-based algorithms are generally used for discovering point outliers. In these algorithms, abrupt changes in the spatial neighborhood of a data point are used to diagnose outliers. Therefore, the first step is to define the concept of a spatial neighborhood. The behavioral values within the spatial neighborhood of a given data point are combined to create an *expected value* of the behavioral attribute. This expected value is then used to compute the deviation of the data point from the expected value. This provides an outlier score. This definition of point outliers in spatial data is similar to that in time series data.

Intuitively, it is unusual for the behavioral attribute value to vary abruptly within a small spatial locality. For example, a sudden variation of the temperature within a small spatial locality will be detected by this method. The neighborhood may be defined in many different ways:

- *Multidimensional neighborhoods:* In this case, the neighborhoods are defined with the use of multidimensional distances between data points. This approach is appropriate when the contextual attributes are defined as coordinates.

- *Graph-based neighborhoods:* In this case, the neighborhoods are defined by linkage relationships between spatial objects. Such neighborhoods may be more useful in cases where the location of the spatial objects may not correspond to exact coordinates (e.g., county or ZIP code). In such cases, graph-based representations provide a more general modeling tool.

Both multidimensional and graph-based methods will be discussed in the following sections.

Multidimensional Methods

While traditional multidimensional methods can also be used to detect outliers in spatial data, such methods do not distinguish between the contextual and the behavioral attributes. Therefore, such methods are not optimized for outlier detection in spatial data. This is because the (contextual) spatial attributes should be treated differently from the behavioral attributes. The basic idea is to adapt the k-nearest neighbor outlier detection methods to the case of spatial data.

The spatial neighborhood of the data is defined with the use of multidimensional distances on the spatial (contextual) attributes. Thus, the contextual attributes are used for determining the k nearest neighbors. The average of the behavioral attribute values provides an expected value for the behavioral attribute. The difference between the expected and true value is used to predict outliers. A variety of distance functions can be used on the multidimensional spatial data for the determination of proximity. The choice of the distance function is important because it defines the choice of the neighborhood that is used for computing the deviations in behavioral attributes. For a given spatial object o, with behavioral attribute value $f(o)$, let $o_1 \ldots o_k$ be its k-nearest neighbors. Then, a variety of methods may be used to compute the predicted value $g(o)$ of the object o. The most straightforward method is the mean:

$$g(o) = \sum_{i=1}^{k} f(o_i)/k$$

Alternatively, $g(o)$ may be computed as the median of the surrounding values of $f(o_i)$, to reduce the impact of extreme values. Then, for each data object o, the value of $f(o) - g(o)$ represents a deviation from predicted values. The extreme values among these deviations may be computed using a variety of methods for univariate extreme value analysis. These are discussed in Chap. 8. The resulting extreme values are reported as outliers.

Graph-Based Methods

In graph-based methods, spatial proximity is modeled with the use of links between nodes in a graph representation of the spatial region. Thus, nodes are associated with behavioral attributes, and strong variations in the behavioral attribute across neighboring nodes are recognized as outliers. Graph-based methods are particularly useful when the individual nodes are not associated with point-specific coordinates, but they may correspond to regions of arbitrary shape. In such cases, the links between nodes correspond to neighborhood relationships between the different regions. Graph-based methods define spatial relationships in a more general way because semantic relationships can also be used to define neighborhoods. For example, two objects could be connected by an edge if they are in the same *semantic* location, such as a building, restaurant, or an office. In many applications, the links may be weighted on the basis of the strength of the proximity relationship. For example, consider a disease outbreak application in which the spatial objects correspond to county regions. In such a case, the strength of the links could correspond to the length of the boundary between two regions. Multidimensional data is a special case, where links correspond to distance-based proximity. Thus, graph representations allow more generic interpretations of the contextual attribute.

Let S be the set of neighbors of a given node i. Then, the concept of spatial continuity can be used to create a *predicted* value of the behavioral attribute based on those of its (spatial) neighbors. The strength of the links between i and its neighbors can also be used to compute the predicted values as either the weighted mean or median on the behavioral attribute of the k nearest spatial neighbors. For a given spatial object o with the behavioral attribute value $f(o)$, let $o_1 \ldots o_k$ be its k linked neighbors based on the relationship graph. Let the weight of the link (o, o_i) be $w(o, o_i)$. Then, the linkage-based weighted mean may be used to compute the predicted value $g(o)$ of the object o.

$$g(o) = \frac{\sum_{i=1}^{k} w(o, o_i) \cdot f(o_i)}{\sum_{i=1}^{k} w(o, o_i)}$$

Alternatively, the weighted median of the neighbor values may be used for predictive purposes. Since the true value of the behavioral attribute is known, this can be used to model the deviations of the behavioral attributes from their predicted values. As in the case of multidimensional methods, the value of $f(o) - g(o)$ represents a deviation from the predicted values. Extreme value analysis can be used on these deviations to determine the spatial outliers. This process is identical to that in the multidimensional case. The nodes with high values of the normalized deviation may be reported as outliers.

16.2.5.2 Shape Outliers

Shape-based outliers are relatively easy to determine in spatial data, with the use of the transformation from spatial data to time series described in Sect. 16.2.1. After the transformation has been performed, a k-nearest neighbor outlier detector can be applied to the resulting time series. The distance to the kth-nearest neighbor can be reported as the outlier score. A few key issues need to be kept in mind, while computing the outlier score.

1. The distance function needs to be modified to account for the rotation invariance of shape matching. This is achieved by comparing all cyclic shifts of one time series to the other. The rotation invariant distance can be captured by Eq. 16.1.

2. In some applications, mirror image invariance also needs to be accounted for. In such cases, all cyclic shifts and their reversals need to be included in the aforementioned comparison. The outliers are determined with respect to this enhanced database.

While a vanilla k-nearest neighbor detector can determine the outliers correctly, the approach can be made faster by pruning. The basic idea is similar to the *Hotsax* method discussed in Chap. 14, where a nested loop structure is used to maintain the top-n outliers. The outer loop corresponds to the selection of different candidates, and the inner loop corresponds to the computation of the k-nearest neighbors of each of these candidates. The inner loop can be terminated early, when the k-nearest neighbor value is less than the nth best outlier found so far. For optimal performance, the candidates in the outer loop and the computations in the inner loop need to be ordered appropriately.

This ordering is performed as follows. A combination of the SAX representation and LSH-hashing is used to create clusters on the candidates. Candidates which map to clusters with few members are examined first in the outer loop to discover high quality outliers early in the algorithm execution. Objects which appear in the same cluster as the outer loop candidate are examined first in the inner loop to ensure fast termination of the inner loop. This facilitates better pruning performance. The bibliographic notes contain pointers to specific details of the creation of SAX-based clusters in shape outlier detection.

16.2.6 Classification of Shapes

It is assumed that a set of N labeled shapes are used to conduct the training. This trained model is used to perform classification of test instances, for which the label is unknown. The transformation from spatial into time series data is a useful tool for distance-based classification algorithms. As in the case of clustering and outlier detection, the first step of the process is to transform the shapes into time series. This transforms the problem to the time series classification problem. A number of methods for the classification of time series are discussed in Sect. 14.7 of Chap. 14. The main difference is that the rotation invariance of the shapes needs to be accounted for. Any of the distance-based methods proposed in Sect. 14.7 of Chap. 14 for time series classification may be used after the shapes have been transformed into time series. This is because distance-based methods can be easily made rotation-invariant by using Eq. 16.1. The two main distance-based methods for time series classification include the nearest neighbor method and the graph-based collective classification approach. While the nearest-neighbor method is straightforward, the graph-based method is discussed in some detail below.

The graph-based method is transductive because it requires the test instances to be available at the time of training. When a larger number of test instances are available along with the training data, the latter method may be used. Therefore, the different methods may be more suitable in different scenarios. The overall approach for graph-based classification may be described as follows:

1. Transform both the training and test shapes into time series, by using the centroid sweep method described in Sect. 16.2.1.

2. Use any of the distance functions described in Sect. 3.4.1 of Chap. 3 to construct a neighborhood graph on the shapes. If needed, use a rotation-invariant version of the distance function, as discussed in Eq. 16.1. Each shape represents a node, which is connected to its k-nearest neighbors with edges. The labeled shapes correspond to labeled nodes. The collective classification method described in Sect. 19.4 of Chap. 19 is used to assign labels to the unlabeled nodes (i.e., the test shapes).

In some cases, rotation invariance may not be an application-specific need. In such cases, the efficiency of distance computation is improved.

16.3 Trajectory Mining

Trajectory data arises in a wide variety of spatial applications. The proliferation of GPS-enabled devices, such as mobile phones, has enabled the large-scale collection of trajectory data. Trajectory data can be analyzed for a very wide variety of insights, such as determining co-location patterns, clusters and outliers. Trajectory data is different from the other kinds of spatial data discussed in this chapter in the following respects:

1. In the spatial data applications addressed so far in this chapter, spatial attributes are contextual, whereas other types of attributes (e.g., temperature in a meteorological application) are behavioral. In the case of trajectory data, spatial attributes are behavioral.

2. The *only* contextual attribute in trajectory data is time. Therefore, trajectory data can be considered *spatiotemporal data*. While the scenarios discussed in previous sections may also be generalized further by *including* time among the contextual attributes, the spatial attributes are not behavioral in those cases. For example, when sea surface temperatures are tracked over time, both spatial and temporal attributes are contextual.

Trajectory analysis is typically performed in one of two different ways:

1. *Online analysis:* In online analysis, the trajectories are analyzed in *real time*, and the patterns in the trajectories at a given time are most relevant to the analysis.

2. *Shape-based analysis:* In shape-based analysis, the time variable has already been removed from the analysis. For example, two similar trajectories, formed at different periods, can be meaningfully compared to one another. For example, a cluster of trajectories is based on their shape, rather than the simultaneity in their movement.

The two kinds of analysis in trajectory data are similar to time series data. This is not particularly surprising because trajectory data is a form of time series data.

16.3.1 Equivalence of Trajectories and Multivariate Time Series

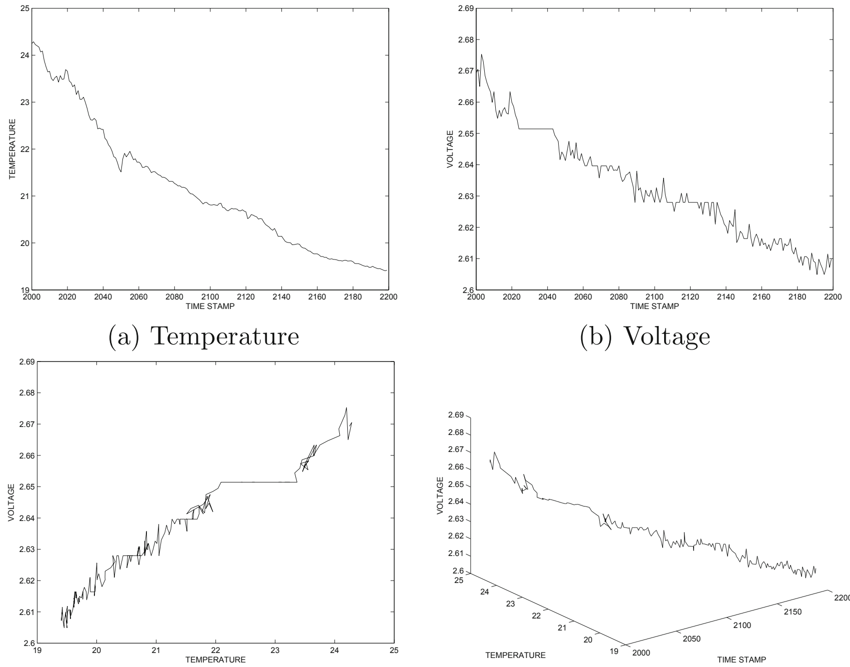
Trajectory data is a form of multivariate time series data. For a trajectory in two dimensions, the X-coordinate and Y-coordinate of the trajectory form two components of the multivariate series. A 3-dimensional trajectory will result in a trivariate series.

Because of the equivalence between multivariate time series and trajectory data, the transformation can be performed in either direction to facilitate the use of the methods designed for each domain. For example, trajectory mining methods can be utilized for applications that are nonspatial. In particular, any n-dimensional multivariate time series can be converted into trajectory data. In multivariate temporal data, the different behavioral attributes are typically measured with the use of multiple sensors simultaneously. Consider the example of the *Intel Research Berkeley Sensor data* [556] that measures different behavioral attributes, such as temperature, pressure, and voltage, in the Intel Berkeley laboratory over time. For example, the behavior of the temperature and voltage sensors in the same segment of time are illustrated in Figs. 16.7a, b, respectively.

It is possible to visualize the variation of the two behaviorial attributes by eliminating the common time attribute, or by creating a 3-dimensional trajectory containing the time and the other two behaviorial attributes. Examples of such trajectories are illustrated in Fig. 16.7c, d, respectively. The most generic of these trajectories is illustrated in Fig. 16.7d. This figure shows the simultaneous variation of all three attributes. In general, a multivariate time series with n behavioral attributes can be mapped to an $(n + 1)$-dimensional trajectory. Most of the trajectory analysis methods are designed under the assumption of 2 or 3 dimensions, though they can be generalized to n dimensions where needed.

16.3.2 Converting Trajectories to Multidimensional Data

Because of the equivalence between trajectories and multivariate time series, trajectories can also be converted to multidimensional data. This is achieved by using the wavelet transformation on the time series representation of the trajectory. The wavelet transformation for time series is described in detail in Sect. 2.4.4.1 of Chap. 2. In this case, the time series is multivariate, and therefore has two behavioral attributes. The wavelet representation for



(a) Temperature (b) Voltage

(c) Temperature-Voltage Trajectory (d) Time-Temperature-Voltage Trajectory

Figure 16.7: Multivariate time series can be mapped to trajectory data

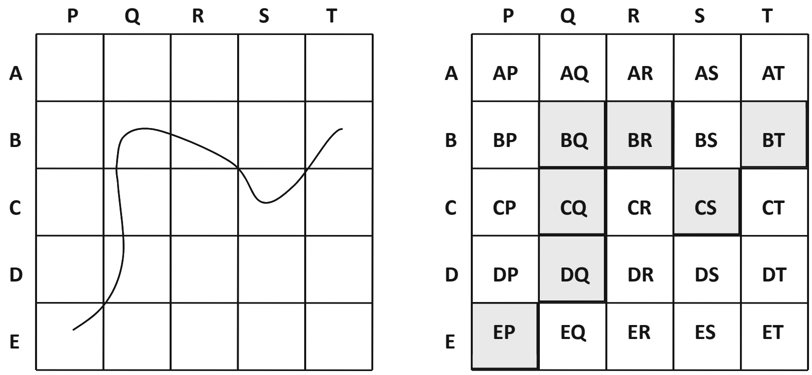
each of these behavioral attributes is extracted independently. In other words, the time series on the X-coordinate is converted into a wavelet representation, and so is the time series on the Y-coordinate. This yields two multidimensional representations, one of which is for the X-coordinate, and the other is for the Y-coordinate. The dimensions in these two representations are combined to create a single higher-dimensional representation for the trajectory. If desired, only the larger wavelet coefficients may be retained to reduce the dimensionality. The conversion of trajectory data to multidimensional data is an effective way to use the vast array of multidimensional methods for trajectory analysis.

16.3.3 Trajectory Pattern Mining

There are many different ways in which the problem of trajectory pattern mining may be formulated. This is because of the natural complexity of trajectory data that allows for multiple ways of defining patterns. In the following sections, some of the common definitions of trajectory pattern mining will be explored. These definitions are by no means exhaustive, although they do illustrate some of the most important scenarios in trajectory analysis.

16.3.3.1 Frequent Trajectory Paths

A key problem is that of determining frequent sequential paths in trajectory data. To determine the frequent sequential paths from a set of trajectories, the first step is to convert the multidimensional trajectory (with numeric coordinates) to a 1-dimensional discrete



(a) Trajectory (b) Relevant grid regions

Figure 16.8: Grid-based discretization of trajectories

sequence. Once this conversion has been performed, any sequential pattern mining algorithm can be applied to the transformed data.

The most effective way to convert a multidimensional trajectory to a discrete sequence is to use grid-based discretization. In Fig. 16.8a, a trajectory has been illustrated, together with a 4×4 grid representation of the underlying data space. The grid ranges along one of the dimensions are denoted by A, B, C, D, and E. The grid ranges along the other dimension are denoted by P, Q, R, S, and T. The 2-dimensional tiles are denoted by a combination of the ranges along each of the dimensions. For example, the tile AP represents the intersection of the grid range A with the grid range P. Thus, each tile has a distinct (discrete) identifier, and a trajectory can be expressed in terms of the sequence of discrete identifiers through which it passes. The shaded tiles for the trajectory in Fig. 16.8a are illustrated in Fig. 16.8b. The corresponding 1-dimensional sequential pattern is as follows:

$EP, DQ, CQ, BQ, BR, CS, BT$

This transformation is referred to as the *spatial tile transformation*. In principle, it is possible to enhance the discretization further, by imposing a minimum time spent in each grid square, though this is not considered here. Consider a database containing N different trajectories. The frequent sequential paths can be determined from these trajectories by using a two-step approach:

1. Convert each of the N trajectories into a discrete sequence, using grid-based discretization.

2. Apply the sequential pattern mining algorithms discussed in Sect. 15.2 of Chap. 15 to discover frequent sequential patterns from the resulting data set.

By incorporating different types of constraints on the sequential pattern mining process, such as time-gap constraints, it is also possible to apply these constraints on the trajectories. One advantage of this transformation-based approach is that it can take advantage of the power of all the different variations of sequential pattern mining. Sequential pattern mining has a rich body of literature on constraint-based methods.

Another interesting aspect is that this formulation can be modified to address situations in which the patterns of movements occur at the *same period in time*. The time period over which the movement occurs is discretized into m periods denoted by $1 \ldots m$. For example, the intervals could be $[8AM, 9AM]$, $[9AM, 10AM]$, $[10AM, 11AM]$, and so on. Thus, for each time-interval, the grid identifier is tagged with the relevant time-period identifier. A time period identifier is tagged to a grid region if a minimum amount of time from that time period range was spent in that region by the trajectory. This results in a set of patterns defined on a new set of discrete symbols of the form $< GridId >:< TimeId >$. In the trajectory of Fig. 16.8a, a possible sequence that appends the time period identifier is as follows:

$$EP:1, EP:2, DQ:2, DQ:3, DQ:4, CQ:5, BQ:5, BR:5, CS:6, CS:7, BT:7$$

This transformation is referred to as the *spatiotemporal tile transformation*. Note that this sequence is longer than that in Fig. 16.8b because the trajectory may spend more than one interval in the same grid region. A set of N different sequences are extracted, corresponding to the N different trajectories. The sequential pattern mining can be performed on this new representation. Because of the addition of the time identifiers, the resulting patterns will correspond to *simultaneous* movements in time. Thus, the sequential pattern mining approach has significant flexibility in terms of either detecting patterns of similar shapes, or patterns of simultaneous movements. Furthermore, because the sequential pattern mining formulation does not require successive symbols in a frequent sequential pattern to be contiguous in the original sequence, it can ignore noisy gaps in the underlying trajectories, while mining patterns. Furthermore, by using different constrained sequential pattern mining formulations, different kinds of constrained trajectory patterns can be discovered.

One drawback of the approach is that the granularity level of the discretization may affect the quality of the results. It is possible to address this issue by using a multigranularity discretization of the spatial regions. A different approach for conversion to symbolic representation is the use of spatial clustering on the different temporal snapshots of the object positions. The cluster identifiers of each object over different snapshots may be used to construct its sequence representation. The bibliographic notes contain pointers to several algorithms for transformation and pattern discovery from trajectories. The broader idea of many of these methods is to convert to a symbolic sequence representation for more effective pattern mining.

16.3.3.2 Colocation Patterns

Colocation patterns are designed to discover social connections between the trajectories of different individuals. The basic idea of colocation patterns is that individuals who frequently appear at the same point at the same time are likely to be related to one another. Colocation pattern mining attempts to discover patterns of individuals, rather than patterns of spatial trajectory paths. Because of the complementary nature of this analysis, a *vertical* representation of the sequence database is particularly convenient.

A similar grid discretization (as designed for the case of frequent trajectory patterns) can be used for preprocessing. However, in this case, a somewhat different (vertical) representation is used for the locations of the different individuals in the grid regions at different times. For each grid region and time-interval pair, a list of person identifiers (or trajectory identifiers) is determined. Thus, for the grid region EP and time interval 5, if the persons 3, 9, and 11 are present, then the corresponding set is constructed:

$EP : 5 \Rightarrow \{3, 9, 11\}$

Note that this is an *unordered* set, since it represents the individuals present in a particular $(space, time)$ pair. A similar set can be constructed over all the $(space, time)$ pairs that are populated with at least two individuals. This can be viewed as a *vertical representation* of the sequence database.

Any frequent pattern mining algorithm, discussed in Chap. 4, can be applied to the resulting database of sets. The frequent patterns correspond to the frequent sets of *colocated* individuals. These individuals are often likely to be socially related individuals.

16.3.4 Trajectory Clustering

In the following, a detailed discussion of the different kinds of trajectory clustering algorithms will be provided. Trajectory clustering algorithms are naturally related to trajectory pattern mining because of the close relationship between the two problems. Trajectory clustering methods are of two types.

1. The first type of methods use conventional clustering algorithms, with the use of a distance function between trajectories. Once a distance function has been designed, many different kinds of algorithms, such as k-medoids or graph-based methods, may be used.

2. The second type of methods use data transformation and discretization to convert trajectories into sequences of discrete symbols. Different types of transformations, such as segment extraction or grid-based discretization, may be applied to the trajectories. After the transformation, pattern mining algorithms are applied to the extracted sequence of symbols.

In addition, a number of other *ad hoc* methods have also been designed for trajectory clustering. This section will focus only on the systematic techniques. The bibliographic notes contain pointers to the *ad hoc* methods.

16.3.4.1 Computing Similarity Between Trajectories

A key aspect of trajectory clustering is the ability to compute similarity between different trajectories. At first sight, similarity function computation seems to be a daunting task because of the spatial and temporal aspects of trajectory analysis. However, in practice, similarity computation between trajectories is not very different from that of time series data. As discussed at the beginning of Sect. 16.3, trajectory data is equivalent to multivariate time series data. Several dynamic programming algorithms are discussed in Chap. 3 for similarity computation in univariate time series data. These algorithms can be generalized to the multivariate case. In the following, the extension of the *dynamic time warping* algorithm to the multivariate case will be discussed. A similar approach can be used for other dynamic programming methods. The reader is advised to revisit Sect. 3.4.1.3 of Chap. 3 on dynamic time warping before reading further.

First, the discussion on univariate time series distances will be revisited briefly. Let $DTW(i, j)$ be the optimal distance between the first i and first j elements of two *univariate* time series $\overline{X} = (x_1 \dots x_m)$ and $\overline{Y} = (y_1 \dots y_n)$ respectively. Then, the value of $DTW(i, j)$

is defined recursively as follows:

$$DTW(i,j) = distance(x_i, y_j) + \min \begin{cases} DTW(i, j-1) & \text{repeat } x_i \\ DTW(i-1, j) & \text{repeat } y_j \\ DTW(i-1, j-1) & \text{otherwise} \end{cases} \quad (16.2)$$

In the case of a 2-dimensional trajectory, we have a multivariate time series for each trajectory, corresponding to the two coordinates of each trajectory. Thus, the first trajectory has two coordinates corresponding to $\overline{X1} = (x1_1 \ldots x1_m)$ and $\overline{X2} = (x2_1 \ldots x2_m)$. The second trajectory has two coordinates, corresponding to $\overline{Y1} = (y1_1 \ldots y1_n)$ and $\overline{Y2} = (y2_1 \ldots y2_n)$. Let $\overline{X_i} = (x1_i, x2_i)$ represent the 2-dimensional position of the first trajectory at the ith timestamp, and let $\overline{Y_j} = (y1_j, y2_j)$ represent the 2-dimensional position of the second trajectory at the jth timestamp. *Then, the only difference from the case of univariate time series data is the substitution of the 1-dimensional distances in the recursion with 2-dimensional distances.* Therefore, the modified multidimensional DTW recursion $MDTW(i,j)$ is as follows:

$$MDTW(i,j) = distance(\overline{X_i}, \overline{Y_j}) + \min \begin{cases} MDTW(i, j-1) & \text{repeat } \overline{X_i} \\ MDTW(i-1, j) & \text{repeat } \overline{Y_j} \\ MDTW(i-1, j-1) & \text{otherwise.} \end{cases} \quad (16.3)$$

Note that the multidimensional DTW recursion is virtually identical to the univariate case, except for the term $distance(\overline{X_i}, \overline{Y_j})$ that is now a multidimensional distance between spatial coordinates. For example, one might use the Euclidean distances. The simplicity of the generalization is a result of the fact that time warping has little to do with the dimensionality of the time series. All the dimensions in the time series are warped in exactly the same way. Therefore, the 1-dimensional distance in the recursion can be substituted with multidimensional distances. It should also be pointed out that this general principle applies to most of the dynamic programming methods for computing distances between temporal series and sequences.

16.3.4.2 Similarity-Based Clustering Methods

Many conventional clustering methods, such as k-medoids and graph-based methods, are based on the similarity between data objects. Therefore, once a similarity function has been defined, these methods can be used directly for virtually any data type. It should be pointed out that these methods are very popular in different data domains, such as time series data or discrete sequence data. It was shown in Chaps. 14 and 15, how these methods may be used for time series and discrete sequence data, respectively. The approach used here is exactly analogos to the description in these chapters, except that multivariate time series similarity measures are used for computation in this case. The reader is referred to Chap. 6 for the basic description of the k-medoids and graph-based methods, as applied to multidimensional data. The main problem with similarity-based methods, is that they work best only when the trajectory segments are relatively short. For longer trajectories, it becomes more difficult to compute the similarity between pairs of objects because many portions of the trajectories may be noisy. Therefore, the choice of similarity function becomes more important. Some of the similarity functions discussed in Sect. 3.4.1 of Chap. 3, allow for gaps in the similarity computation. However, the effectiveness of these methods for multivariate time series and trajectories is highly data-specific. In general, these similarity functions are best used for trajectories of shorter lengths.

16.3.4.3 Trajectory Clustering as a Sequence Clustering Problem

Trajectory clustering methods can be performed with the same grid-based discretization methods that are used for frequent pattern mining in trajectories. A two-step approach is used. The first step is to use grid-based discretization to convert the trajectory into a 1-dimensional discrete sequence. Once this transformation has been performed, any of the sequence clustering methods discussed in Chap. 15 may be used. The overall clustering approach may be described as follows:

1. Use grid-based discretization, as discussed in Sect. 16.3.3.1, to convert the N trajectories to N discrete sequences.

2. Apply any of the sequence clustering methods of Sect. 15.3 in Chap. 15 to create clusters from the sequences.

3. Map the sequence clusters back to trajectory clusters.

As discussed in Sect. 16.3.3.1, the grid-based sequences constructed in the first step can be based on either the grid identifiers (spatial tile transformation) only, or on a combination of grid-identifiers and time-interval identifiers (spatiotemporal tile transformation). In the first case, the resulting clusters correspond to trajectories that are close together in space, but not necessarily in time. In the second case, the trajectories in a cluster will to be close together in space *and* occur at the same time. In other words, such clusters represent objects that are moving together in time.

One advantage of the sequence clustering approach, over similarity-based methods, is that many of the sequence clustering methods can ignore the irrelevant parts of the sequences in the clustering process. This is because many sequence clustering methods, such as subsequence-based clustering (Sect. 15.3.3 of Chap. 15), naturally allow for noisy gaps in the trajectories during the clustering process. This is important because longer trajectories often share significant segments in common, but may have gaps or regions where they are not similar. The ability to account for such nonmatching regions is not quite as effective with similarity function methods that compute distances between trajectories as a whole.

16.3.5 Trajectory Outlier Detection

In this problem, it is assumed that N different trajectories are available, and it is desirable to determine outlier trajectories, as those that differ significantly from the trends in the underlying data. As with all data types, the problem of trajectory outlier detection is closely related to that of trajectory clustering. In particular, both problems utilize the notion of similarity between data objects. As in the case of data clustering, one can use either a similarity-based approach, or a transformational approach to outlier detection.

16.3.5.1 Distance-Based Methods

The ability to design a distance function between trajectories provides a way to define outliers with the use of distance-based methods. In particular, the k-nearest neighbor method, or any distance-based method can easily be generalized to trajectories, once the distance function has been defined. For example, one may use the multidimensional time warping distance function to compute the distance of a trajectory to the $N-1$ other trajectories. The kth nearest neighbor distance is reported as the outlier score. Other distance-based methods such as *LOF* can also be extended to trajectory data because these methods are based only on distance values, and are agnostic to the underlying data type. As in the case of clustering, the major drawback of these methods is that it can be used effectively for shorter trajectories, but not quite as effectively in the case of longer trajectories. This is because longer trajectories will often have many noisy segments that are not truly indicative of anomalous behavior, but are disruptive to the underlying distance function.

16.3.5.2 Sequence-Based Methods

The spatial and spatiotemporal tile transformation discussed at the beginning of Sect. 16.3.3.1 can be used to transform trajectory outlier detection into sequence outlier detection. The advantage of this approach is that many methods are available for sequence outlier detection. As in the case of the other problems such as trajectory pattern mining and clustering, the approach consists of two steps:

1. Convert each of the N trajectories to sequences using either spatial tile transformation or spatiotemporal tile transformation, discussed at the beginning of Sect. 16.3.3.1.

2. Use any of the sequence outlier detection methods discussed in Sect. 15.4 of Chap. 15, to determine the outlier sequences.

3. Map the sequence outliers onto trajectory outliers.

This approach is particularly rich in terms of the types of the outliers it can find, by varying on the specific subroutines used in each of the aforementioned steps. Some examples of such variations are as follows:

- In the first step of sequence transformation, either spatial or spatiotemporal tiles may be used. When spatial tiles are used, the discovered outliers are based only on the shape of the trajectory, and they are not based on the objects moving together. From an application-centric perspective, consider the case where trajectories of taxis are tracked by GPS, and it is desirable to determine taxis that take anomalous routes relative to other taxis at any period of time. Such an application can be addressed well with spatial tiles. Spatiotemporal tiles track *online* trends. For example, for a flock of GPS-tagged animals, if a particular animal deviates from its flock, it is reported as an outlier.

- The formulations for sequence outlier detection are particularly rich. For example, sequence outlier detection allows the reporting of either *position* outliers or *combination* outliers. This is discussed in detail in Sect. 15.4 of Chap. 15. Position outliers in the transformed sequences, map onto anomalous positions in the trajectories. For example, a taxi cab making an unusual turn at a junction will be detected. On the other hand, combination outliers will map onto unusual segments of trajectories.

Thus, the sequence-based transformation is particularly useful in being able to detect a rich diversity of different types of outliers. It can determine outliers based on patterns of movements *either* over all periods, or at a particular period. It can also discover small segments of outliers in any portion of the trajectory.

16.3.6 Trajectory Classification

In this problem, it is assumed that a training data set of N labeled trajectories is provided. These are then used to construct a training model for the trajectories. The unknown class label of a test trajectory is determined with the use of this training model. Since classification is a supervised version of the clustering problem, methods for trajectory classification use similar methods to trajectory clustering. As in the case of clustering methods, either distance-based methods, or sequence-based methods may be used.

16.3.6.1 Distance-Based Methods

Several classification methods, such as nearest neighbor methods and graph-based collective classification methods, are dependent only on the notion of distances between data objects. After the distances between data objects have been defined, these classification methods are agnostic to the underlying data type.

The k-nearest neighbor method works as follows. The top-k nearest neighbors to a given test instance are determined. The dominant class label is reported as the relevant one for the test instance. Any of the multivariate extensions of time series distance functions, such as multidimensional DTW, may used for the computation process.

In graph-based methods, a k-nearest neighbor graph is constructed on the data objects. This is a semi-supervised method because the graph is constructed on a mixture of labeled and unlabeled objects. The basic discussion of graph-based methods may be found in Sect. 11.6.3 of Chap. 11. Each node corresponds to a trajectory. An undirected edge is added from node i to node j if either j is among the k nearest neighbors of i or vice versa. This results in a graph in which only a subset of the objects is labeled. The goal is to use the labeled nodes to infer the labels of the unlabeled nodes in the network. This is the collective classification problem that is discussed in detail in Sect. 19.4 of Chap. 19. When the labels on the unlabeled nodes have been determined using collective classification methods, they are mapped back to the original data objects. This approach is most effective when many test instances are simultaneously available with the training instances.

16.3.6.2 Sequence-Based Methods

In sequence-based methods, the first step is to transform the trajectories into sequences with the use of spatial or spatiotemporal tile-based methods. Once this transformation has been performed, any of the sequence classification methods discussed in Chap. 15 may be used. Therefore, the overall approach may be described as follows:

1. Convert each of the N trajectories to sequences using either the spatial tile transformation, or spatiotemporal tile transformation, discussed at the beginning of Sect. 16.3.3.1.

2. Use any of the sequence classification methods discussed in Sect. 15.6 of Chap. 15 to determine the class labels of sequences.

3. Map the sequence class labels to trajectory class labels.

The spatial tile transformation and spatiotemporal tile transformation methods provide different abilities in terms of incorporating different spatial and temporal features into the classification process. When spatial tile transformations are used, the resulting classification is not time sensitive, and trajectories from different periods can be modeled together on the basis of their *shape*. On the other hand, when the spatiotemporal tile transformation is used, the classification can only be performed on trajectories from the same approximate time period. In other words, the training and test trajectories must be drawn from the same period of time. In this case, the classification model is sensitive not only to the shape of the trajectory but also to the precise times in which their motion may have occurred. In this case, even if all the trajectories have exactly the same shape, the labels may be different because of temporal differences in speed at various times. The precise choice of the model depends on application-specific criteria.

16.4 Summary

Spatial data is common in a wide variety of applications, such as meteorological data, trajectory analysis, and disease outbreak data. This data is almost always a contextual data type, in which the data attributes are partitioned into behavioral attributes and contextual attributes. The spatial attributes may either be contextual or behavioral. These different types of data require different types of processing methods.

Contextual spatial attributes arise in the case of meteorological data where different types of spatial attributes such as temperature or pressure are measured at different spatial locations. Another example is the case of image data where the pixel values at different spatial locations are used to infer the properties of an image. An important transformation for shape-based spatial data is the centroid-sweep method that can transform a shape into time series. Another important transformation is the spatial wavelet approach that can transform spatial data into a multidimensional representation. These transformations are useful for virtually all data mining problems, such as clustering, outlier detection, or classification.

In trajectory data, the spatial attributes are behavioral, and the only contextual attribute is time. Trajectory data can be viewed as multivariate time series data. Therefore, time series distance functions can be generalized to trajectory data. This is useful in the development of a variety of data mining methods that are dependent only on the design of the distance function. Trajectory data can be transformed into sequence data with the use of tile-based transformations. Tile-based transformations are very useful because they allow the use of a wide variety of sequence mining methods for applications such as pattern mining, clustering, outlier detection, and classification.

16.5 Bibliographic Notes

The problem of spatial data mining has been studied extensively in the context of geographic data mining and knowledge discovery [388]. A detailed discussion of spatial databases may be found in [461]. The problem of search and indexing, was one of the earliest applications in the context of spatial data [443]. The centroid-sweep method for data mining of shapes is discussed in [547]. A discussion of spatial colocation pattern discovery with nonspatial behavioral attributes is found in [463]. This method has been used successfully for many data mining problems, such as clustering, classification, and outlier detection.

The problem of outlier detection from spatial data is discussed in detail in [5]. This book contains a dedicated chapter on outlier detection from spatial data. Numerous methods have been designed in the literature for spatial and spatiotemporal outlier detection [145, 146, 147, 254, 287, 326, 369, 459, 460, 462]. The algorithm for unusual shape detection was proposed in [510].

The tile-based simplification for pattern mining from trajectories was proposed in [375]. Pattern mining in trajectory data is closely related to clustering. The problem of mining periodic behaviors from trajectories is addressed in [352]. Moving object clusters have been studied as *Swarms* [351], *Flocks* [86] and *Convoys* [290]. Among these, *Swarms* provide the most relaxed definition, in which noisy gaps are allowed. In these noisy gap periods, objects from the same cluster may not move together. An algorithm for maintaining real-time communities from trajectories in social sensing applications was proposed in [429]. A method for partitioning longer trajectories into smaller segments for shape-based clustering was proposed in [338]. Anomaly monitoring from trajectories of moving object streams was studied in [117]. The *Top-Eye* method, an algorithm for monitoring the top-k anomalies in moving object trajectories, was proposed in [226]. The *TRAOD* algorithm, which discovers shape-based trajectory outliers, was proposed in [337]. A method that uses region-based and trajectory-based clustering for classification was proposed in [339].

16.6 Exercises

1. Discuss how to generalize the spatial wavelets to the case where there are n contextual attributes.

2. Implement the algorithm to construct a multidimensional representation from spatial data, with the use of wavelets.

3. Describe a method for converting shapes to a multidimensional representation.

4. Implement the algorithm for converting shapes to time series data.

5. Suppose that you had N different snapshots of sea surface temperature over successive instants in time over a spatial grid. You want to identify contiguous regions over which significant change has occurred between successive time instants. Describe an approach to identify such regions and time instants with the use of spatial wavelets.

6. Suppose the snapshots of Exercise 5 were not from successive instants in time. How would you identify spatial snapshots that were very different from the other snapshots with the use of spatial wavelets? How would you identify specific regions that are very different from the remaining data?

7. Suppose that you used the tile-based approach for finding frequent trajectory patterns. Discuss how the different constraint-based variants of sequential pattern mining map onto different constraint-based variants of sequential trajectory patterns.

8. Propose a snapshot-based clustering approach for converting trajectories to symbolic sequences. Discuss the advantages and disadvantages with respect to the tile-based approach.

9. Implement the different variations for converting trajectories to symbolic sequences with the use of the tile-based technique for frequent trajectory pattern mining.

10. Discuss how to use wavelets to perform different data mining tasks on trajectories.